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# Reynolds Stress Models of Homogeneous Turbulence

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Existing and new models for the rapid and the return terms in the Reynolds stress equations have been tested in two ways. One, by direct comparison of the models with simulation data. The other, by simulating the flows using the models and comparing the predicted Reynolds stresses with the data. We find that existing linear models can be improved and that non-linear models are in better agreement with the simulation data for a wide variety of flows.

## 1. Introduction

Homogeneous flows are considered to be basic flows in the study of complex turbulent flows. These flows are the simplest turbulent flows, yet, the pressure-strain and the dissipation rate terms in the Reynolds stress equations do not vanish in these flows. These terms need to be modeled for closure of the Reynolds stress equations, and are usually recombined into a so-called rapid pressure-strain and a return term. On the other hand, the terms related to turbulence diffusion (for example, the triple correlation tensor and pressure transport terms, which will complicate the turbulence modeling) do not appear in homogeneous flows, this allows us to concentrate on two of the important terms to be modeled. There exist several models developed for these terms, we have for example, for the rapid term, the models of Naot, Shavit and Wolfshtein (1970), Launder, Reece and Rodi (1975, hereafter referred to as LRR), Shih and Lumley (1985, hereafter referred to as SL), Reynolds (1987), and others; for the return term, the models of Rotta (1951), Lumley (1978, hereafter referred to as Lumley) and a second-order form by Shih, Mansour and Moin (1987, hereafter referred to as SMM).

The research conducted at the CTR this summer concentrated on testing some existing and new models developed for the rapid and the return terms. These models were tested in two ways. First, we compared these models directly with numerical simulation data, since direct evaluation of these terms is possible using the full simulation data. Second, we used these models in a finite difference code for the Reynolds stress equations and compared the solutions of the modeled Reynolds stress equations with numerical simulation data.

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## 2. Reynolds stress Closure Models

### 2.1 Reynolds stress equation

For homogeneous turbulence, the Reynolds stress equations read as follows,

$$\begin{aligned}\langle u_i u_j\rangle_{,t} =& -\langle u_j u_k\rangle U_{i,k} - \langle u_i u_k\rangle U_{j,k} \\ & + (\langle p u_{i,j}\rangle + \langle p u_{j,i}\rangle)/\rho - 2\nu\langle u_{i,k} u_{j,k}\rangle\end{aligned}$$

where, $\langle\ \rangle$ stands for ensemble averaging. In this equation, the second line, i.e. the pressure strain correlation tensor and dissipation tensor must be modeled. The usual approach is to recombine the terms in the equations as follows:

$$\langle u_i u_j\rangle_{,t} = P_{ij} + \Pi^1_{ij} + \Pi^2_{ij} - (2/3)\langle\epsilon\rangle\delta_{ij}$$

where $P_{ij}$ is the production term,

$$P_{ij} = -(\langle u_j u_k\rangle U_{i,k} + \langle u_i u_k\rangle U_{j,k})$$

and $\Pi^1_{ij}$ and $\Pi^2_{ij}$ are called the rapid and the return term respectively and are defined as

$$\Pi^1_{ij} = (\langle p^1 u_{i,j}\rangle + \langle p^1 u_{j,i}\rangle)/\rho$$

$$\Pi^2_{ij} = (\langle p^2 u_{i,j}\rangle + \langle p^2 u_{j,i}\rangle)/\rho - 2\nu\langle u_{i,k} u_{j,k}\rangle + (2/3)\langle\epsilon\rangle\delta_{ij}$$

where $\langle\epsilon\rangle = \nu\langle u_{i,k} u_{i,k}\rangle$, the pressures $p^1$ and $p^2$ are solutions to the rapid and the slow Poisson equations (for more details see SL).

### 2.2 Models

Based on realizability, Shih and Lumley (1985) and Reynolds (1987) proposed the following model for the rapid term,

$$\begin{aligned}\Pi^1_{ij} =& (1/5 + 2\alpha_5)\langle q^2\rangle(U_{i,j} + U_{j,i}) \\ & -2/3(1-\alpha_5)(P_{ij} - 2P\delta_{ij}/3) \\ & +(2/3 + 16\alpha_5/3)(D_{ij} - 2P\delta_{ij}/3) \\ & +(6/5)b_{ij}P + (2/15)(P_{ij} - D_{ij}) \\ & +(2/5)[(\langle u_i u_k\rangle U_{j,q} + \langle u_j u_k\rangle U_{i,q})\langle u_k u_q\rangle \\ & -\langle u_i u_p\rangle\langle u_j u_q\rangle(U_{p,q} + U_{q,p})]/\langle q^2\rangle\end{aligned} \tag{2.1}$$

where,

$$D_{ij} = -(\langle u_j u_k\rangle U_{k,i} + \langle u_i u_k\rangle U_{k,j})$$

$P = P_{ii}/2$ is the turbulent kinetic energy production,

$$\alpha_5 = -(1/10)(1 + 0.8F^{1/2}) \qquad \text{(SL)}$$

$$\alpha_5 = -(1/10)\left\{1 + 3.5[1 - (1 - F)^{1/4}]\right\} \qquad \text{(SMM)}$$

If we retain in Eq. (2.1) the first three lines and set $\alpha_5 = -1.45455$, we get the linear model of LRR.

A general form of the model for the return term is suggested by Lumley (1978), and Shih (1984):

$$\Pi^2_{ij} = -\langle\epsilon\rangle\{(2 + C_f F^{\xi})b_{ij} + \gamma[b^2_{ij} + (1/3 + 2II\ )b_{ij} + 2II\ \delta_{ij}/3]\} \qquad (2.2)$$

where

$$C_f = (1/9)\exp(-7.77/\sqrt{Re})\{72/\sqrt{Re} + 80.1\ln[1 + 62.4(-II\ + 2.3III\ )]\}$$

$$II = -b_{ij}b_{ji}/2$$

$$III = b_{ij}b_{jk}b_{ki}/3$$

$$\gamma = \gamma_0(1 - F^{\eta})$$

$$Re = \langle q^2\rangle^2/(9\langle\epsilon\rangle\nu)$$

$$F = 1 + 9II\ + 27III$$

$$\xi = 0, \quad \gamma_0 = 0, \quad C_f = 1 \qquad \text{(Rotta, 1951)}$$

$$\xi = 1, \quad \gamma_0 = 0 \qquad \text{(Lumley)}$$

$$\xi = 17/20, \quad \eta = 1/20, \quad \gamma_0 = -2 \qquad \text{(SMM)}$$

## 3. Model Testing

### 3.1 Direct comparison with simulation data

The data for homogeneous strain of Lee and Reynolds (1985) and for homogeneous shear of Rogers, Moin and Reynolds (1986) were used to directly compare the model expression with the simulation results. For all simulated shear flows, the non-linear rapid model $\Pi^1_{ij}$, Eq. (2.1), and the non-linear return model $\Pi^2_{ij}$, Eq. (2.2), are in good agreement with the simulation data. The linear rapid model ( launder, Reece and Rodi, 1975) and the linear return model (Lumley, 1978) are also included for comparison. Here, we present two typical flows: C128U†(with a moderate shear rate $S = 28.284$) and C128W† (with a high shear rate $S = 56.568$). Fig.1 - Fig.4 show the comparison between the models and the simulation data. We find that the non-linear rapid model works well in each component of the Reynolds stress equations (see Fig. 1 and Fig. 2), while the linear rapid model (LRR) does not work well for the $\langle uu \rangle$ and $\langle vv \rangle$ components. On the other hand, Lumley's linear return model works very well in all simulated shear flows as indicated in Fig. 3 and Fig. 4. But, we find that the non-linear return model, Eq. (2.2), works at least as good as Lumley's linear model in all simulated shear flows, in addition, the non-linear model works better in relaxation from simple strains (typical comparisons are shown in Figs 5-7). The return to isotropy cases (or relaxation cases), after irrotational strain, of Lee and Reynolds (1985) provide a critical evaluation of the return model. We find that the return term models work well in all relaxation flows from axisymmetric contractions, but the agreement between the model expression and the data deteriorates in some relaxation flows from plane strains and axisymmetric expansions. Fig.5 shows a typical relaxation from an axisymmetric contraction. Fig.6 and Fig.7 show relaxations from the plane strain and axisymmetric expansion respectively. The failure of the return term models in some relaxation flows from plane strain (Fig. 6.1) and axisymmetric expansion (Fig. 7.1) is due to the inability of the current models to reflect the effect of the initial condition on the relaxation process.

### 3.2 Predictions using the modeled Reynolds stress equations

In this section, we choose the homogeneous shear case (C128W, high shear $S = 56.568$) of Rogers, Moin and Reynolds (1986) to evaluate the performance of the rapid and return term models in predicting the Reynolds stresses. However, in order to integrate the Reynolds stress equations, we need a model equation for the dissipation rate $\langle \epsilon \rangle$. A standard transport model equation for $\langle \epsilon \rangle$ (Lumley,1978) was used in conjunction with the models of SL, Lumley, and SMM. The $\langle \epsilon \rangle$ equation of LRR was used in conjunction with the LRR model.

Figures 8.1 and 8.2 show the Reynolds stresses and dissipation rate as predicted using the models of SMM (Eqs. 2.1 and 2.2). The model predicts well the shear stress and the streamwise component of the Reynolds stress but slightly overpredicts the cross stream components. Similar results were obtained using the models of SL and lumley for the rapid and return terms (See Fig. 9b). Figure 9a shows the results using the linear models of LRR. In this case the Reynolds stresses are overpredicted by a significant amount.

## 4. Future Work

From this study, we conclude that the models given by Eq.(2.1) and Eq.(2.2) are appropriate at least for homogenous turbulent shear flows. The linear models are unable to predict the high shear case, and are expected to have severe limitations for more general cases. The nonlinear models were developed based on a general

† The name of the flow cases are those of Rogers, Moin and Reynolds (1986)

realizability conditions, we should be able to use them to model other flows. In particular, these models should be extended to inhomogeneous flows and should be evaluated in a similar manner.

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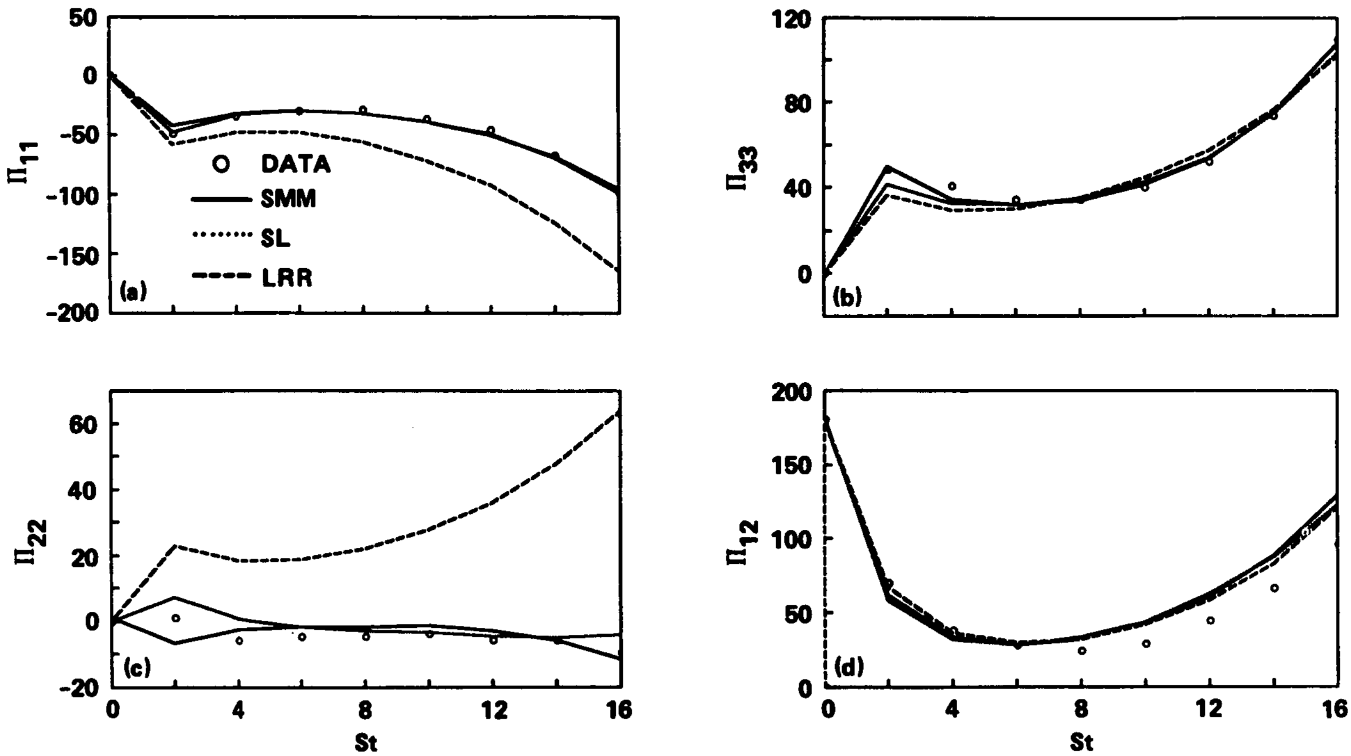


FIGURE 1. Rapid Pressure strain terms for homogeneous shear, $S = 28.284$ (case C128U of Rogers, Moin and Reynolds, 1986). Comparison of models with data.

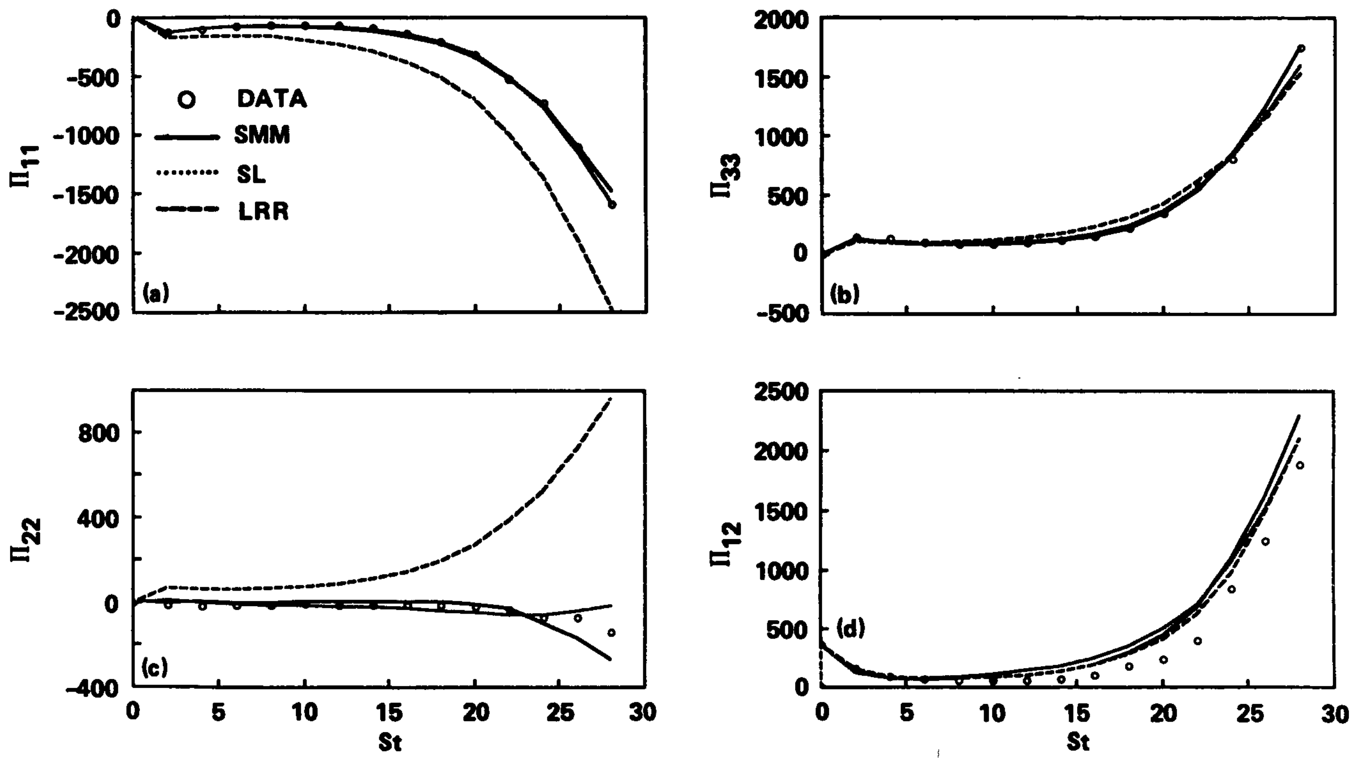


FIGURE 2. Return Pressure strain terms for homogeneous shear, $S = 28.284$ (case C128U of Rogers, Moin and Reynolds, 1986). Comparison of models with data.

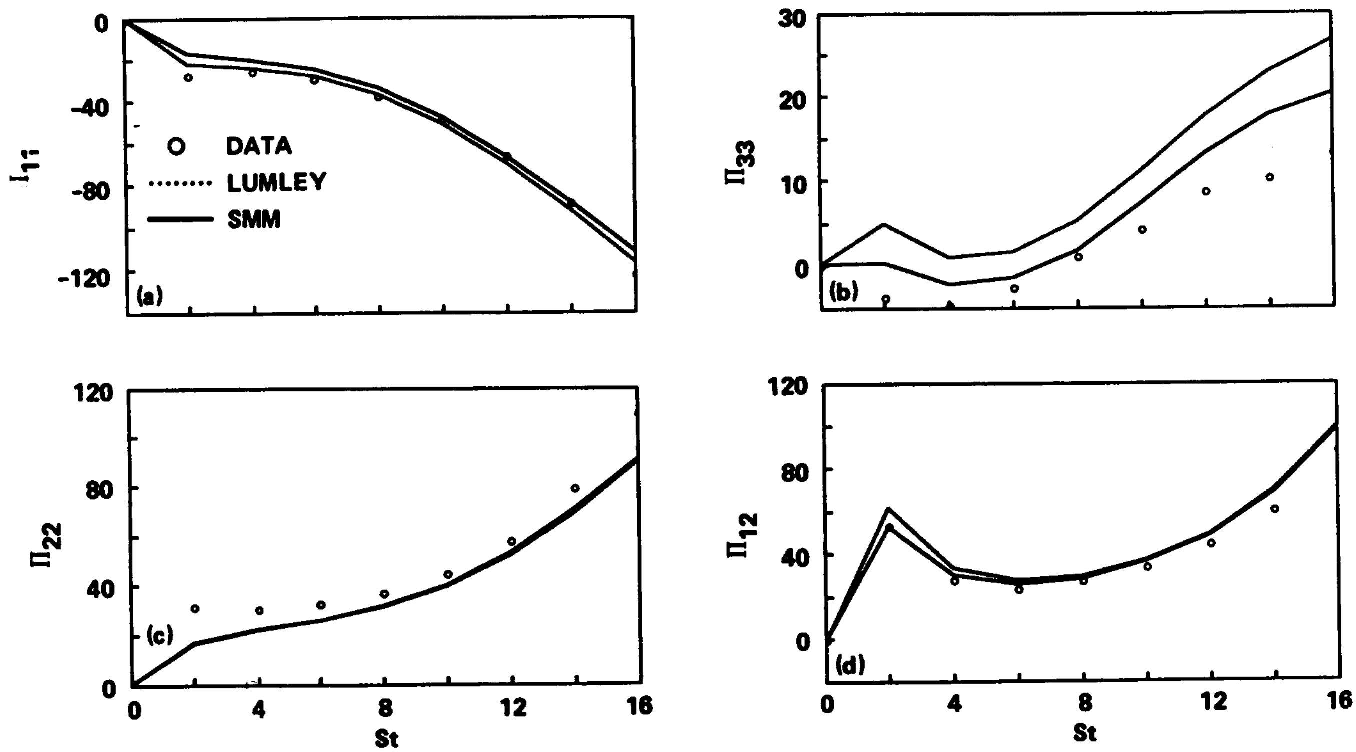


FIGURE 3. Rapid Pressure strain terms for homogeneous shear, $S = 56.568$ (case C128U of Rogers, Moin and Reynolds, 1986). Comparison of models with data.

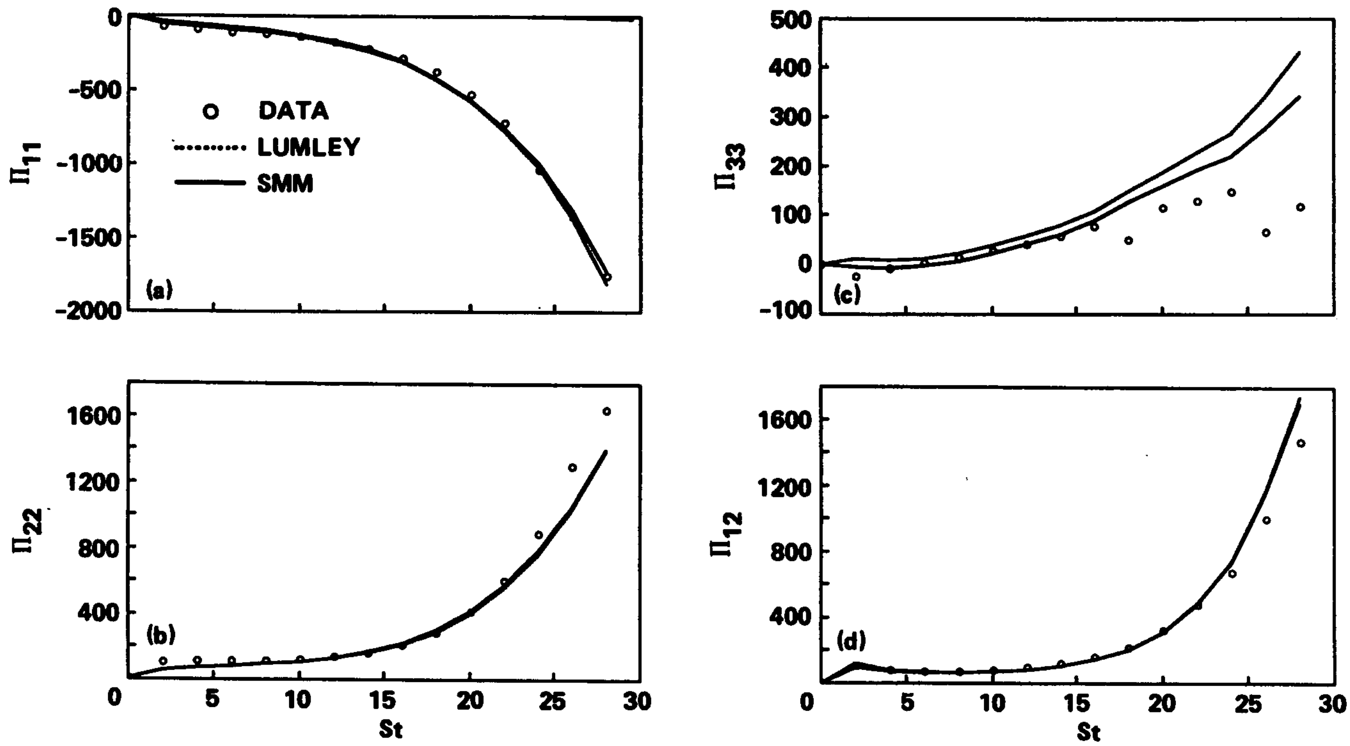


FIGURE 4. Return Pressure strain terms for homogeneous shear, $S = 56.568$ (case C128U of Rogers, Moin and Reynolds, 1986). Comparison of models with data.

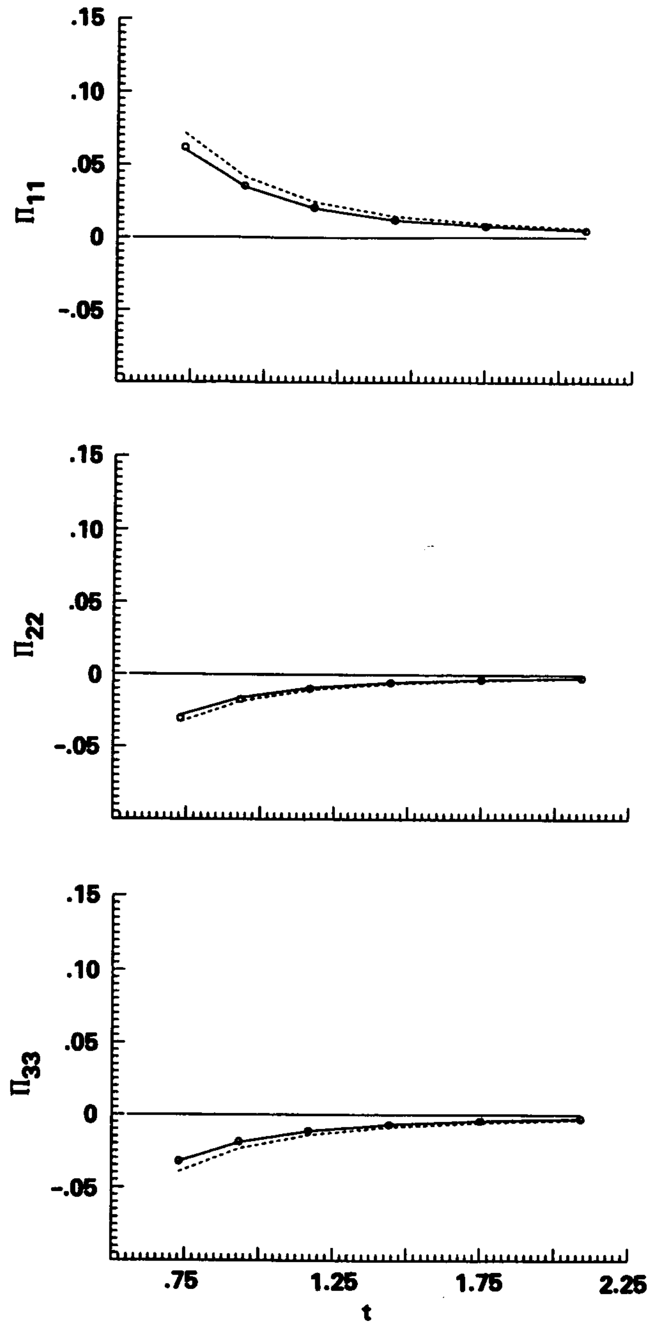


FIGURE 5. Return Pressure strain terms for return from axisymmetric contraction (case M2R of Lee and Reynolds, 1985). Comparison of models with data. o Data, ---- SMM, ······· Lumley.

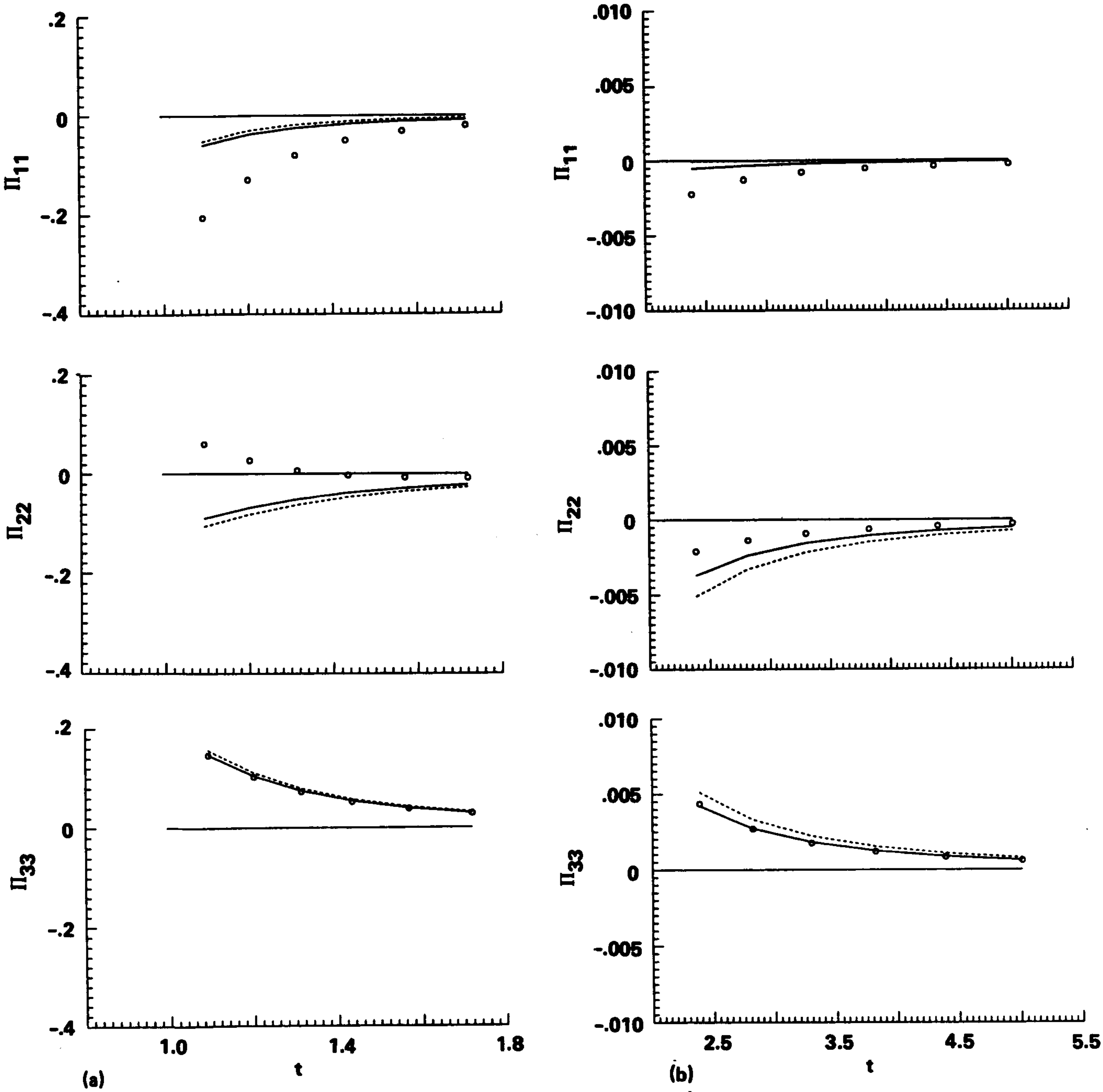


FIGURE 6. Return Pressure strain terms for return from plane strain (cases E4R, a), and A2R, b), of Lee and Reynolds, 1985). Comparison of models with data. o Data, ---- SMM, ······· Lumley.

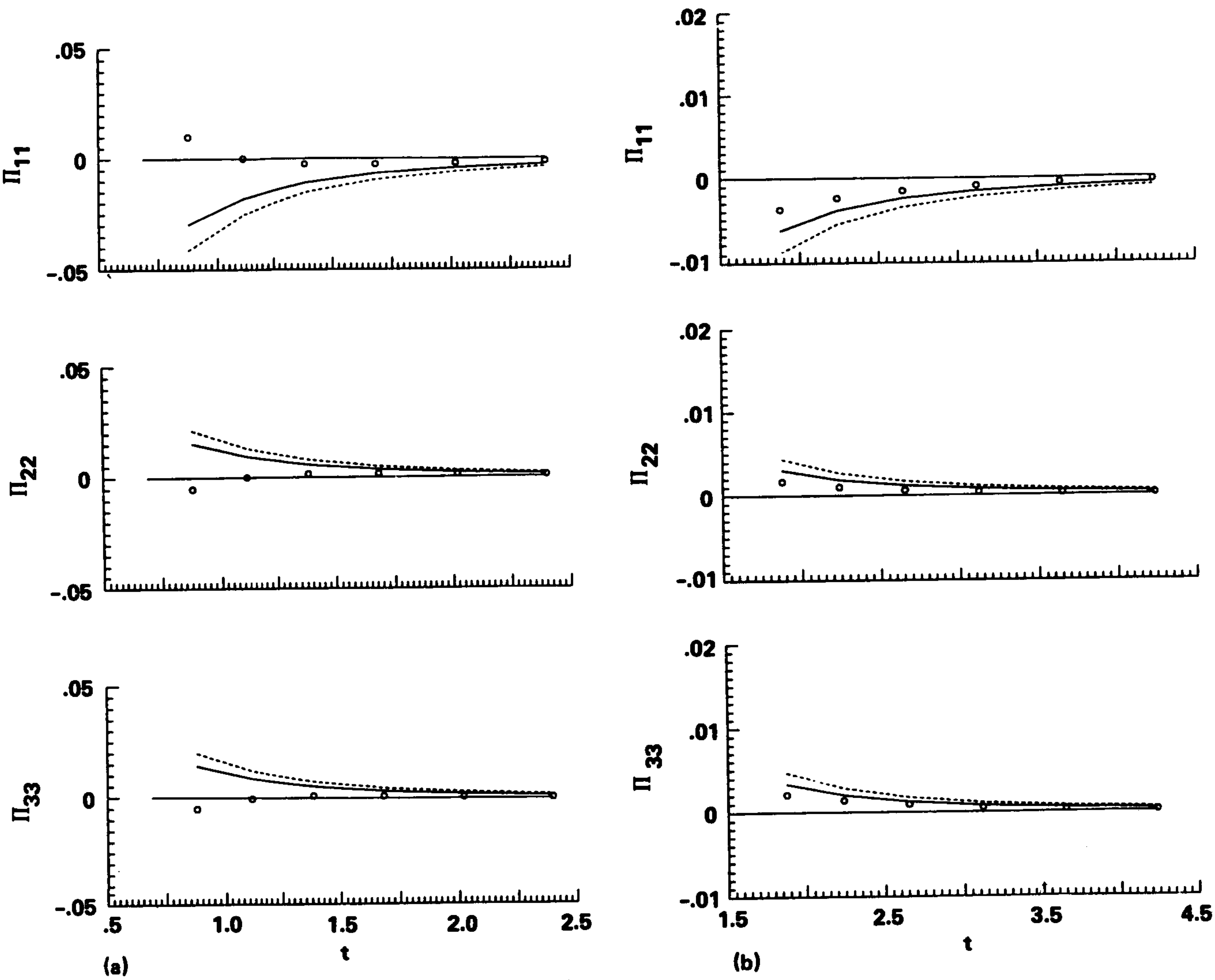


FIGURE 7. Return Pressure strain terms for return from axisymmetric expansion (cases P3R, a), and O3R, b), of Lee and Reynolds, 1985). Comparison of models with data. o Data, ---- SMM, ········ Lumley.

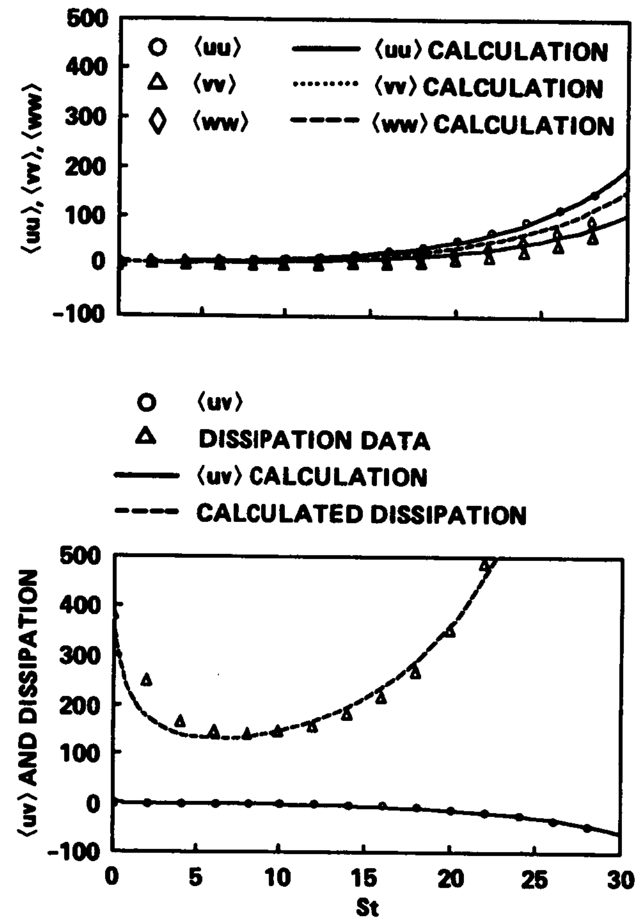


FIGURE 8. Reynolds stresses and dissipation rate for homogeneous shear, $S =$ 56.568 (case C128W of Roger, Moin and Reynolds, 1986). Comparison of prediction using SMM with data.

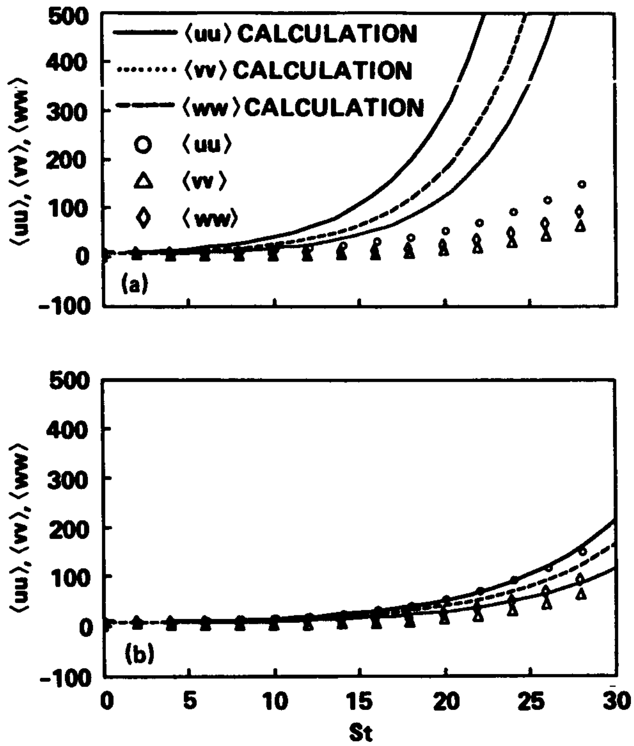


FIGURE 9. Reynolds stresses development for homogeneous shear, $S = 56.568$ (case C128W of Roger, Moin and Reynolds, 1986).

a) Comparison of prediction using LRR with data.

b) Comparison of prediction using SL with data.