

A low dimensional dynamical system for the wall layer

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Low dimensional dynamical systems which model a fully developed turbulent wall layer, for $y^+ \leq 40$ have been derived (Aubry *et al.* 1986, Aubry, 1987). The model is based on the optimally fast convergent proper orthogonal decomposition, or Karhunen–Loeve expansion, proposed by Lumley, 1967. This decomposition provides a set of eigenfunctions which are derived from the autocorrelation tensor at zero time lag. Those used in the previous studies were experimentally determined in a pipe flow at a Reynolds number 8750 based on the mean centerline velocity and the diameter of the pipe. (Herzog, 1986).

Via Galerkin projection, low dimensional sets of ordinary differential equations in time, for the coefficients of the expansion, were derived from the Navier Stokes equations. The energy loss to the unresolved modes was modeled by an eddy viscosity representation, analogous to Heisenberg's spectral model. In the previous work the equations of a ten dimensional system, consisting of one eigenfunction per wave number for the zero streamwise wave number, and six spanwise wave numbers corresponding to a periodic length of 333 wall units, were examined. The solution, which consisted of longitudinal rolls, exhibited an intermittent behavior (the zero mode decays to zero). The rolls are initially steady, but then oscillate with slowly growing amplitude until they "burst" into much more complicated features before recovering their initial state. The whole sequence then repeats. This is suggestive of the bursting event observed in visualization experiments (Kline *et al.*, 1967).

This approach may shed light on the basic dynamical mechanism of the fundamental bursting event. However, until recently, it was limited to the specific experimental flow (Herzog, 1986).

Another set of eigenfunctions and eigenvalues have been obtained from direct numerical simulation of a plane channel at a Reynolds number of 6600, based on the mean centerline velocity and the channel width (Moin & Moser, 1987). This new set of eigenfunctions is compared to those of Herzog (1986). The expansion still converges very quickly, since 75% of the kinetic energy is contained in the first eigenmode. Thus it still seems quite reasonable to truncate the expansion at the first mode. The energy content at the first eigenmode still drops faster with streamwise wave numbers than with spanwise wavenumbers, justifying, in a first approximation, no variations in the streamwise direction. However, the ratio between the streamwise and cross-stream length scales is not as large as was observed in the decomposition of Herzog, suggesting that few elongated patterns

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are present. The energy in cross-stream wavenumbers is slightly larger than in the experimental case and the peak in the spectrum is shifted to a higher wavenumber. Also, the peak magnitude of the numerically generated spectrum for $k_x = 0$ seems anomalously large. The contribution of the first eigenmode to the variance of the velocity fluctuations in the three directions is very similar in both cases. However, while the contribution of Herzog's first mode to the Reynolds shear stress is 50% near the wall, 95% at $y^+ = 20$, 78% at the upper edge of the layer, that of Moin and Moser exceeds 100% in the region $13 \leq y^+ \leq 25$ (120% at $y^+ = 25$). This apparent paradox occurs because the contribution of higher order modes to the Reynolds shear stress is negative in that region. The eigenfunctions themselves are quite similar in both cases, at least amongst those selected for inclusion in the dynamical system.

Using the new eigenvalues and eigenfunctions, a new ten dimensional set of ordinary differential equations has been derived using five non-zero cross-stream Fourier modes with a periodic length of 377 wall units. The coefficients in the equations are similar to those of the previous study. As in the previous work, the evolution equations are globally stable, since all the coefficients of the cubic terms are negative. This is due to the positive contribution of the first eigenmode to the Reynolds shear stress. The coefficients of the cubic terms are larger with the present data than in the previous studies. This appears to be caused by the higher proportion of Reynolds shear stress carried by the first mode.

The new dynamical system has been integrated for a range of the eddy viscosity parameter α . For large values, the solution goes to a stable fixed point, involving only the second and fourth Fourier mode. When α decreases, this fixed point undergoes a bifurcation to a limit cycle. As α decreases more, the solution becomes much more complicated and intermittent. In contrast, the results of the previous work showed a transition directly from a fixed point to an intermittent solution exhibiting the bursts discussed above. The intermittent solution in the present work exhibits the same basic features previously observed. In the previous work the solution cyclically visited the neighborhood of two different fixed points, being attracted to a double homoclinic orbit which connected them. In the new system the solution switches back and forth in a similar way between different orbits and limit cycles. When α decreases more, we observe much more disorganized motion.

This work is encouraging. Although we could not analyze in detail the bifurcation diagram during the short period of the program, we observed intermittency of the solution for some values of the parameter α . The appearance of limit cycles introduces periodic motions superposed to the intermittency. We plan more investigations in future work.

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