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A General Form for the Dissipation Length Scale in Turbulent Shear Flows

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It has been found that, for a wide range of turbulent wall- bounded shear flows with mean velocity profile $U(y)$, the length scale L_ϵ determining the dissipation is approximately described in terms of distance from the wall y, the mean shear dU/dy, and the variance of the normal component of turbulence $\overline{v^2}$, by the formula

$$L_\epsilon^{-1} \approx \frac{A_B}{y} + A_S \frac{dU/dy}{\sqrt{\overline{v^2}}}$$

where $L_\epsilon = \epsilon/(\overline{v^2})^{3/2}$. To match with shear-free boundary layers, $A_B \simeq 0.27$, and with the log layer, $A_S \simeq 0.46$. The shear flows tested here were: boundary layers over a flat plate, sink flow, oscillatory flow and channel flow. The use of $\sqrt{\overline{v^2}}$ as a velocity scale minimizes the effects of Reynolds number. However, the formula fails within a distance of order L_ϵ for the regions where $dU/dy = 0$.

1. Introduction

The estimation of the rate of dissipation of turbulent energy ϵ is a critical feature of many computations of turbulent shear flows. However, current methods based on a heuristic differential equation for ϵ are not always accurate and almost never understood in physical terms. In particular, the relative effects of the distance (y) from a boundary and the shear dU/dy on the eddies is not clear.

The essential point in thinking about the rate of dissipation ϵ is that it is controlled by the steepest gradients of the energy-containing eddies. Therefore we need to be able to define the smallest integral or macroscales.

The aim of the research described here is to specify the relevant velocity components and macroscale L_ϵ that enable ϵ to be estimated. (For previous discussion, see Hunt, Stretch & Britter, 1986.)

Recent theoretical and experimental research on shear-free turbulent boundary layers (Hunt, 1984) ("SFBL", where $dU/dy \simeq 0$), has demonstrated how the

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smallest "macro" scale is that of the normal velocity component $L_{11}^{(2)}$, and that $L_{11}^{(2)} \simeq 1.7y$. If we are to use this length to estimate ϵ, we recall that in a SFBL. ϵ is approximately invariant with distance from the wall $(\partial\epsilon/\partial y \simeq 0)$. It is also found by theoretical (or scaling) arguments that $\overline{v^2} = C_B \epsilon^{2/3} y^{2/3}$, where C_B is a constant. The linear analysis of Hunt (1984) gives a value for $C_B = 1.8$, while the measurements in the atmospheric convective boundary layer give $C_B = 2.5$.

Therefore in the SFBL it is natural to define the dissipation scale as

$$L_\epsilon \equiv \frac{(\overline{v^2})^{3/2}}{\epsilon} \simeq C_B^{3/2} y \tag{1.1}$$

It is convenient to express (1.1) as

$$L_\epsilon^{-1} = A_B y^{-1} \tag{1.2}$$

So we take A_B as ranging from $(1/2.5)^{3/2} = 0.25$ to $(1/2.0)^{3/2} = 0.35$. Note that the horizontal scale of the vertical fluctuations $L_{11}^{(2)}$ in the SFBL is about $1.7y$, so L_ϵ is about $L_{11}^{(2)}$.

By contrast in a uniform shear, the length scales, including L_ϵ, are largely determined by the shear dU/dy and the velocity fluctuations, so that

$$L_\epsilon = C_s \sqrt{\overline{v^2}} / (dU/dy)$$

where C_s is a dimensionless parameter of order unity. Recent numerical simulations on the time evolution of turbulence in a homogeneous shear flow by Rogers et al. (1986), Lee, Kim & Moin (1987), and Rogallo (1981) show that the parameter C_s depends on the initial conditions (in particular $L_{11}^{(2)}(dU/dy)/\sqrt{\overline{v^2}}$, and the total strain ($=t\ dU/dy$). However, for a wide class of actual shear flows the effective value of $t\ dU/dy$ only varies over a range of about 3 (Townsend, 1976). So we can expect that there is an approximately constant value for

$$C_s = \frac{L_\epsilon \mid dU/dy \mid}{(\overline{v^2})^{1/2}} = \frac{1}{A_S} \tag{1.3}$$

What happens in a shear flow near a boundary? Dissipation of turbulent energy is driven by the straining of small eddies by slightly larger eddies. So the dissipation length scale L_ϵ depends on the smaller of the two effects of the boundary and the shear. So we take the harmonic mean of (1.2) and (1.3)

$$L_\epsilon^{-1} = \frac{A_B}{y} + A_S \frac{\mid dU/dy \mid}{\sqrt{\overline{v^2}}} \tag{1.4}$$

Taking $A_B \simeq 0.27$, then A_S can be calculated from the log layer (assuming at high Reynolds number $\sqrt{\overline{v^2}} = 1.3u_*$), and a local equilibrium between production and dissipation of turbulence energy. We obtain

$$A_S \simeq 0.46 \tag{1.5}$$

In the research performed at the CTR, L_ϵ has been computed using the data from direct simulations of a number of wall-bounded flows. We make a comparison here with (1.4), using (1.1) to define L_ϵ

2. Preliminary results

In Fig. 1a L_ϵ/δ is plotted against y/δ for the flat plate boundary layer (Spalart, 1986b); in Fig. 1b, for the sink boundary layer (Spalart, 1986a); in Fig. 2 for the oscillating boundary layer (Spalart & Baldwin, 1987), where the flow reverses, and in Fig. 3 for the channel flow (Mansour, Kim & Moin, 1987).

Where the results for L_ϵ have been computed for different values of the Reynolds number (e.g., Fig. 1), the normalization (1.1) reduces the profiles of L_ϵ/δ to a form that is approximately independent of Re. If L_ϵ were defined on the basis of $\epsilon/(\overline{u_i^2})^{3/2}$, this would not be so. In Figs. 1 through 3, we have used the technique of extrapolating the values of $\overline{u^2}$ to their values as $Re \to \infty$, by suitable extrapolation of the high wave number spectrum (Spalart, 1986b; Perry et al., 1986). This method apparently works well even for transitional turbulence, such as occurs in the oscillating boundary layer near reversal.

All the results agree well in the log regions (where L_ϵ is proportional to y) for which the coefficient A_B was defined. But the results show that the formula (1.4) applies well beyond the log region. This implies that dU/dy is controlling the scale. The structure of turbulence must be rather similar if the constant is so good! But, note that, at the edge of the boundary layer or in a reversing boundary layer, where $dU/dy = 0$, the model is not satisfactory. The local turbulent scale is determined by advection of evolution from previous time. (Effects that are approximately incorporated in the ϵ equation!)

3. Further work

Apparently the proposed formula (1.4) has some generality. But we still do not know what aspect of the turbulent structure exactly corresponds to the length L_ϵ. The research of Lee and Hunt (in progress) on R.D.T. near a wall may help explain more about the relative role of blocking and shear, as will the related research on two-point correlations.

The use of R.D.T. to study the linear interactions will not really tell us how shear and blocking affect the nonlinear transfer. That may only come from more detailed computations and models of the spectra (e.g., by the studies of scale transfer by Schiestel, D.I.A. by Yoshizawa, or the large-scale/small-scale interactions using R.D.T. by Kida & Hunt).

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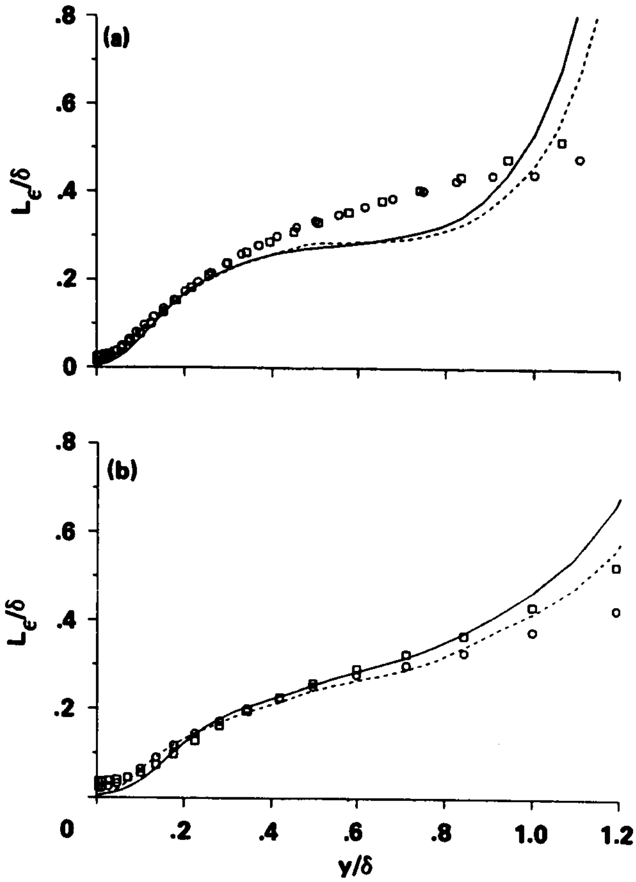

FIGURE 1. Distribution of the dissipation length scale as function of the distance to the wall.

a) Flat-plate boundary layer.

formula Eq. 1.4: —— $R_\theta = 670$, ---- $R_\theta = 1410$.

Simulations $(\overline{v^2}^{3/2}/\epsilon)$: □ $R_\theta = 670$, ○ $R_\theta = 1410$.

b) Sink flow boundary layer.

formula Eq. 1.4: —— $K = 2.5 \times 10^{-6}$, ---- $K = 1.5 \times 10^{-6}$.

Simulations $(\overline{v^2}^{3/2}/\epsilon)$: □ $K = 2.5 \times 10^{-6}$, ○ $K = 1.5 \times 10^{-6}$.

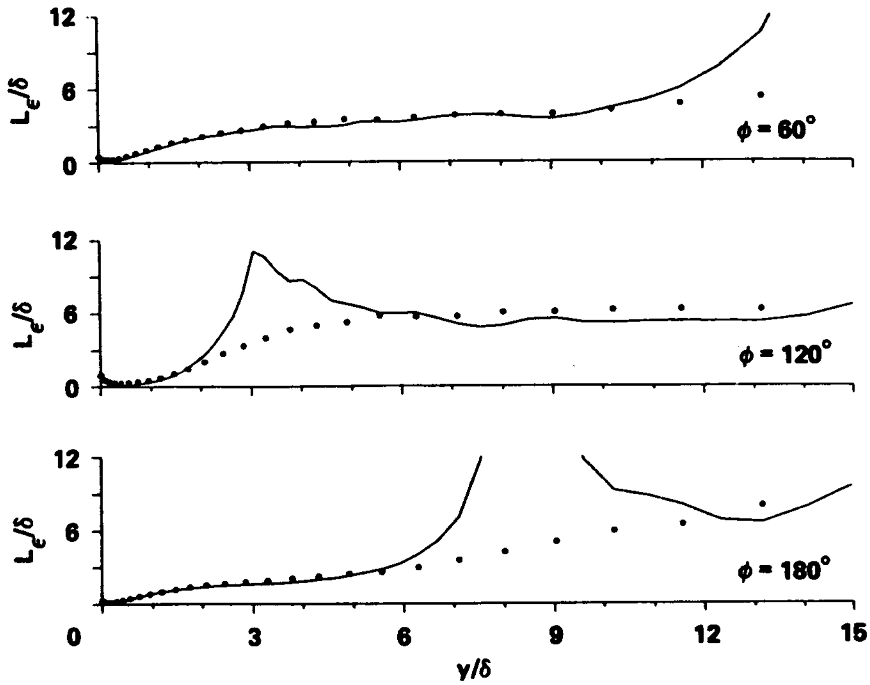

FIGURE 2. Oscillating boundary layer, Re = 1000. Formula Eq. 1.4: —— ; Simulations $(\overline{v^2}^{3/2}/\epsilon)$: •

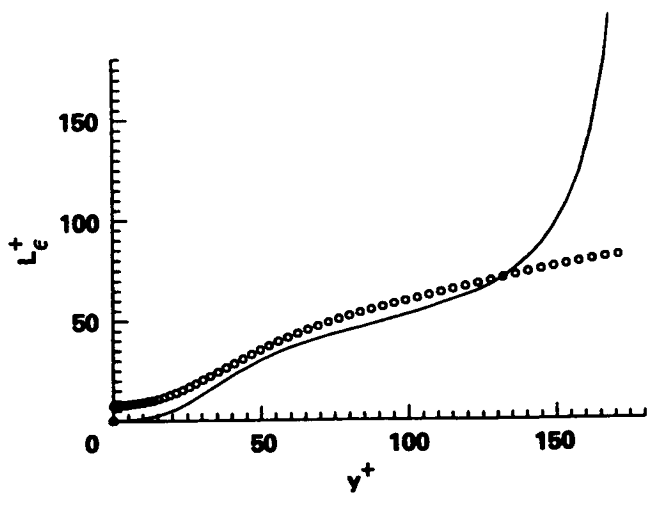

FIGURE 3. Channel Flow, Re_δ = 3,300. Formula Eq. 1.4: —— ; Simulations $(\overline{v^2}^{3/2}/\epsilon)$: ○