Principal modes in multimode waveguides

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We generalize the concept of principal states of polarization and prove the existence of principal modes in multimode waveguides. Principal modes do not suffer from modal dispersion to first order of frequency variation and form orthogonal bases at both the input and the output ends of the waveguide. We show that principal modes are generally different from eigenmodes, even in uniform waveguides, unlike the special case of a single-mode fiber with uniform birefringence. The difference is most pronounced when different eigenmodes possess similar group velocities and when their field patterns vary as a function of frequency. This work may provide a new basis for analysis and control of dispersion in multimode fiber systems.

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Multimode fibers (MMFs) are widely used in local-area networks because of the ease of optical coupling and alignment and the low cost of related components.[1] The bandwidth–distance product of MMF links, however, is strongly limited by modal dispersion.[2] Hence it is essential to provide a rigorous and systematic analysis of modal dispersion in MMF.

Here we show that the concept of principal states of polarization (PSPs), originally developed to describe polarization-mode dispersion in single-mode fibers,[3,4] can be generalized to modal dispersion in MMFs. We prove the existence of principal modes in a MMF. These modes, like eigenmodes, are orthogonal at both input and output ends of the fiber but, unlike eigenmodes, do not suffer from modal dispersion to first order. Principal modes form a natural basis for a theoretical description of modal dispersion. Also, selective excitation or detection of principal modes may be an effective means to control modal dispersion or achieve spatial multiplexing in MMF systems.

We consider a narrowband optical signal, centered at a frequency ω, propagating in a fiber that supports N propagating modes. This fiber may be uniform along the axis of propagation (an ideal fiber) or may be nonuniform (a nonideal fiber). Suppose that a normalized electric field pattern $|a\rangle$ with an amplitude ϵ_a at the fiber input propagates to a field pattern $|b\rangle$ with an amplitude ϵ_b at the output. We represent this propagation by

$$\epsilon_b|b\rangle = T\epsilon_a|a\rangle\,, \tag{1}$$

where $T(\omega)$ is an $N \times N$ matrix representing propagation (both loss and phase change), as well as mode coupling. If we neglect mode-dependent loss, we can represent $T(\omega)$ as

$$T(\omega) = \exp[\phi(\omega)]U(\omega)\,, \tag{2}$$

where $\phi(\omega)$ is a scalar representing overall loss and phase change and $U(\omega)$ is an $N \times N$ unitary matrix representing lossless propagation and mode coupling.

The eigenmodes of the fiber are defined to be eigenmodes of the operator U; i.e., they are field patterns that, launched at the input, propagate to identical field patterns at the output, except for an overall phase shift. By analogy with the PSPs,[3] the principal modes are defined so that given an input principal mode $|a\rangle$, the corresponding output principal mode $|b\rangle$ is independent of frequency ω to first order. Consequently, a pulse-modulated optical signal transmitted with modal shape $|a\rangle$ retains its integrity and is received in modal shape $|b\rangle$. Motivated by such considerations, we take the derivative with respect to frequency ω on both sides of Eq. (1), with the assumptions that $\epsilon_a = 1$ and that $\partial|a\rangle/\partial\omega = 0$, to obtain

$$\frac{\partial\epsilon_b}{\partial\omega}|b\rangle + \epsilon_b\frac{\partial|b\rangle}{\partial\omega} = e^{\phi}\left(\frac{\partial\phi}{\partial\omega}U + \frac{\partial U}{\partial\omega}\right)|a\rangle \tag{3}$$

Requiring that $\partial|b\rangle/\partial\omega = 0$, and noting also from Eq. (1) that $|b\rangle = e^{\phi}U|a\rangle/\epsilon_b$, we have

$$F(\omega)|a\rangle = \tau|a\rangle\,, \tag{4}$$

where

$$F(\omega) \equiv -iU^{+}(\omega)\frac{\partial U(\omega)}{\partial\omega} \tag{5}$$

defines the group-delay operator, and

$$\tau = i\left(\frac{\partial\phi}{\partial\omega} - \frac{1}{\epsilon_b}\frac{\partial\epsilon_b}{\partial\omega}\right)$$

represents group delay.

The group delay operator F is Hermitian. This is evident by noting that, since U is unitary, $U^{+}U = 1$, and therefore $0 = (\partial U^{+}/\partial\omega)U + U^{+}(\partial U/\partial\omega)$, which proves $F^{+} = F$. Consequently all eigenvalues of the F operator are real, which is required for the group delay of a physical system. The set of eigenmodes $\{|a\rangle\}$ of the F operator, which is a set of the input principal modes, forms an orthogonal basis. The set of output principal modes $\{|b\rangle\} \equiv \{U|a\rangle\}$ also forms an orthogonal basis. Thus the input and output principal modes can be used to expand any input or output electromagnetic field pattern and provide a convenient basis for describing dispersion properties in multimode systems.

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The principal mode model, as represented by Eq. (4), is applicable to general cases of nonideal MMFs that are not uniform along the propagation direction, and in which strong modal mixing and coupling occurs. Building on the analogy between the PSPs and the principal modes defined here, one might be able to systematically generalize the statistical theory of polarization mode dispersion[5] to the more complex cases of MMFs. The principal mode model also suggests new approaches for modal dispersion control[6] and mode-division multiplexing in MMF systems.[7]

Despite the formal similarity between the PSPs and the principal modes, in practice, there are important differences between the two. One can model a single-mode fiber with polarization-mode dispersion by cascading multiple fiber segments, each segment having a uniform but differently oriented birefringence that is frequency independent. Within each uniform segment, the principal modes (the eigenmodes of the F operator describing the segment) are identical to the eigenmodes (the eigenmodes of the operator U describing the segment). In the general case, however, operators F and U do not commute ($[F, U] \neq 0$), and the principal modes differ from the eigenmodes, even for a fiber that is uniform along the propagation direction. It it thus of interest to relate the principal modes to the eigenmodes in the case of a general uniform system.

At a given frequency ω, an eigenmode $|n\rangle$ and its corresponding wave vector β_n can be obtained by solving an eigenvalue problem[8,9]:

$$\Theta(\omega)\,|n\rangle = \beta_n|n\rangle\,, \tag{6}$$

where the Θ matrix is calculated directly from the Maxwell equations. The propagation matrix U for a fiber that is uniform along the propagation direction is related to the Θ matrix by $U = \exp[i\Theta(\omega)z]$, where z is the length of the fiber. To determine the principal modes of such a uniform fiber in terms of the eigenmodes $\{|n\rangle\}$, we first note that $(\mathrm{d}U/\mathrm{d}\omega)\,|n\rangle = \mathrm{d}/\mathrm{d}\omega(U|n\rangle) - U(\mathrm{d}|n\rangle/\mathrm{d}\omega)$ and therefore from Eq. (5), in the basis of the eigenmodes $\{|n\rangle\}$, the F operator can be represented as

$$\begin{aligned}\langle m|F|n\rangle &= i\left(\langle m|\frac{\mathrm{d}}{\mathrm{d}\omega}|n\rangle - \langle m|U^{+}\frac{\mathrm{d}}{\mathrm{d}\omega}U|n\rangle\right)\\ &= z\,\frac{\mathrm{d}\beta_n}{\mathrm{d}\omega}\,\delta_{mn} + i\{1 - \exp[i(\beta_n - \beta_m)z\}\\ &\quad\times\langle m|\frac{\mathrm{d}}{\mathrm{d}\omega}|n\rangle\,. \end{aligned}\tag{7}$$

The first term in Eq. (7) is a diagonal element of the F matrix and is the usual group delay for individual eigenmodes. The second term represents an off-diagonal element and arises from the variations of the eigenmode field patterns as a function of frequency. To evaluate $\langle m|\,\mathrm{d}/\mathrm{d}\omega\,|n\rangle$, we take the derivative of Eq. (6) on both sides with respect to ω to obtain

$$\frac{\mathrm{d}\langle n|}{\mathrm{d}\omega} = -(\beta_n I - \Theta)^{-1}\left(\frac{\mathrm{d}\beta_n}{\mathrm{d}\omega}I - \frac{\mathrm{d}\Theta}{\mathrm{d}\omega}\right)\langle n|\,. \tag{8}$$

Inserting Eq. (8) into Eq. (7), and noticing that the second term in Eq. (7) is nonzero only for off-diagonal elements, we have

$$\begin{aligned}\langle m|F|n\rangle &= z\,\frac{\mathrm{d}\beta_n}{\mathrm{d}\omega}\,\delta_{mn} + i\left\{\frac{1 - \exp[i(\beta_n - \beta_m)z]}{\beta_n - \beta_m}\right\}\\ &\quad\times\langle m|\frac{\mathrm{d}\Theta}{\mathrm{d}\omega}|n\rangle(1 - \delta_{mn})\\ &= z\,\frac{\mathrm{d}\beta_n}{\mathrm{d}\omega}\,\delta_{mn} + z\,\mathrm{sinc}\left[\frac{(\beta_n - \beta_m)z}{2}\right]\\ &\quad\times\exp\left[i\,\frac{(\beta_n - \beta_m)z}{2}\right]\langle m|\frac{\mathrm{d}\Theta}{\mathrm{d}\omega}|n\rangle(1 - \delta_{mn})\,,\end{aligned}\tag{9}$$

where $\mathrm{sinc}(x) \equiv \sin(x)/x$. The off-diagonal terms become important when the differences between the group velocities of eigenmodes are sufficiently small (e.g., this may occur in graded-index fibers). In such cases there will be significant differences between the eigenmodes and the principal modes even for a uniform fiber. If the group-velocity differences are large, on the other hand, the off-diagonal terms can be safely ignored, and the principal modes can be well approximated by the eigenmodes.

As a simple example of a uniform multimode system where the off-diagonal terms in Eq. (9) dominate, we consider the coupling of two different single-mode waveguides in the vicinity of the phase-matching frequency ω_0, as shown in the inset of Fig. 1. The waveguides have group velocities ν_1 and ν_2. For such a system, the Θ matrix can be represented as[8]

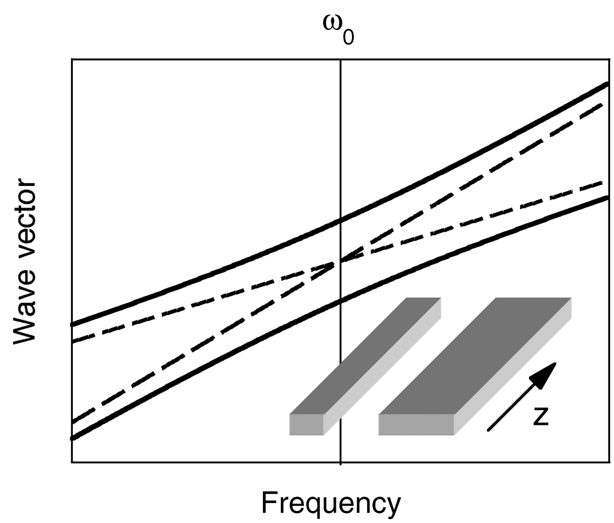

Fig. 1. Schematic of the dispersion relations for a multimode waveguide system in the vicinity of the phase-matching frequency. The system, shown in the inset consists of two single-mode waveguides coupled together. The dashed lines represent the dispersion relations of the individual waveguides. The solid curves represent the dispersion relations of the two eigenmodes of the coupled system. ω_0 is the frequency at which phase-matched coupling between the two waveguides occurs.

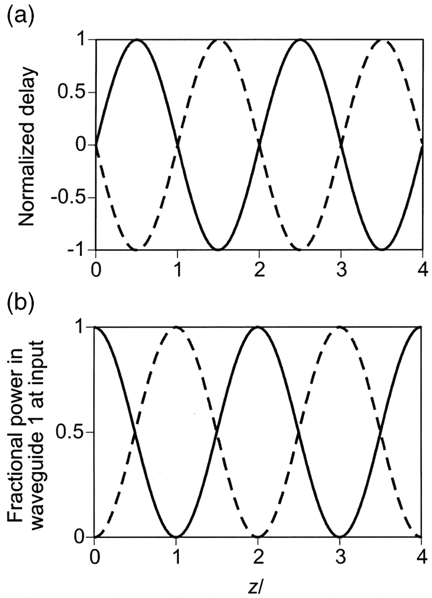

Fig. 2. Properties of the two principal modes, represented as solid and dashed curves, as a function of propagation distance z, for the system shown in Fig. 1. κ is the coupling constant between the waveguides. (a) Normalized group delay. The vertical axis corresponds to $(\tau/2\kappa)(1/\nu_1 - 1/\nu_2)$, where τ is the group delay and ν_1 and ν_2 are the group velocities of the individual waveguides. (b) Fraction of the total optical power that is localized in waveguide 1 at the input. Notice that the principal modes vary as a function of propagation distance, unlike eigenmodes in this uniform system.

$$\Theta = \begin{bmatrix} \beta_0 + \delta & \kappa \\ \kappa & \beta_0 - \delta \end{bmatrix}, \tag{10}$$

where κ is the coupling constant. The parameters

$$\beta_0 \equiv \frac{1}{2}\left(\frac{\omega - \omega_0}{\nu_1} + \frac{\omega - \omega_0}{\nu_2}\right),$$

$$\delta \equiv \frac{1}{2}\left(\frac{\omega - \omega_0}{\nu_1} - \frac{\omega - \omega_0}{\nu_2}\right)$$

represent the average of and difference between the wave vectors of the two waveguides. The eigenmodes of the coupled system have wave vectors $\beta_{1,2} = \beta_0 \pm \sqrt{\delta^2 + \kappa^2}$, which are shown as solid curves in Fig. 1. At the phase-matching frequency $\omega = \omega_0$, $\delta = 0$, the eigenmodes become

$$|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{11}$$

The eigenmodes are independent of propagation distance z and have wave vectors $\beta_{1,2} = \beta_0 \pm \kappa$. The group velocities of the eigenmodes are identical. To determine the principal modes of this system, noting that

$$\frac{\mathrm{d}\Theta}{\mathrm{d}\omega} = \frac{1}{2}\left(\frac{1}{\nu_1} - \frac{1}{\nu_2}\right)\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{12}$$

and inserting Eqs. (11) and (12) into Eq. (9), we have

$$F = \frac{z}{2}\left(\frac{1}{\nu_1} - \frac{1}{\nu_2}\right)\mathrm{sinc}(\kappa z)\begin{bmatrix} \cos(\kappa z) & i\sin(\kappa z) \\ -i\sin(\kappa z) & -\cos(\kappa z) \end{bmatrix}. \tag{13}$$

Diagonalizing F, we find that the principal modes have group delays (after subtracting the average) given by

$$\tau_{1,2} = \pm\frac{z}{2}\left(\frac{1}{\nu_1} - \frac{1}{\nu_2}\right)\mathrm{sinc}(\kappa z)$$
$$= \pm\frac{1}{2\kappa}\left(\frac{1}{\nu_1} - \frac{1}{\nu_2}\right)\sin(\kappa z)\cdot \tag{14}$$

At the input, the principal modes are represented by

$$|p_1\rangle = \begin{bmatrix} \cos\left(\frac{\kappa z}{2}\right) \\ -i\sin\left(\frac{\kappa z}{2}\right) \end{bmatrix}, \qquad |p_2\rangle = \begin{bmatrix} \sin\left(\frac{\kappa z}{2}\right) \\ i\cos\left(\frac{\kappa z}{2}\right) \end{bmatrix}. \tag{15}$$

Plots of the group delays for the two principal modes (after subtracting the average) and the fractional powers of the two principal modes in the first waveguide are shown in Fig. 2. It is evident that in this system the principal modes are dependent on the propagation distance and are thus markedly different from the eigenmodes of this uniform system.

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