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Interscale energy transfer in numerically simulated turbulence

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Energy transfer is investigated for flows obtained by direct numerical simulations of low Reynolds number homogeneous-shear and isotropic turbulence and by large-eddy simulations of high Reynolds number isotropic turbulence. The transfer in spectral space is found to be local but results from interaction between separated scales. The transfer among small scales is highly intermittent in physical space. The measurements suggest an important correlation between transfer among small scales and the energy of large scales.

1. Introduction

Using results of low-Reynolds-number direct numerical simulations (DNS), Domaradzki and Rogallo (1988, 1990) analyzed the energy transfer in isotropic turbulence and concluded that beyond the energy containing range the energy was transferred among scales of motion similar in size but that the interactions responsible for this local energy transfer were nonlocal in k-space. The same transfer mechanism was also found when the eddy-damped quasinormal Markovian (EDQNM) approximation was applied to high Reynolds number flows which are inaccessible to the DNS technique.

The conclusions concerning the apparent universality of this transfer mechanism are extended in this work to homogeneous shear flows and to high Reynolds number isotropic flows obtained by large-eddy simulation. We also devise a physical-space representation of the spectral energy transfer calculated in k space that allows us to estimate the spatial intermittency of the energy transfer and the spatial correlation between quantities defined using only large-scales flow information and the dynamically important energy transfer among different scales. In particular, this is useful in evaluating the performance of subgrid-scale models formulated in physical space, e.g. the classical Smagorinsky eddy viscosity model.

2. Numerical Velocity Fields

We have used velocity fields generated by numerical simulations that were run for sufficiently long times to fully establish nonlinear interactions.

The velocity field C128U8 is the result of a DNS of uniformly sheared homogeneous turbulence performed by Rogers (1986), and LES128 is the result of a

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large-eddy simulation of forced isotropic turbulence, at nominally infinite Reynolds number, performed by Chasnov (1990). The energy spectrum of LES128 exhibits a $k^{-5/3}$ law over the entire range of simulated wavenumbers. The field K128 is obtained from a DNS of isotropic turbulence performed by Rogallo (unpublished). Its use is motivated primarily by the fact that the two dynamically important processes that determine the evolution of the energy spectrum, i.e. viscous dissipation and nonlinear transfer, are very well resolved. This resolution is obtained at the expense of lowering the Reynolds number as compared with the two other cases.

3. Basic Quantities

The Navier-Stokes equations, in the Fourier spectral representation, for the fluctuating velocity field u_n subjected to uniform shear $\mathbf{U} = (sx_2, 0, 0)$ are

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) u_n(\mathbf{k},t) = (-i/2)P_{nlm}(\mathbf{k}) \int u_l(\mathbf{p},t)u_m(\mathbf{k}-\mathbf{p},t)d\mathbf{p}$$

$$+2s\frac{k_1 k_n}{k^2}u_2(\mathbf{k},t) + sk_1\frac{\partial}{\partial k_2}u_n(\mathbf{k},t) - s\delta_{n1}u_2(\mathbf{k},t) \tag{1}$$

$$ik_n u_n = 0 \tag{2}$$

where

$$P_{nlm}(\mathbf{k}) = k_m(\delta_{nl} - k_n k_l/k^2) + k_l(\delta_{nm} - k_n k_m/k^2), \tag{3}$$

ν is the kinematic viscosity, and the summation convention is assumed. In subsequent formulae explicit time dependence will be omitted.

The equation for the energy amplitudes $\frac{1}{2}|u(\mathbf{k})|^2 = \frac{1}{2}u_n(\mathbf{k})u_n^*(\mathbf{k})$ is obtained from (1)

$$\frac{\partial}{\partial t}\frac{1}{2}|u(\mathbf{k})|^2 = -2\nu k^2\frac{1}{2}|u(\mathbf{k})|^2 + T(\mathbf{k}) + sk_1\frac{\partial}{\partial k_2}\frac{1}{2}|u(\mathbf{k})|^2 - sRe\{u_1(\mathbf{k})u_2^*(\mathbf{k})\} \tag{4}$$

where the asterisk denotes complex conjugate.

The nonlinear energy transfer is

$$T(\mathbf{k}) = \frac{1}{2}Im\left[u_n^*(\mathbf{k})P_{nlm}(\mathbf{k})\int u_l(\mathbf{p})u_m(\mathbf{k}-\mathbf{p})d\mathbf{p}\right] \tag{5}$$

and the following two terms in (4) containing s describe energy transfer due to the mean shearing deformation of turbulent eddies and turbulent energy production by the mean shear respectively. A detailed description of these effects is given by Deissler (1961), Fox (1964), and Lumley (1964), and is summarized in Hinze's (1975) monograph. Note that the corresponding equations for isotropic turbulence are obtained from (1) by taking $s = 0$. In particular, the nonlinear transfer term (5) has the same form for both homogeneous shear turbulence and isotropic turbulence.

The principal quantity of interest here is the energy exchange between a given mode $\mathbf{k}$ and all pairs of modes $\mathbf{p}$ and $\mathbf{q} = \mathbf{k} - \mathbf{p}$ that form a triangle having $\mathbf{k}$ as one of the legs and where $\mathbf{p}$ and $\mathbf{q}$ lie in prescribed regions $\mathcal{P}$ and $\mathcal{Q}$ of the spectral space respectively. For a given $\mathbf{k}$, confining $\mathbf{p}$ and $\mathbf{q}$ to $\mathcal{P}$ and $\mathcal{Q}$ is equivalent to selecting a specific set of triangles from all of the possible triangles contributing to the energy transfer at the wavevector $\mathbf{k}$ in (5).

In this work, we choose $\mathcal{P}$ and $\mathcal{Q}$ as shells in the wavenumber space $k - \frac{1}{2}\Delta k < |\mathbf{k}| < k + \frac{1}{2}\Delta k$ with a shell thickness Δk. This choice is natural for isotropic turbulence and is also convenient for other homogeneous fields as first suggested by Batchelor (1953).

The net nonlinear energy transfer to wavenumber band k is denoted by $T(k)$, and the contribution to this transfer resulting from nonlinear interactions between wavenumbers in the band k and wavenumbers in the bands p and q is denoted by $T(k|p,q)$. According to this definition

$$T(k) = \sum_p \sum_q T(k|p,q) = \sum_p P(k|p) \tag{6}$$

where the $P(k|p)$ is the result of summation of $T(k|p,q)$ over all bands q and is interpreted as the contribution to the net energy transfer into band k due to all interactions involving band p.

The functions $T(k)$, $P(k|p)$, and $T(k|p,q)$ give progressively more detailed information about energy transfer among different scales of motion in a turbulent field. The method of computing these functions is described by Domaradzki and Rogallo (1990).

4. Analysis of Energy Transfer in Spectral Space

All of the contributing terms of (4), computed for the field C128U8 and averaged over spherical shells with thickness $\Delta k = 1$, are plotted in figure 1. The calculation of the linear transfer $sk_1 \frac{\partial}{\partial k_2} \frac{1}{2}|u(\mathbf{k})|^2$ suffers from low accuracy due to the coarse resolution of $\mathbf{k}$, and we believe that this term is close to zero for $k > 40$, contrary to the plotted results. Despite this numerical error, a few important conclusions can be drawn from these results. Nonlinear transfer, viscous dissipation, and mean shear all make significant contributions to the energy balance for wavenumbers $k < 40$ which comprise the energy containing range and a significant fraction of the dissipation range. Energetics of the smaller eddies ($k > 40$) is affected only by nonlinear transfer and viscous dissipation which are roughly in balance. Thus, the energetics of turbulence in about half of the spectral domain ($k > 40$) is not affected directly by the large scale mean shear.

The triad structure of the nonlinear energy transfer term is illustrated by plotting $P(k|p)$ in figure 2a as a function of k for p fixed in a wavenumber band beyond the peak of the energy spectrum. The contributions $T(k|p,q)$ to $P(k|p)$, from all significant bands q, are also included. The peaks of $P(k|p)$ are located in the vicinity of the band p, indicating that the energy transfer is primarily between comparable scales of motion. However, the decomposition into functions $T(k|p,q)$ reveals that

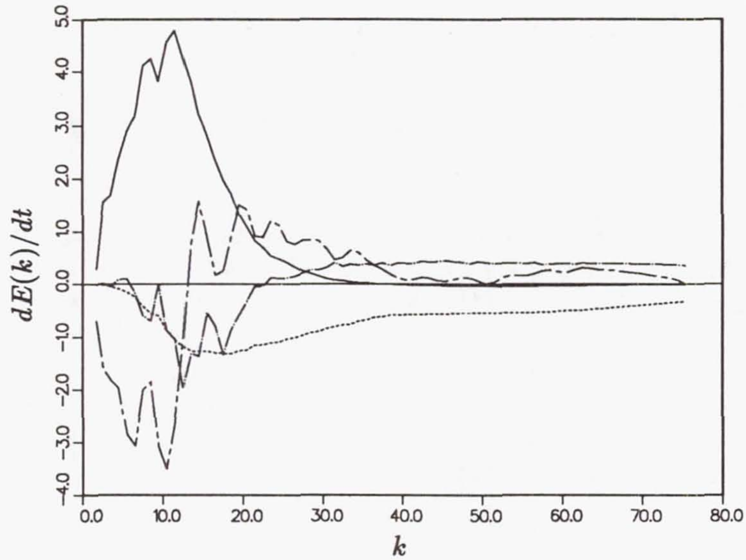


FIGURE 1. Spectral energy balance for the field C128U8. —— production, ---- dissipation, —·— nonlinear transfer, —-— linear transfer. The linear transfer data has been smoothed.

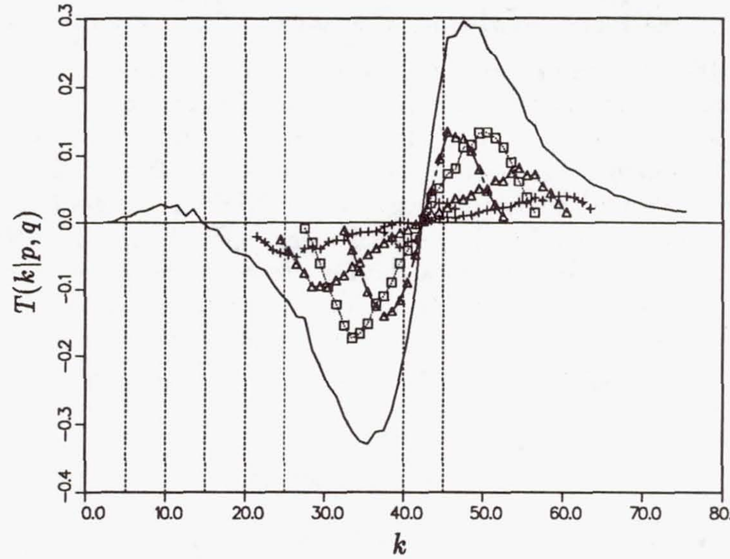
the largest contributions to this local transfer come from the interactions involving a scale in the energy containing range $5 < q < 20$. Thus, for homogeneous shear flow we obtain the same result as previously reported by Domaradzki and Rogallo (1988, 1990) for isotropic flows: local energy transfer between two scales beyond the energy containing range results from nonlocal interactions with scales in the energy containing range.

Analysis of the nonlinear transfer for the two remaining velocity fields, LES128 and K128, provided the same qualitative results.

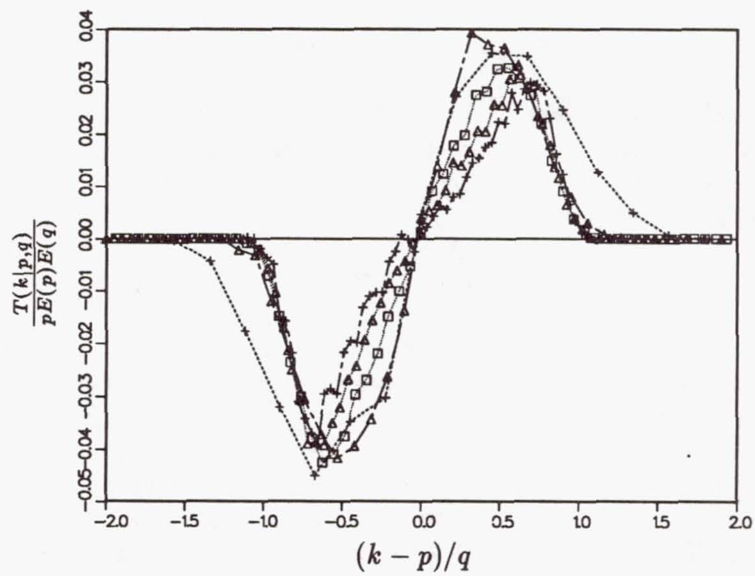
An attempt was made to find a similarity scaling for the functions $T(k|p,q)$. For a given energy spectrum, the following transformation collapses reasonably well all curves $T(k|p,q)$ for a band p beyond the energy containing range.

$$T(k|p,q) = pE(p)E(q)T_s(\frac{k-p}{q}) \tag{7}$$

The similarity variable $\xi = (k-p)/q$ is deduced from geometric relations for a triad with legs k, p, and q and the scaling factor $pE(p)E(q)$ is *ad hoc* (but is found in the EDQNM theory for power-law spectra in the disparate-scale limit). In figure 2b we show the result of scaling (7) applied to the measured functions $T(k|p,q)$ of figure 2a. Interestingly, the transfer scales with the energy $E(q)$ of the large eddies rather than with their rate-of-strain $qE(q)^{1/2}$ which is the scaling postulated by a number of classical closure hypotheses (Monin and Yaglom, 1975). We have not been able to propose a convincing dynamical model of transfer processes which would provide scaling (7).



(a)



(b)

FIGURE 2. Detailed triad contributions to energy transfer for case C128U8: (a) unscaled, (b) scaled by (7). The transfer spectra $T(k|p,q)$ are shown for band $40 < p < 45$, and all bands q that make a significant contribution to $P(k|p)$. +---- $0 < q < 5$, △ — - — $5 < q < 10$, □ ········ $10 < q < 15$, △ ········ $15 < q < 20$, +— - — $20 < q < 25$, —— $P(k|p)$.

5. Physical Space Representation of Spectral Energy Transfer

Let us denote by $N_n^{\mathcal{PQ}}(\mathbf{k})$ the contribution to the integral (the nonlinear term) in (1) from only those interactions between modes $\mathbf{p}$ and $\mathbf{q} = \mathbf{k} - \mathbf{p}$ such that each of them is confined to one of the two prescribed wavenumber bands $\mathcal{P}$ and $\mathcal{Q}$. This quantity is computed using the method described by Domaradzki and Rogallo (1990). Its Fourier transform to physical space, $N_n^{\mathcal{PQ}}(\mathbf{x})$ say, gives the contribution to the rate of change of velocity in physical space $u_n(\mathbf{x}, t)$ caused by the nonlinear interactions involving two scales from the respective wavenumber bands $\mathcal{P}$ and $\mathcal{Q}$ in the spectral space. Note that these interactions influence all modes $\mathbf{k}$ that can form a triangle with modes such that one is in $\mathcal{P}$ and the other in $\mathcal{Q}$. Consider next a velocity field truncated to a prescribed wavenumber band $\mathcal{K}$, i.e.

$$u_n^{\mathcal{K}}(\mathbf{k}) = \begin{cases} u_n(\mathbf{k}), & \text{if } \mathbf{k} \epsilon \mathcal{K} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The Fourier transform of (8) to physical space, $u_n^{\mathcal{K}}(\mathbf{x})$ say, represents the contribution in physical space that scales from band $\mathcal{K}$ make to the total velocity. The contracted product of these two physical space quantities

$$T^{\mathcal{KPQ}}(\mathbf{x}) = u_n^{\mathcal{K}}(\mathbf{x}) N_n^{\mathcal{PQ}}(\mathbf{x}) \tag{9}$$

gives a physical space representation of the energy transfer to/from modes in the k-band due to their nonlinear interactions with modes in the p- and q-bands.

An interesting case is obtained by dividing wavenumber space into two disjoint regions $\mathcal{K}$ ($k < k_c$) and $\mathcal{P}$ ($k > k_c$). The quantity

$$T_{SGS}(\mathbf{x}|k_c) = T^{\mathcal{KPP}}(\mathbf{x}) + T^{\mathcal{KKP}}(\mathbf{x}) \tag{10}$$

provides a physical space representation of the rate of change of energy of large scales $k < k_c$ due to nonlinear interactions involving small scales $k > k_c$. This is precisely the energy transfer process which is the subject of subgrid-scale modeling.

We have computed transfer functions (9) and (10) for various wavenumber bands of the field K128. The low wavenumber band $\mathcal{Q}$ is chosen to cover the entire energy containing range $0 < q < 10$. Figure 3a shows one plane from the full transfer (9) representing in physical space the energy transfer to eddies in the band $23 < k < 28$ caused by their interactions with eddies in the bands $20 < p < 25$ and $0 < q < 10$. The transfer function is spatially intermittent and is predominantly positive, indicating a flow of energy from the larger scales p to the smaller scales k. In figure 4b, we plot the same function for $17 < k < 22$. The transfer is now predominantly negative as expected and occurs at roughly the same locations as the transfer of figure 3a. We thus conclude that the local energy transfer between similar wavenumber modes in spectral space is intermittent in physical space.

We have attempted to correlate this spatial distribution of energy transfer with a number of simpler quantities (rate-of-strain, dissipation, energy, etc.) calculated from the velocity field truncated to contain only either large or small scales. In

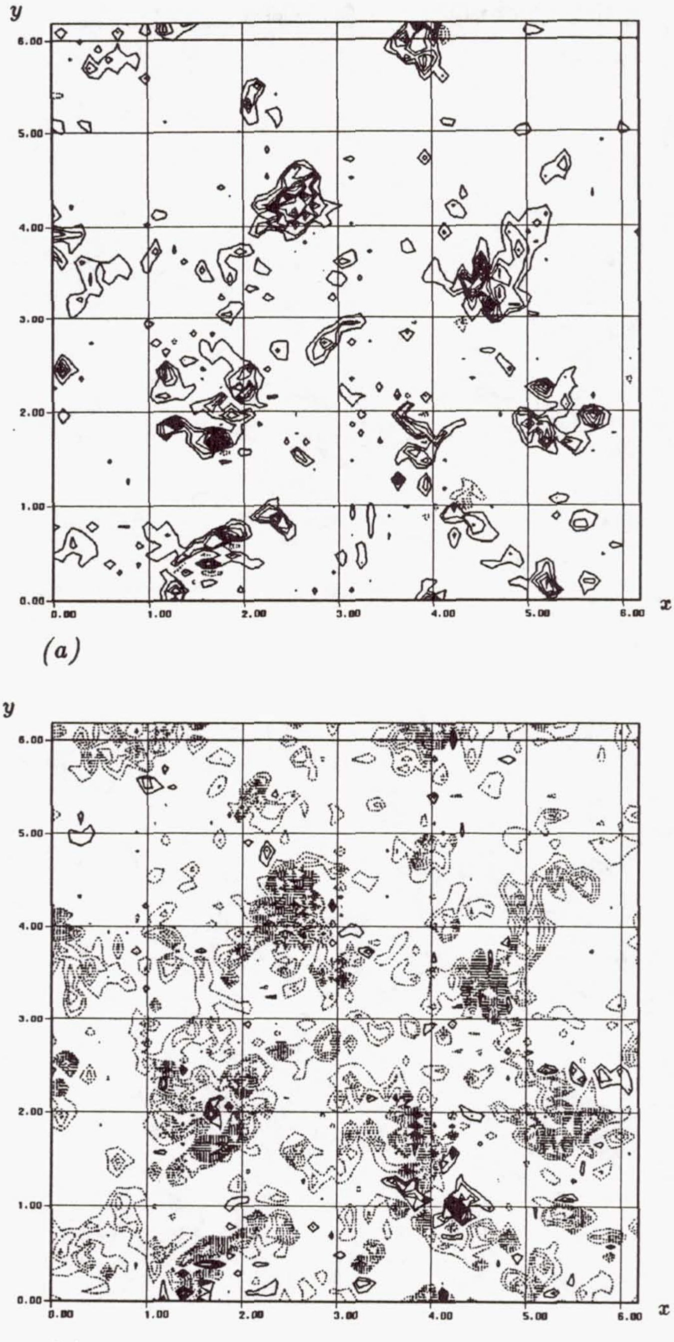
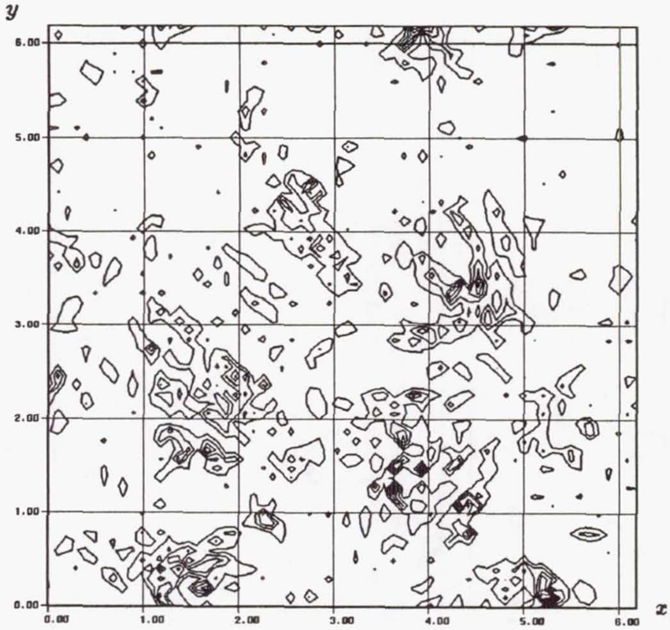
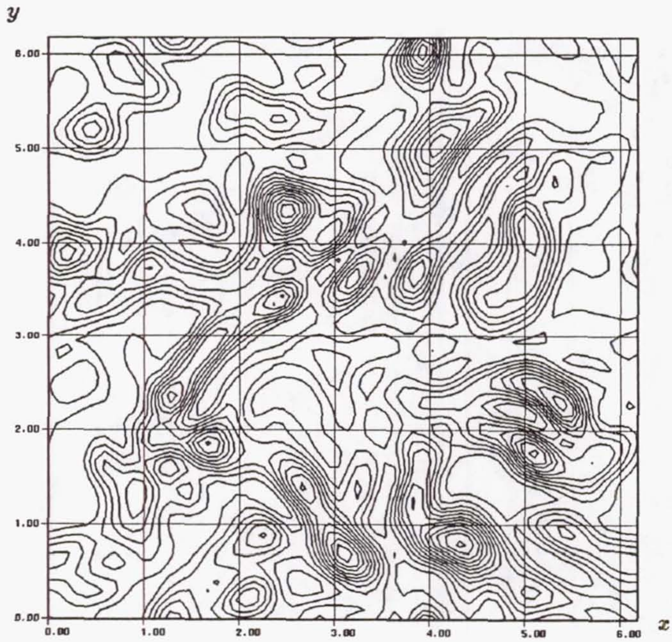


FIGURE 3. Energy transfer $T^{\mathcal{K}\mathcal{P}\mathcal{Q}}(\mathbf{x})$ of K128 in physical space for $20 < p < 25, 0 < q < 10$: (a) $23 < k < 28$, (b) $17 < k < 22$.



(a)



(b)

FIGURE 4. Turbulent energy in physical space for the velocity field truncated in spectral space to wavenumber band: (a) $23 < k < 28$, (b) $0 < k < 10$.

figures 4a and 4b, we show the physical-space distribution of energy for the velocity field truncated to $23 < k < 28$ and $0 < k < 10$, respectively. Both energy fields correlate very well with the energy transfer among small scales shown in figure 3. Correlation of other calculated quantities with the energy transfer, notably the square of the rate-of-strain tensor, was generally much worse. Therefore, we conclude that the energy transfer among small scales occurs mostly at those physical locations that contain large amounts of turbulent energy rather than at the locations of high strain rate. This correlation is the physical space counterpart of the observed importance of the nonlocal triads in the energy transfer process in spectral space.

We have used formula (10) to calculate subgrid-scale (SGS) energy transfer for the field K128 with the cutoff wavenumber $k_c = 10$. The full SGS transfer field, plotted in figure 5a for a typical plane, is characterized by the presence of both negative and positive regions. These indicate energy flux from and to the large scales respectively due to subgrid-scale interactions. The classical Smagorinsky model (Smagorinsky, 1963) for this transfer, based on the velocity field truncated to the large scales $0 < k < 10$, is plotted in figure 5b. Note that the model captures properly the locations of the regions where the transfer is most intense but fails completely to predict the inverse energy transfer from small to large scales.

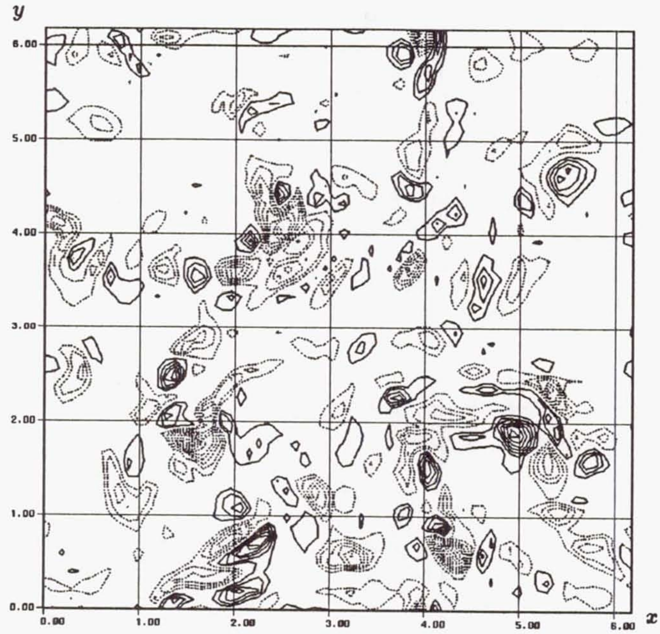
6. Conclusions

Using results of direct numerical simulations of homogeneous shear turbulence, we have shown that the nonlinear energy transfer in spectral space beyond the energy containing range has the same character as reported previously for isotropic turbulence: *local* energy transfer caused by *nonlocal* triad interactions. The same conclusion was reached for velocity fields obtained in large-eddy simulations of isotropic turbulence at high Reynolds numbers.

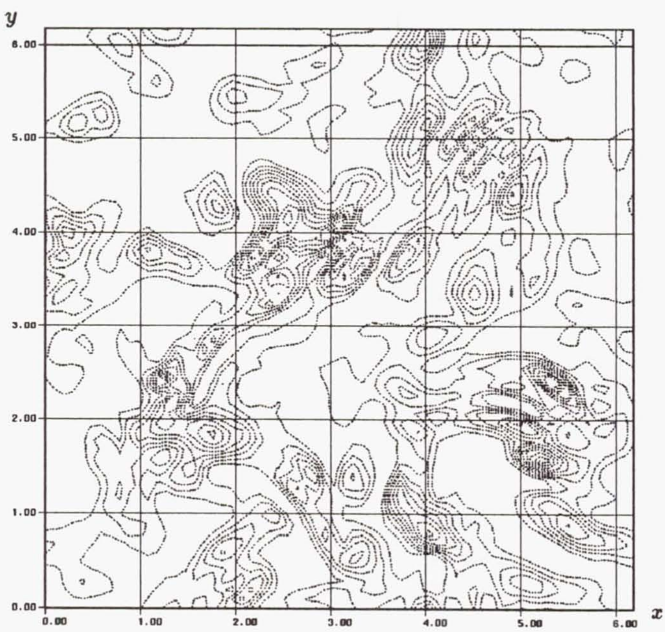
An *ad hoc* scaling roughly collapses the transfer $T(k|p,q)$ to a self-similar form. This scaling implies an important role which the energetic scales play in the energy transfer among small scales, but the process does not appear to be simply straining of the small scales by the large ones.

We have devised a physical space representation of the energy transfer processes among scales of motion belonging to three distinct wavenumber bands in spectral space and conclude from it that the energy transfer among small scales is highly intermittent in physical space. Furthermore, regions of significant transfer appear to correlate better with regions of significant large-scale energy than with those of significant large-scale strain rate.

As a particular case, we have calculated the subgrid-scale energy transfer in isotropic turbulence. This SGS transfer exhibits regions of energy drain from large to small scales as well as significant regions of reversed energy transfer from small to large scales. The Smagorinsky eddy viscosity model captures the locations of the most intense transfer but predicts that it is always from large to small scales, contrary to the measurements from direct calculations.



(a)



(b)

FIGURE 5. Subgrid scale energy transfer in physical space $T_{SGS}(\mathbf{x}|k_c)$ for $k_c = 10$: (a) measured, (b) computed using the Smagorinsky eddy viscosity model for the velocity field truncated to wavenumber band $0 < k < 10$.

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