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Role of pressure diffusion in non-homogeneous shear flows

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A non-local model is presented for approximating the pressure diffusion in calculations of turbulent free shear and boundary layer flows. It is based on the solution of an elliptic relaxation equation which enables local diffusion sources to be distributed over lengths of the order of the integral scale. The pressure diffusion model was implemented in a boundary layer code within the framework of turbulence models based on both the $k-\varepsilon-\overline{v^2}$ system of equations and the full Reynolds stress equations. Model computations were performed for mixing layers and boundary layer flows. In each case, the pressure diffusion model enabled the well-known free-stream edge singularity problem to be eliminated. There was little effect on near-wall properties. Computed results agreed very well with experimental and DNS data for the mean flow velocity, the turbulent kinetic energy, and the skin-friction coefficient.

1. Introduction

In higher-order turbulence models 'pressure diffusion' is usually neglected, or at best added to 'turbulent diffusion' (Launder 1984) and the two modeled in aggregate. Pressure diffusion refers to the term $\partial_i\overline{u_j p}$ in the Reynolds stress budget; turbulent diffusion refers to $\partial_k\overline{u_i u_j u_k}$. The latter represents the ensemble averaged effect of turbulence convection and can often be modeled as a diffusion process; the former, however, is harder to explain as diffusion. Turbulent diffusion is usually considered to be the dominant diffusion mechanism and pressure diffusion is considered to be negligible. However, Lumley (1975) showed that, for homogeneous turbulence, the application of symmetry constraints to the exact equation for the "slow" or non-linear part of the pressure diffusion led to the result that its magnitude is 20% of that of the triple velocity correlation. In the present study, DNS databases for several shear flows were examined, namely: the mixing layer simulation of Rogers and Moser (1994); the wake simulation of Rogers (private communication); the boundary layer simulation of Spalart (1988); and the backward facing step simulation of Le & Moin (1994). These confirm that, in the main shear regions, pressure diffusion is roughly 20-30% of turbulent diffusion. However, it appears to be mostly counter-gradient transport, so that it merely reduces the effect of turbulent diffusion, which is mostly gradient transport. Thus, the current practice of absorbing pressure diffusion and turbulent diffusion into a single model

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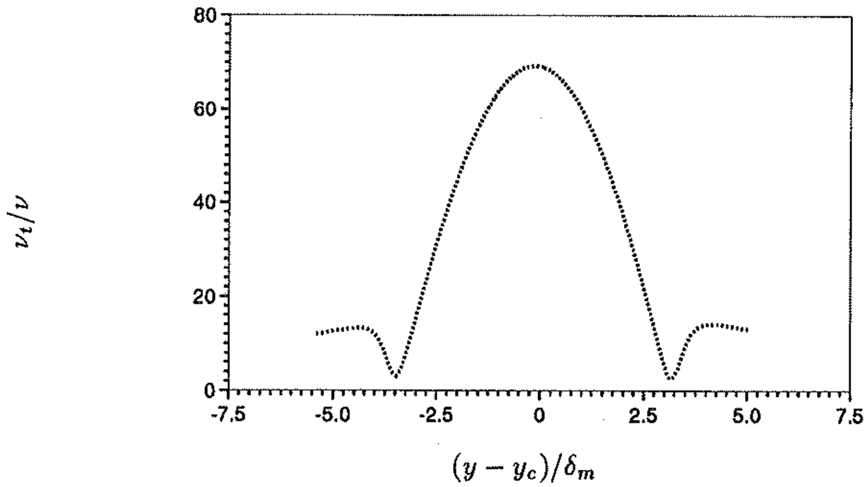

FIGURE 1. Computed eddy viscosity distribution in a two-stream mixing layer.

term appears reasonable, as far as the main shear regions are concerned. But the DNS data show that near the edges of the shear layers, turbulent diffusion decreases rapidly to zero, while pressure diffusion decreases only very gradually, so the latter then becomes dominant. Thus, the budgets show that near the free-stream edge the balance is between pressure transport and mean convection, or temporal drift, rather than between turbulent transport and the latter. When applied to one or two-equation models (Coles 1968, Cazalbou *et al.* 1994), an assumed balance between evolution and turbulent diffusion leads to an unsteady non-linear diffusion problem whose solution has a propagating front at the edge of which the eddy viscosity ν_t, the turbulent kinetic energy k, and its dissipation rate ε all go sharply to zero. Fig. 1 illustrates how the eddy viscosity drops abruptly at the edge of the shear layer, even though a non-zero value was imposed in the free stream. This singular solution is not in agreement with experimental data, which show that all these properties asymptote gradually to free-stream values. A consequence of the singularity is that computations with these turbulence models are unable to properly account for free-stream turbulence effects since the free stream and the main shear regions are decoupled, except where insufficient grid resolution produces smearing via numerical diffusion. This result would also apply to second moment closures which use gradient diffusion models for turbulent transport.

The free-stream edge singularity can be removed by introducing a model for the pressure diffusion which does not vanish abruptly at the edge of the main shear flow. Since pressure transport should be non-local, a new model for pressure diffusion is introduced based on the elliptic relaxation concept introduced by Durbin (1991, 1993) for modeling the pressure-strain correlation in non-homogeneous turbulent flows. Computations with the new model are compared to experimental data for the plane mixing layer and the boundary layer flows.

2. Mathematical model

In the present study only simply shear flows (mixing layers, wakes, boundary layers) are considered, so that boundary layer forms of the governing equations are applicable. For simplicity, we shall only present equations for the $k - \varepsilon - \overline{v^2}$ model (Durbin 1991, 1994). Though the $\overline{u_i u_j} - \overline{f_{ij}}$ model was also utilized, the results are virtually the same for the simple shear flows considered here.

The mean flow equations are:

$$D_t U = \partial_y[(\nu + \nu_t)\partial_y U] \tag{1}$$

$$\nabla \cdot U = 0 \tag{2}$$

for incompressible thin shear layers. (D_t represents the total derivative and ∂_y the cross-stream partial derivative.)

2.1 $k - \varepsilon - \overline{v^2}$ model

The usual $k - \varepsilon - \overline{v^2}$ model (Durbin 1994) is modified by the addition of a pressure diffusion term to the k-equation and the $\overline{v^2}$ equation. Thus, we have:

$$D_t k = P_k - \varepsilon + \partial_y[(\nu + \nu_t/\sigma_k)\partial_y k] - \partial_y(\overline{pv}) \tag{3}$$

$$D_t \varepsilon = \frac{C_{\epsilon_1} P_k - C_{\epsilon_2}\varepsilon}{T} + \partial_y[(\nu + \nu_t/\sigma_\epsilon)\partial_y \varepsilon] \tag{4}$$

$$D_t \overline{v^2} = k f_{22} - \overline{v^2}\frac{\varepsilon}{k} + \partial_y[(\nu + \nu_t/\sigma_k)\partial_y \overline{v^2}] - 2\partial_y(\overline{pv}) \tag{5}$$

where the rate of turbulent energy production is

$$P_k = \nu_t(\partial_y U)^2 \tag{6}$$

and the eddy viscosity is given by

$$\nu_t = C_\mu \overline{v^2} T \tag{7}$$

The term $k\overline{f_{22}}$ is the source of $\overline{v^2}$ via redistribution from the streamise component $\overline{u^2}$. This is the pressure strain term in homogeneous turbulent flow. In non-homogeneous turbulent flows, non-local effect are introduced through the elliptic relaxation equation

$$L^2 \partial_y^2 f_{22} - f_{22} = (1 - C_1)\frac{2/3 - \overline{v^2}/k}{T} - C_2 \frac{P_k}{k} \tag{8}$$

The length and time scales which appear in the equations are given by

$$L = C_L \max\left[\frac{k^{3/2}}{\varepsilon}, C_\eta \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right] \tag{9}$$

$$T = \max\left[\frac{k}{\varepsilon}, C_T\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right] \tag{10}$$

In free shear flows, equations (9) and (10) are modified by removing the Kolmogorov limits which are strictly only relevant to wall bounded flow. The boundary conditions follow Durbin (1994) and are not repeated. Their main effect is to produce the proper behavior of k, ε, and $\overline{v^2}$ near no-slip boundaries.

The empirical coefficients, are $C_L = 0.3$, $C_\mu = 0.19$, $C_{\epsilon_1} = 1.3$ and $C_{\epsilon_2} = 1.90, C_T = 6.0$, $C_\eta = 70.0$, $C_1 = 1.4$, $C_2 = 0.3$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$.

2.2 Pressure diffusion model

The pressure transport is modeled as:

$$-\partial_y \overline{pv} = \partial_y f \tag{11}$$

Non-local effects are introduced via the elliptic relaxation equation

$$L^2 \partial_y^2 f - f = -f^L \tag{12}$$

where f^L is a local gradient diffusion model

$$f^L = C_s \frac{\nu_t}{\sigma_k} \partial_y k \tag{13}$$

It is assumed that the same length scale which governs the non-locality in the pressure redistribution would also govern the non-locality in the pressure transport. C_s is a free parameter which is determined by optimization. The boundary conditions are

$$\begin{aligned} f &= 0 \qquad &&; \text{free stream} \\ \partial_y f &= 0 \qquad &&; \text{no-slip wall} \end{aligned} \tag{14}$$

3. Results and discussion

Model computations were performed for a two-stream mixing layer with the ratio of low to high free-stream velocities of 0.6 chosen to coincide with the experimental study of Bell and Mehta (1990). A parabolic forward marching code was utilized. Starting from hyperbolic tangent profiles, the solution was marched until self-similar results were obtained. Computations were performed with 3 values of C_s, namely 0.0, 0.5, and 1.0. $C_s = 0.0$ represents the case with no pressure diffusion. Computed results of ν_t/ν and k are compared in Figs. 2(a) and 2(b), respectively. It is clearly seen that, at the higher values of C_s, the edge singularity present with $C_s = 0$ has been removed. k profiles now go gradually to zero exhibiting long tails. Also ν_t profiles gradually approach the small values set in the free stream, though

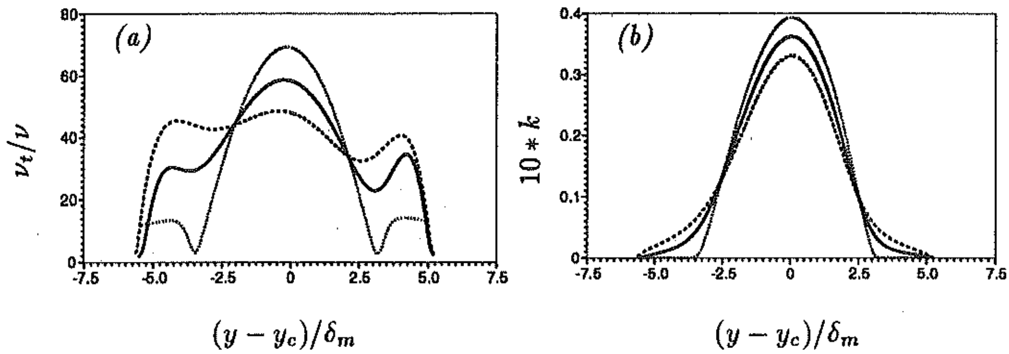

FIGURE 2. Effect of diffusion coefficient C_s on computations of the two-stream mixing layer: ······· , $C_s = 0.0$; ——— , $C_s = 0.5$; ---- , $C_s = 1.0$. *(a)* eddy viscosity *(b)* turbulent kinetic energy.

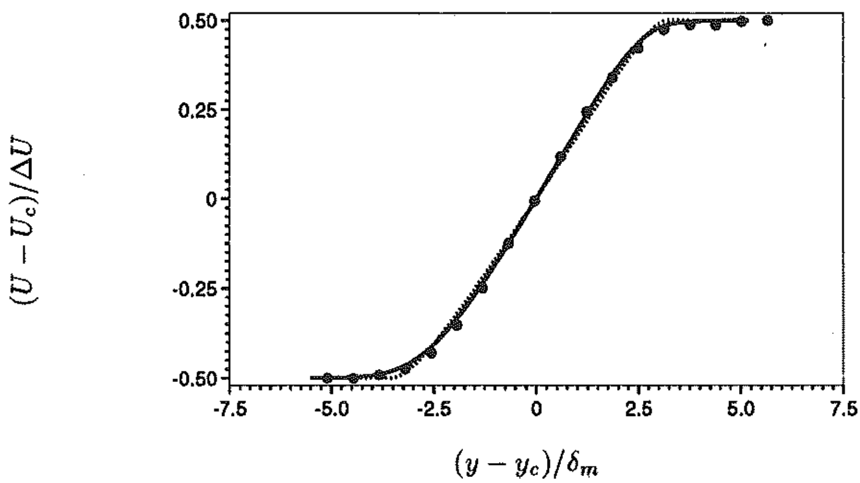

FIGURE 3. Comparison of velocity profiles in the two-stream mixing layer to • experimental data of Bell & Mehta (1990). (See figure 2 for legends.)

not monotonically. The ν_t profile obtained with $C_s = 0.5$ is quite similar to that given by the DNS data of Rogers and Moser (1994) with equation (7) as definition. The peak value computed with $C_s = 0.5$ was closest to that measured by Bell and Mehta (1990); hence $C_s = 0.5$ was chosen for all subsequent computations. Fig. 3 shows a comparison of computed velocity profiles with $C_s = 0.0$, 0.5 versus experimental data of Bell and Mehta (1990). Both model computations give the correct spread rate but the computation with the pressure diffusion model shows smoother profiles near the free-stream edges in agreement with experimental data. Similarly, k profiles are compared in Fig. 4. In this case, the differences between model computations with and without pressure diffusion are more dramatic. The non-local role of pressure diffusion in transporting turbulent kinetic energy from the center of the layer to the edges of the free stream is apparent.

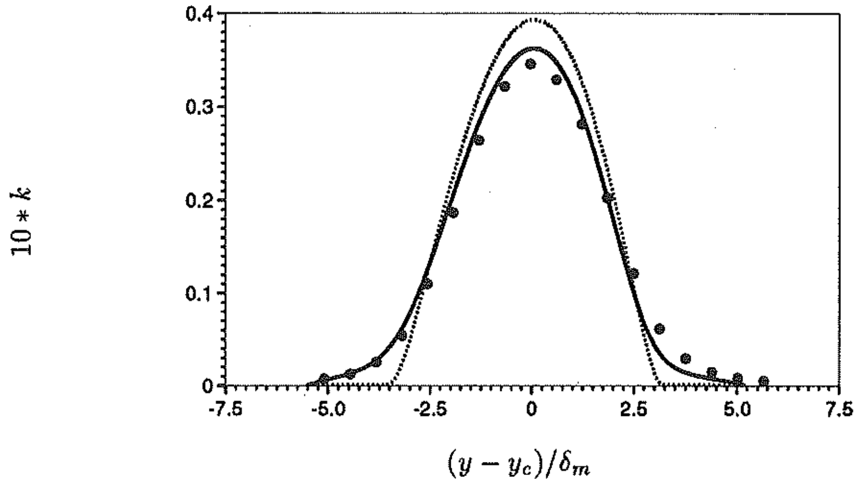

FIGURE 4. Comparison of k profiles in the two-stream mixing layer to • experimental data of Bell & Mehta (1990). (See figure 2 for legends.)

With the pressure diffusion model thus calibrated, the question is whether the improved agreement with data near free-stream edges would also be reproduced in a wall boundary layer flow without producing an adverse effect on near-wall agreement, where the role of pressure diffusion should be minimal. Computations were performed for a developing wall boundary layer using the DNS data of Spalart (1988) as inlet conditions. The solution was then marched until self-similar results were established. Profiles of ν_t/ν across the boundary layer are compared for $C_s =$ 0.0, 0.5 and 1.0 in figure 5. The main effect of pressure diffusion is near the free-stream edge, with little or no effect on near wall values. Skin friction coefficients are compared to experimental data of Coles and Hirst (1968) in figure 6. This confirms that the pressure diffusion produces negligible effect on near-wall properties. Finally k profiles at $R_\theta = 7{,}500$, normalized with the local friction velocity, are compared to experimental data of Klebanoff (1955) in figure 7. Again, pressure diffusion has negligible effect in the near-wall region, but produces more gradual decay near the free-stream edge.

Thus, the sharp free-stream edge problem can be cured by introducing a non-local model for the pressure diffusion. In the present study a gradient diffusion model was utilized for f^L. Although it enables the edge singularity problem to be overcome, it appears too simple to reproduce all features of the pressure transport through the layer and near the free stream. It can be argued that we are only modeling the deviation from the usual model treatment, i.e., the part which is not directly proportional to turbulent diffusion. Nevertheless, it is desirable to explore more general forms of f^L. Consideration is being given to such models. To aid in this study, we hope to obtain a splitting of the DNS mixing layer data of Rogers and Moser (1994) for the pressure diffusion into slow and rapid parts. We are also studying the form of the pressure diffusion term in the shearless mixing layer simulation of Briggs *et al.* (1994). Initial computations of the shearless flows studied

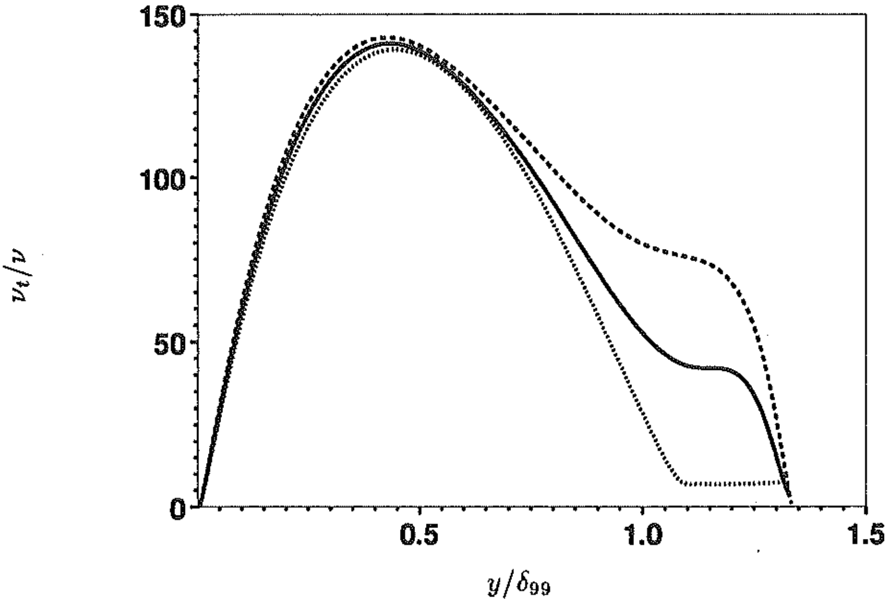

FIGURE 5. Effect of diffusion coefficient C_s on eddy viscosity distribution in boundary layer. (See figure 2 for legends.)

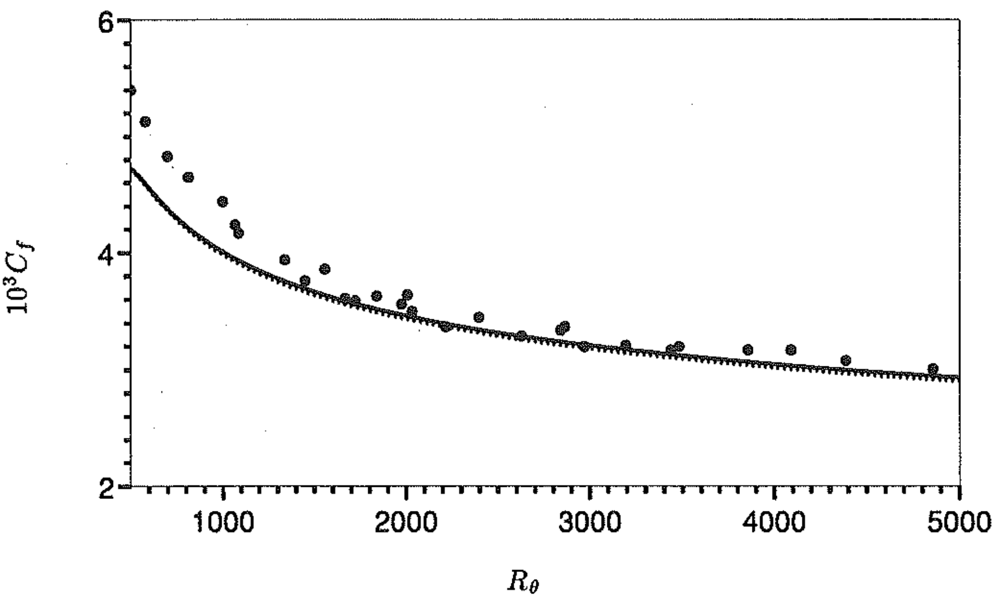

FIGURE 6. Comparison of skin friction coefficient in a boundary layer to • experimental data of Coles & Hirst (1968). (See figure 2 for legends.)

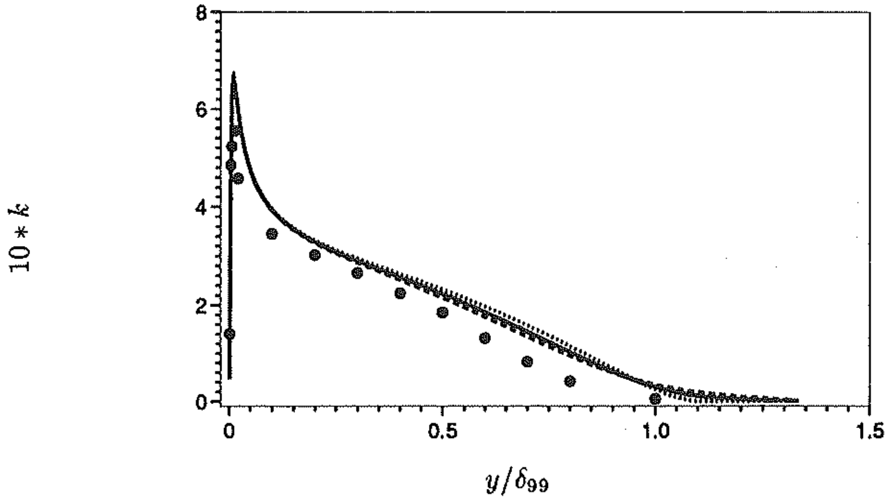

FIGURE 7. Comparison of k profiles in a boundary layer to • experimental data of Klebanoff (1955). (See figure 2 for legends.)

experimentally by Veeravalli and Warhaft (1989) show that most of the current turbulent diffusion models with or without the present diffusion model significantly underpredict the shear layer spread rate.

4. Conclusions

A non-local model for pressure diffusion has been developed based on the solution of an elliptic relaxation equation. This enabled the free-stream edge singularity problem, which most current turbulence models suffer from, to be eliminated. The pressure diffusion model did not produce any undesirable effects on near-wall properties. Current turbulent diffusion models with and without the new pressure diffusion model are unable to predict observed growth rates in shearless turbulence mixing layer.

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