Overview of the K-space Group Projects

The projects of the K-space group consisted of six independent lines of inquiry with a common theme of the interaction among scales of motion in turbulence. The studies were almost entirely limited to homogeneous turbulence. In the following paragraphs I will give a brief overview of the six projects.

The invited participants were:
Jean-Pierre Bertoglio (Ecole Centrale de Lyon)
Julian Hunt (University of Cambridge)
Paolo Orlandi (University of Rome)
Roland Schiestel (Institut de Mecanique Statistique de la Turbulence)
Akira Yoshizawa (University of Tokyo)
The local participants were:
Jeffrey Buell (NASA Ames)
Gary Coleman (Stanford University)
Joel Ferziger (Stanford University)
Robert Rogallo (NASA Ames)
Alan Wray (NASA Ames)

Bertoglio wishes to test the accuracy of the assumptions within the EDQNM closure at a deeper level than has been previously done. An important step in the closure is the estimation of a Lagrangian time scale which is assumed to be a functional of E(k). Bertoglio judges several candidate functionals by the degree to which they collapse the two-time velocity auto-correlations in direct simulations of isotropic turbulence. This approach appears very promising, and good collapse has been achieved at the higher wave numbers. This work is the continuation of a joint effort by Bertoglio, co-workers in France, and Squires and Ferziger at Stanford.

The EDQNM theory is purely statistical in nature, whereas many important turbulence problems are dominated by the presence of persistent coherent structures. Bertoglio's second goal is to determine how well EDQNM copes with such structures. An experimental program has been started in France using propellers to inject known coherent structures into a turbulent flow, and to conduct a parallel numerical simulation. Comparisons between experiment, simulation, and theory for this flow should illuminate any difficulties that the statistical theory has with imbedded coherency. During the workshop Bertoglio attempted to define initial conditions for such a simulation. The energy in the simulation peaked at the blade-passing frequency of the propellers while that in the experiment peaked at their rotation frequency. It should be a simple matter to solve that mystery, but there was simply not enough time during the workshop to do so.

Orlandi's project concerns the use of EDQNM as a subgrid (or supergrid) model for a large-eddy simulation. At a minimum, such models must accurately account for the energy transfer between the computationaly resolved and unresolved scales.

In EDQNM, the net transfer into wave number k is calculated as the integral over interacting triads (k,p,q) of a functional of E(k), E(p), and E(q). Orlandi has demonstrated that EDQNM accurately produces the transfer measured in a direct simulation of isotropic turbulence both when the full set of interactions is considered, and also when a subset (truncated spectrum) of interactions is considered. Then, if the spectrum of the unresolved scales can be estimated, EDQNM can account for their contribution to the energy transfer. In the simulation however, the effect of the unresolved scales must appear as additional terms in the *momentum* equations. The subgrid term is usually modeled by a gradient diffusion form with an eddy viscosity determined from the subgrid transfer (see the works of Kraichnan, Chollet, and Lesieur) . The supergrid term is presumably some sort of forcing but a gradient diffusion form, with negative eddy viscosity, does not seem physically correct. Another possibility is the application of a mean strain that is uniform in space but random in time. The art of driving simulations at the large scales, as was done by Hunt et al above, is currently not well understood (see the works of Siggia, Kerr, Pope, etc.). Orlandi also attempted to extend the EDQNM closure to homogeneous shear, but encountered numerical accuracy problems in the calculation of the five-dimensional integrals required.

While the EDQNM model appears to be reasonably accurate and tractable in isotropic turbulence, in anisotropic flows it is far less tractable, requires additional assumptions, and is much more expensive to compute. Because of this, the theory has not received much attention for anisotropic flows. These are however very important in the context of subgrid models for LES calculations because as the grid resolution increases, the subgrid contribution approaches homogeneity much more rapidly than it approaches isotropy. Coleman and Ferziger consider the possibility of a Galerkin approach to simplify the EDQNM calculation. The angular distribution of velocity correlations over spherical shells (which are, in the proper variables, uniform in isotropic flow) would be represented by the weighted sum of a small number of smooth basis functions. Coleman and Ferziger attempted to estimate the number of basis functions required by inspecting the angular distribution of the Reynolds stress spectrum tensor in a direct simulation of homogeneous sheared turbulence. The distributions were quite smooth and the authors speculated that they could be represented by a sum of two or three functions. There appear to be several important issues that were not covered in a general way: the choice of the coordinate system and the choice of the dependent variables. In any expansion technique the choice of variables, both dependent and independent, is crucial. In this case some clues might be extracted from rapid-distortion theory, from consideration of the principal axes of the stress and mean strain rate tensors, and from the manner in which the mean flow gradient enters the EDQNM equations.

The project of Hunt, Buell, and Wray concerns the relation between space and time spectra (or correlations), and their dependence on the reference frame (Eulerian or Lagrangian). In particular, they wanted to determine how the advection of small-weak scales by large-strong ones influences the Eulerian time spectra at high frequency and wave number. The results support the assertion of Tennekes that such advection dominates the frequency spectrum at high frequency, but indicates that Hunt's earlier proposal relating the time-space spectrum to the space spectrum was over simplified. During the project Hunt reworked the analysis using a more realistic p.d.f. for the advecting velocity and derived a simple relation which agrees well with the simulation. Some anomalies were observed in the computed data; these appear to be a consequence of the small statistical sample of forced modes, the small range of spatial scales that could be retained, and the low Reynolds number that was required for adequate numerical resolution.

In Schiestel's model, the equation for the Reynolds stress spectrum tensor is integrated, in wave number space, over spherical shells rather than over the entire space as done in classical one-point Reynolds stress formulations. As a result, some scale information is retained. Within each shell, one must model the same quantities as in classical one-point closures (pressure-strain, dissipation, etc.) and in addition, model transfers between shells. These transfers are globally conservative and do not appear in the one-point approaches. Schiestel's goal during the summer program was to compare the models for these terms with data from direct simulations. He hoped to determine the accuracy of the models he is currently using, and to get some clues to aid in their improvement. As one would expect, some of the terms were modeled rather well while others were not, and, unfortunately, there was not enough time to consider improvements. The statistics taken from the simulations were rather noisy at the larger scales because of the small sample and sometimes biased at the small scales due to mesh anisotropy. But they are simply not available elsewhere, and appear to be precisely what Schiestel needs.

Over the past several years, Yoshizawa has worked out a formal two-scale expansion of the Navier-Stokes equations in which the interaction between the scales is weak. The scales are disparate in both space and time, and are separated by formally averaging over an intermediate scale at which Taylor series expansion of the large scales is assumed to be valid and averages of the small scales are assumed to be statistically converged. The interaction terms in general depend upon deterministic features of the large-scale field (its derivatives) and statistical features of the small-scale field (local correlations). The disparity of the spatial scales leads , at small scale, to homogeneous turbulence at lowest order, and the time scale disparity leads to its isotropy. The required statistics of the small scales are in turn modeled by the DIA formalism. At higher order the small scales become anisotropic. Within this framework (TSDIA) it is possible to find the form (formally, the asymptotic expansion) of terms that must be modeled in one-point closures, for example the diffusion of kinetic energy. A model is then postulated by replacing the gauge functions in the expansion with "model constants". It was Yoshizawa's hope to be able to test several of these models and to estimate the contribution of the higher-order terms, using simulation data to determine the constants. Unfortunately some model terms could not be computed because the required statistics had not been extracted from the database prior to the summer program and during the program no one was available to do so. In addition, the data that was available was not really adequate for the simultaneous determination of several constants. The sample was too small, as was the Reynolds number. Yoshizawa was then forced to omit most of the new terms suggested by TSDIA. When this was done the models fit the data well (with one notable exception) with constants close to the values previously predicted. For example, the fact that rotation reduces the dissipation rate was correctly predicted. The exception was the case of turbulent diffusion in homogeneous shear of a passive scalar having a mean gradient in the stream direction. In this case the mean gradient itself changes with time but was in fact held fixed in the simulation used by Yoshizawa. A later simulation treated the case of changing mean gradient but it is not clear which case is appropriate for testing Yoshizawa's model.

Summary

All of the projects are, in my opinion, worthwhile and feasible, and will hopefully be pursued further. The progress made during the summer session was limited by the number of local participants available to interact directly with the computers. The group benefited greatly from discussions with Robert Kraichnan and Evgeny Novikov, and several projects were directly influenced by their suggestions. We at Ames benefited from each of these projects because in every case we were exposed to areas of research and ideas that we had not pursued in-house before.

Bob Rogallo