Mode-Coupling Analysis of Multipole Symmetric Resonant Add/Drop Filters

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***Abstract*— Time-dependent mode-coupling theory is used to analyze a type of resonant add/drop filter based on the excitation of degenerate symmetric and antisymmetric modes. Flat-top transfer functions are achieved with higher order filters that utilize multiple resonator pairs, designed to satisfy the degeneracy conditions. The resulting analytic expressions lead to an equivalent circuit and the transfer characteristics of the filter are related to standard L–C circuit designs.**

I. Introduction

ADD/DROP filters that access one channel of a wavelength-division-multiplexed (WDM) system without disturbing other channels are very important components for WDM communications. Filters based on resonators side coupled to waveguides have been considered for this application [1]–[4]. Among their advantages is their small size and the fact that a number of resonators can be combined to synthesize desirable higher order filter responses within a small area.

A type of channel dropping filter based on the excitation of degenerate symmetric and antisymmetric resonant modes has been first described in [3], [5], and [6]. The analysis has been recast into coupling of modes in time in [4] and the conditions for degeneracy for a system employing a pair of identical single mode standing wave resonators were derived. The response of these filters was shown to be Lorentzian (single pole). In order to achieve improved transfer characteristics such as low crosstalk from other channels and flattened resonance peaks, higher order filters are needed. Here, the coupling of modes in time is simply extended to the case of higher order filters consisting of multiple resonator pairs. The resonators are treated as lumped elements and the resulting continued fraction expressions provide a one-to-one correspondence with standard L–C filter design. An equivalent circuit is thus derived and a rough layout of the structure is based on handbook filter designs of circuit theory. A similar approach has already been used for cascaded resonators in [2] and [7].

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II. nth–Order Filter

In the case of a symmetric system consisting of two identical single-mode resonators placed between two waveguides as shown in the schematic of Fig. 1(a), the symmetric and antisymmetric modes of the structure are degenerate if the coupling of the two resonators via the signal waveguides exactly balances their direct coupling. The mode-coupling analysis of [4] shows that the two conditions to be satisfied are that the waveguide sections between the two resonator reference planes must be an odd multiple of a quarter guided wavelength and that the sum of the inverse decay rates into the two waveguides must be equal to the direct coupling coefficient. An example of the filter response when the degeneracy has been achieved and losses are neglected is shown in Fig. 1(b). Assuming that the phase planes defining the incoming and outgoing waves have been appropriately chosen and that the resonators couple equally to the two waveguides, the response at the drop port (in this case, port 4 with input from port 1) was found as

$$\frac{s_{-4}}{s_{+1}} \equiv D_R = \frac{\dfrac{2}{\tau_e}}{j(\omega - \omega_o) + \dfrac{2}{\tau_e}} \tag{1}$$

where ω_o is the degenerate frequency and τ_e is the decay rate, associated with power lost to either guide, of the symmetric and antisymmetric modes. Fig. 1(b) also shows the response at the remaining ports of the device where R, T, and D_L are defined as

$$D_L \equiv \frac{s_{-3}}{s_{+1}} \tag{2}$$

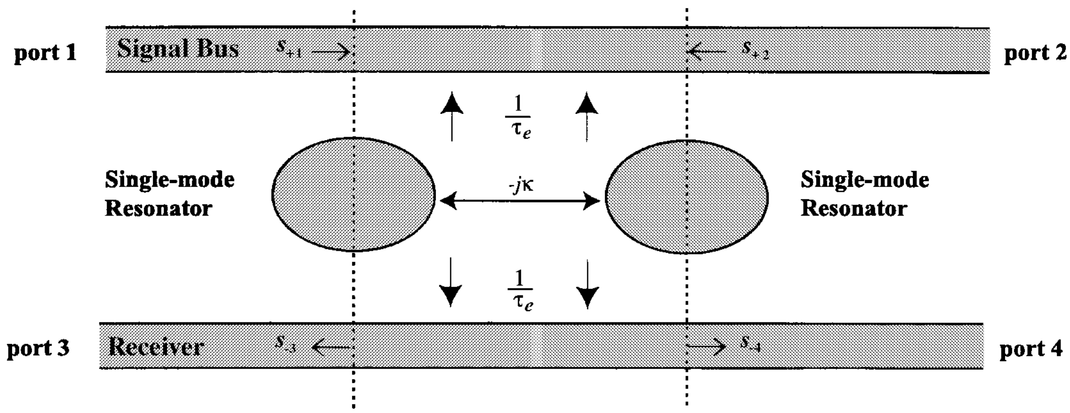
$$R \equiv \frac{s_{-1}}{s_{+1}} \tag{3}$$

and

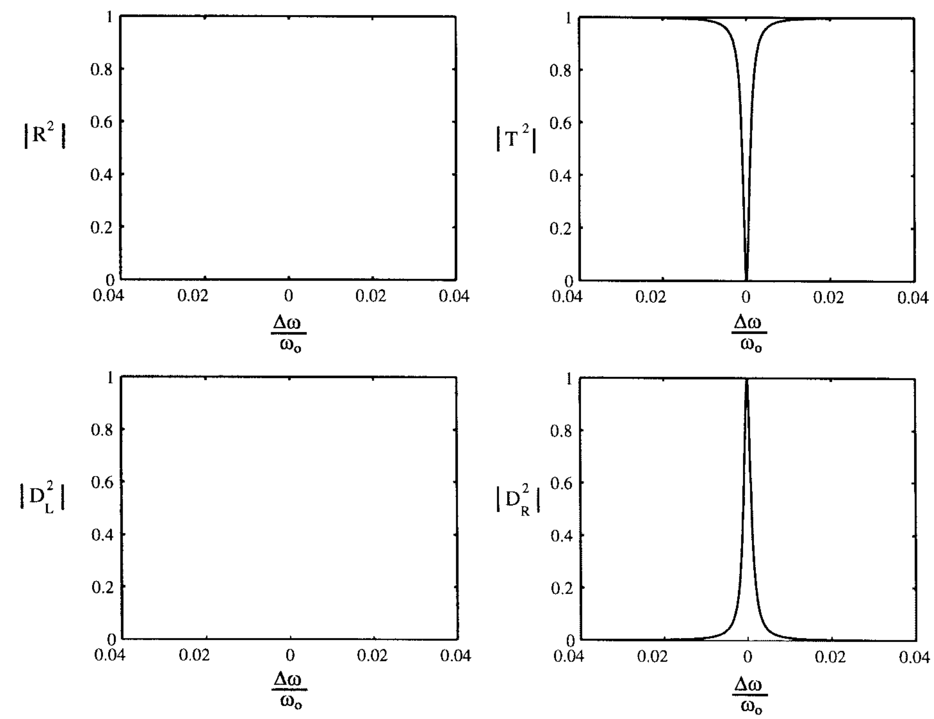
$$T \equiv \frac{s_{-2}}{s_{+1}}. \tag{4}$$

The spectral response of the dropped signal at port 4 is a Lorentzian. At high frequencies, the roll-off is at best 20 dB/decade, which may not be fast enough to meet the crosstalk specifications for adjacent channels. In this case, higher order or multipole filters are needed.

A higher order filter is made by generalizing the scheme described above. Instead of using one pair of resonators placed between two waveguides, n pairs of resonators are evanescently coupled to each other, as shown in Fig. 2. The resulting system behaves as an nth-order filter that is capable of completely transferring the input power from the bus to the receiver waveguide. The nth-order filter can be described in terms of coupled symmetric and antisymmetric modes of the



(a)



(b)

Fig. 1. (a) Symmetric add/drop filter based on two coupled identical single-mode resonators. (b) Filter response when all degeneracy conditions are satisfied, obtained by coupling of modes in time.

pairs of resonators

$$\frac{da_{1s}}{dt} = \left(j\omega_{1s} - \frac{1}{\tau_{es}}\right)a_{1s} + \kappa_s(s_{+1} + s_{+2}) - j\mu_{1s}a_{2s} \tag{5}$$

$$\frac{da_{2s}}{dt} = j\omega_{2s}a_{2s} - j\mu_{1s}a_{1s} - j\mu_{2s}a_{3s} \tag{6}$$

$$\vdots$$

$$\frac{da_{(n-1)s}}{dt} = j\omega_{(n-1)s}a_{(n-1)s} - j\mu_{(n-2)s}a_{(n-2)s} - j\mu_{(n-1)s}a_{ns} \tag{7}$$

$$\frac{da_{ns}}{dt} = \left(j\omega_{ns} - \frac{1}{\tau'_{es}}\right)a_{ns} - j\mu_{(n-1)s}a_{(n-1)s} + \kappa'_s(s_{+3} + s_{+4}) \tag{8}$$

$$\frac{da_{1a}}{dt} = \left(j\omega_{1a} - \frac{1}{\tau_{ea}}\right)a_{1a} + \kappa_a(s_{+1} - s_{+2}) - j\mu_{1a}a_{2a} \tag{9}$$

$$\frac{da_{2a}}{dt} = j\omega_{2a}a_{2a} - j\mu_{1a}a_{1a} - j\mu_{2a}a_{3a} \tag{10}$$

$$\vdots$$

$$\frac{da_{(n-1)a}}{dt} = j\omega_{(n-1)a}a_{(n-1)a} - j\mu_{(n-2)a}a_{(n-2)a} - j\mu_{(n-1)a}a_{na} \tag{11}$$

$$\frac{da_{na}}{dt} = \left(j\omega_{na} - \frac{1}{\tau'_{ea}}\right)a_{na} - j\mu_{(n-1)a}a_{(n-1)a} + \kappa'_a(s_{+3} - s_{+4}) \tag{12}$$

where a_{is} and a_{ia} represent the symmetric and antisymmetric mode amplitude of the ith resonator pair normalized

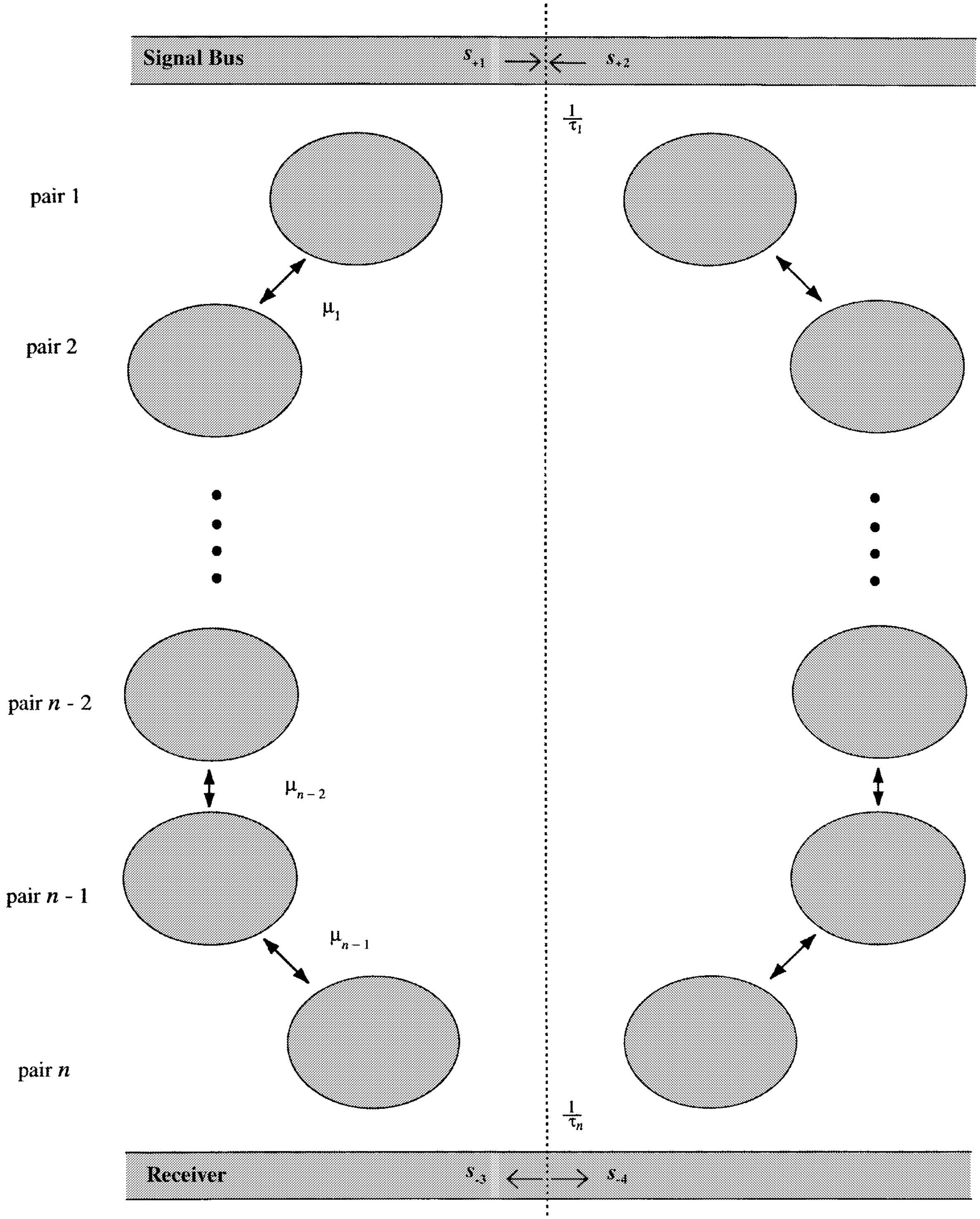


Fig. 2. Schematic of an nth-order filter made of n pairs of coupled resonators.

to the energy in the mode, respectively. Likewise, ω_{is} and ω_{ia} represent the resonance frequency of the ith resonator pair, $1/\tau_{es,\,a} 1/\tau'_{es,\,a}$ are the decay rates associated with the power lost by the symmetric or antisymmetric modes to the waveguides adjacent to the first and last pair of resonators, and $\mu_{is,\,a}$ is the coupling between the symmetric modes and the antisymmetric modes, respectively, of the ith and $(i\text{th}+1)$ resonator pair, real by power conservation. We note that the symmetric modes do not couple to the antisymmetric modes and vice versa. The coefficients $\kappa_{s,\,a}$ and $\kappa'_{s,\,a}$ associated with the coupling to the bus and the receiver, respectively, are found by power conservation to have the form

$$\kappa_{s,\,a} = \sqrt{\frac{1}{\tau_{es,\,a}}}\, e^{j\theta_{s,\,a}} \qquad \kappa'_{s,\,a} = \sqrt{\frac{1}{\tau'_{es,\,a}}}\, e^{j\theta'_{s,\,a}}. \tag{13}$$

The outgoing waves are described by

$$s_{-1} = s_{+2} - \kappa_s^* a_{1s} + \kappa_a^* a_{1a} \tag{14}$$

$$s_{-2} = s_{+1} - \kappa_s^* a_{1s} - \kappa_a^* a_{1a} \tag{15}$$

$$s_{-3} = s_{+4} - {\kappa'}_s^* a_{ns} + {\kappa'}_a^* a_{na} \tag{16}$$

$$s_{-4} = s_{+3} - {\kappa'}_s^* a_{ns} - {\kappa'}_a^* a_{na}. \tag{17}$$

In the following analysis, we will assume for simplicity that the coupling coefficients are real. This can be accomplished by the proper choice of the reference planes. A detailed analysis on how the phase of the coupling coefficients affects the filter response can be found in [4].

For the case in which the system is excited only from one side of the bus, i.e., $s_{+2} = s_{+3} = s_{+4} = 0$ and s_{+1} has a $e^{j\omega t}$ time dependence, we can find the mode amplitudes $a_{is,a}$ using the systems of (5)–(8) and (9)–(12) as shown in (18)–(21), at the bottom of the page, where D_{is} is defined as the denominator associated with the expression for a_{is}. Note that the different D_{is} are continued fractions of different order. Identical expressions exist for the antisymmetric mode amplitudes a_{ia}, where the subscript s is simply replaced by the subscript a, everything else remaining the same. Using (18)–(21) and (13) in (14) and (15), we get the reflected and transmitted waves on the bus guide

$$\frac{s_{-2}}{s_{+1}} = 1 - \frac{1}{D_{1s}\tau_{es}} - \frac{1}{D_{1a}\tau_{ea}} \tag{22}$$

$$\frac{s_{-1}}{s_{+1}} = -\frac{1}{D_{1s}\tau_{es}} + \frac{1}{D_{1a}\tau_{ea}}. \tag{23}$$

Solving for a_{ns} and a_{na}, we find

$$a_{ns} = \left(\frac{-j\mu_{(n-1)s}}{D_{ns}}\right)\left(\frac{-j\mu_{(n-2)s}}{D_{(n-1)s}}\right)\left(\frac{-j\mu_{(n-3)s}}{D_{(n-2)s}}\right)\cdots \left(\frac{-j\mu_{1s}}{D_{2s}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{es}}}s_{+1}}{D_{1s}}\right) \tag{24}$$

$$a_{na} = \left(\frac{-j\mu_{(n-1)a}}{D_{na}}\right)\left(\frac{-j\mu_{(n-2)a}}{D_{(n-1)a}}\right)\left(\frac{-j\mu_{(n-3)a}}{D_{(n-2)a}}\right)\cdots \left(\frac{-j\mu_{1a}}{D_{2a}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{ea}}}s_{+1}}{D_{1a}}\right). \tag{25}$$

Use of the above yields the following response at the remaining output ports:

$$\frac{s_{-3}}{s_{+1}} = -\left(\frac{-j\mu_{(n-1)s}}{D_{ns}}\right)\left(\frac{-j\mu_{(n-2)s}}{D_{(n-1)s}}\right)\cdots \left(\frac{-j\mu_{1s}}{D_{2s}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{es}\tau'_{es}}}}{D_{1s}}\right) + \left(\frac{-j\mu_{(n-1)a}}{D_{na}}\right)\left(\frac{-j\mu_{(n-2)a}}{D_{(n-1)a}}\right)\cdots \left(\frac{-j\mu_{1a}}{D_{2a}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{ea}\tau'_{ea}}}}{D_{1a}}\right) \tag{26}$$

$$a_{ns} = \frac{-j\mu_{(n-1)s}a_{(n-1)s}}{j(\omega-\omega_{ns}) + \frac{1}{\tau'_{es}}} \equiv \frac{-j\mu_{(n-1)s}a_{(n-1)s}}{D_{ns}} \tag{18}$$

$$a_{(n-1)s} = \frac{-j\mu_{(n-2)s}a_{(n-2)s}}{j(\omega-\omega_{(n-1)s}) + \frac{\mu_{(n-1)s}^2}{j(\omega-\omega_{ns}) + \frac{1}{\tau'_{es}}}} \equiv \frac{-j\mu_{(n-2)s}a_{(n-2)s}}{D_{(n-1)s}} \tag{19}$$

$$a_{(n-2)s} = \frac{-j\mu_{(n-3)s}\,a_{(n-3)s}}{j(\omega-\omega_{(n-2)s}) + \frac{\mu_{(n-2)s}^2}{j(\omega-\omega_{(n-1)s}) + \frac{\mu_{(n-1)s}^2}{j(\omega-\omega_{ns}) + \frac{1}{\tau'_{es}}}}} \equiv \frac{-j\mu_{(n-3)s}a_{(n-3)s}}{D_{(n-2)s}} \tag{20}$$

$$\vdots$$

$$a_{1s} = \frac{\kappa_s s_{+1}}{j(\omega-\omega_{1s}) + \frac{1}{\tau_{es}} + \frac{\mu_{1s}^2}{j(\omega-\omega_{2s}) + \frac{\mu_{2s}^2}{j(\omega-\omega_{3s}) + \cdots \frac{\mu_{(n-1)s}^2}{j(\omega-\omega_{ns}) + \frac{1}{\tau'_{es}}}}}} \equiv \frac{\kappa_s s_{+1}}{D_{1s}} \tag{21}$$

and

$$\frac{s_{-4}}{s_{+1}} = -\left(\frac{-j\mu_{(n-1)s}}{D_{ns}}\right)\left(\frac{-j\mu_{(n-2)s}}{D_{(n-1)s}}\right)\cdots \left(\frac{-j\mu_{1s}}{D_{2s}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{es}\tau'_{es}}}}{D_{1s}}\right) - \left(\frac{-j\mu_{(n-1)a}}{D_{na}}\right)\left(\frac{-j\mu_{(n-2)a}}{D_{(n-1)a}}\right)\cdots \left(\frac{-j\mu_{1a}}{D_{2a}}\right)\left(\frac{\sqrt{\frac{1}{\tau_{ea}\tau'_{ea}}}}{D_{1a}}\right). \tag{27}$$

We consider the case where the resonators are designed such that the symmetric and antisymmetric modes are all degenerate at frequency ω_o, i.e.,

$$\omega_{is} = \omega_{ia} \equiv \omega_o, \qquad i = 1, 2, \cdots, n$$

and the decay rates of the symmetric and antisymmetric modes of the first and last pair of resonators are the same, i.e.,

$$\tau_{es} = \tau_{ea} \equiv \tau_e$$

and

$$\tau'_{es} = \tau'_{ea} \equiv \tau'_e.$$

The degeneracy condition for a pair of resonators adjacent to a waveguide can be satisfied by balancing the direct coupling between the resonators with the indirect coupling via the waveguide and by choosing the distance between the resonators to be an odd multiple of a quarter guided wavelength. For the resonator pairs that are not next to a waveguide (i.e., $i = 2, 3, \cdots, n-1$), the degeneracy can be achieved by placing the two resonators of each pair sufficiently far apart so that they are essentially uncoupled. The coupling between the symmetric modes and antisymmetric modes of adjacent pairs of resonators can also be made equal, i.e.,

$$\mu_{is} = \mu_{ia} \equiv \mu_i, \qquad i = 1, 2, \cdots, n.$$

This is possible if there is no cross coupling between resonators on either side of the symmetry plane that belong to different pairs. The above relationships imply that

$$D_{is} = D_{ia} \equiv D_i, \qquad i = 1, 2, \cdots, n.$$

For this highly degenerate case, it is obvious, using (22) and (26), that the signal reflected on the bus and that dropped in port 3 of the receiver guide are identically zero over the entire bandwidth of the resonance, i.e.,

$$s_{-1} = s_{-3} = 0. \tag{28}$$

Also,

$$\frac{s_{-2}}{s_{+1}} = 1 - \frac{2}{D_1\tau_e} \tag{29}$$

and

$$\frac{s_{-4}}{s_{+1}} = -2\left(\frac{-j\mu_{n-1}}{D_n}\right)\left(\frac{-j\mu_{n-2}}{D_{n-1}}\right)\cdots \left(\frac{-j\mu_1}{D_2}\right)\left(\frac{\sqrt{\frac{1}{\tau_e\tau'_e}}}{D_1}\right). \tag{30}$$

We note that the leading frequency term in the product $D_nD_{n-1}D_{n-2}\cdots D_1$ is $(-j)^n(\omega-\omega_o)^n$. Thus, for high frequencies, the magnitude of s_{-4} rolls off approximately as

$$\left|\frac{s_{-4}}{s_{+1}}\right| \approx \frac{\mu_1\mu_2\cdots\mu_{n-1}\sqrt{1/\tau_e\tau'_e}}{(\omega-\omega_o)^n}$$

as expected for an nth-order filter. It is possible to design the system to transfer the signal completely to the receiver guide on resonance. Moreover, it is possible to shape the frequency response of higher order filters. In this case, the spectral response can be engineered by choosing the appropriate coupling between adjacent resonators and the decay rates of the pairs of resonators next to the bus and receiver guides. In general, the selection of the appropriate amount of couplings between resonators for a higher order filter, with $n > 2$, to achieve a desired spectral response is a tedious and nontrivial task and becomes increasingly harder as the order increases. If we are somehow able to map the coupled resonator system to a standard circuit used for implementing higher order filters, this task is reduced to looking up tabulated values of impedances to figure out the appropriate optical couplings and decay rates.

III. Equivalent Circuit

An equivalent circuit attempting to model the behavior correctly at all four ports of the coupled resonator system must be a four-port device. Such a circuit description would be difficult to work with. Instead, we concentrate on the port of primary interest, namely the receiver port s_{-4} and derive a partial "equivalent" circuit which models the behavior of this port correctly. We are justified in following this approach as, in the degenerate case which is the case of interest, we already know the response at two ports, s_{-1} and s_{-3} to be identically zero over the bandwidth of interest and are really only interested in engineering the spectral response of s_{-4}.

The purpose of deriving the equivalent circuit of the stacked resonator system is to facilitate filter design by utilizing the extensive work already done on L–C ladder circuits [8]–[10]. Consider the ladder circuit shown in Fig. 3, consisting of alternating sections of series and parallel L–C circuits. This is a standard circuit used for designing higher order filters. Z_1 is the impedance of the circuit looking into the ladder and Y_2 is the admittance of the ladder circuit beyond the first series L–C subcircuit. Likewise, Z_3 is the impedance looking beyond the first parallel L–C subcircuit. In a similar fashion, we define additional impedances and admittances, Z_i and Y_i with diminishing number of elements in them. The choice of this notation will become clear shortly. Near resonance, we obtain (31)–(34), shown at the bottom of the next page. For definiteness, we have assumed that the order of the filter n is an even number. We would follow similar procedures in the

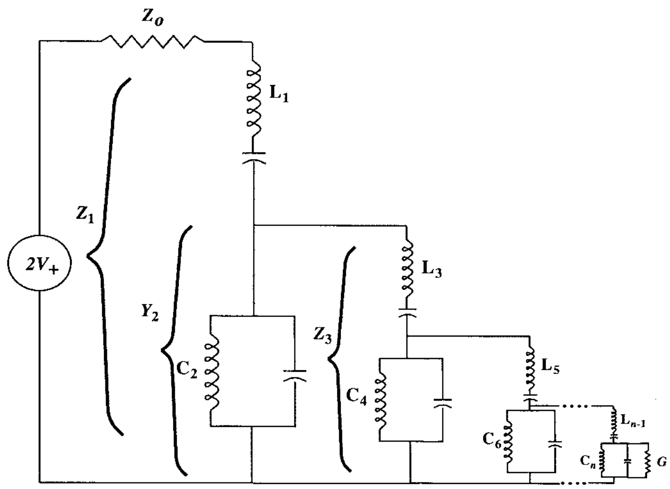


Fig. 3. Postulated equivalent circuit for an nth-order filter made of n pairs of coupled resonators.

case of odd n. Moreover, the impedances and admittances are expanded near the resonance frequencies of the L–C circuits which are assumed to be equal, i.e., $L_iC_i = \omega_o^2$ for all i. The power P_d dissipated in the conductance G of the last parallel L–C branch is given by

$$P_d = \frac{1}{2}\Re\{V_d I_d\}$$

where V_d and I_d are the voltage and current, respectively, across the conductance G. V_d may be found using the voltage divider relationship repeatedly. We find

$$V_d = 2V_+\left(\frac{Z_1}{Z_o+Z_1}\right)\left(\frac{\frac{1}{Y_2}}{Z_1}\right)\left(\frac{\frac{1}{Y_4}}{Z_3}\right)\cdots\left(\frac{\frac{1}{Y_n}}{Z_{n-1}}\right) \tag{35}$$

and

$$I_d = V_d G. \tag{36}$$

Hence,

$$P_d = \frac{2|V_+|^2 G}{|(Z_o+Z_1)Y_2Z_3Y_4\cdots Y_n|^2} \tag{37}$$

which can be normalized to

$$P_d = \frac{(GZ_o)}{\left|\left(1+\frac{Z_1}{Z_o}\right)(Y_2Z_o)\left(\frac{Z_3}{Z_o}\right)(Y_4Z_o)\cdots(Y_nZ_o)\right|^2} \cdot 2|V_+|^2 Y_o \tag{38}$$

so that each factor in the fraction is dimensionless. The power captured by the receiver guide is given by

$$|s_{-4}|^2 = \frac{\frac{1}{\tau_e\tau_e'}\,\mu_1^2\mu_2^2\mu_3^2\cdots\mu_{n-2}^2\mu_{n-1}^2}{|D_1D_2D_3\cdots D_{n-1}D_n|^2}\,|s_{+1}|^2 \tag{39}$$

which can be rewritten as (40), shown at the bottom of the page, again so that each factor in the fraction is dimensionless. Comparing the above expressions for P_d and $|s_{-4}|^2$, we see that they are similar in form provided we draw the following correspondence:

$$1+\frac{Z_1}{Z_o} \longleftrightarrow D_1\tau_e \tag{41}$$

$$Y_2Z_o \longleftrightarrow D_2\,\frac{1}{\mu_1^2\tau_e} \tag{42}$$

$$Z_1 \approx 2j\Delta\omega L_1 + \cfrac{1}{2j\Delta\omega C_2 + \cfrac{1}{2j\Delta\omega L_3 + \cdots + \cfrac{1}{2j\Delta\omega L_{n-1} + \cfrac{1}{2j\Delta\omega C_n + G}}}} \tag{31}$$

$$Y_2 \approx 2j\Delta\omega C_2 + \cfrac{1}{2j\Delta\omega L_3 + \cdots + \cfrac{1}{2j\Delta\omega L_{n-1} + \cfrac{1}{2j\Delta\omega C_n + G}}} \tag{32}$$

$$Z_3 \approx 2j\Delta\omega L_3 + \cfrac{1}{2j\Delta\omega C_4 + \cdots + \cfrac{1}{2j\Delta\omega L_{n-1} + \cfrac{1}{2j\Delta\omega C_n + G}}} \tag{33}$$

$$\vdots$$

$$Y_n \approx 2j\Delta\omega C_n + G \tag{34}$$

$$|s_{-4}|^2 = \frac{\frac{\mu_2^2}{\mu_1^2}\,\frac{\mu_4^2}{\mu_3^2}\cdots\frac{1}{\mu_{n-1}^2}\,\frac{1}{\tau_e\tau_e'}}{\left|(D_1\tau_e)\left(D_2\,\frac{1}{\mu_1^2\tau_e}\right)\left(D_3\,\frac{\mu_1^2\tau_e}{\mu_2^2}\right)\left(D_4\,\frac{\mu_2^2}{\mu_1^2\mu_3^2\tau_e}\right)\cdots\left(D_n\,\frac{\mu_2^2}{\mu_1^2}\,\frac{\mu_4^2}{\mu_3^2}\cdots\frac{1}{\mu_{n-1}^2}\,\frac{1}{\tau_e}\right)\right|^2}\,|s_{+1}|^2 \tag{40}$$

$$\frac{Z_3}{Z_o} \longleftrightarrow D_3 \frac{\mu_1^2 \tau_e}{\mu_2^2} \tag{43}$$

$$\vdots$$

$$Y_n Z_o \longleftrightarrow D_n \frac{\mu_2^2}{\mu_1^2} \frac{\mu_4^2}{\mu_3^2} \cdots \frac{1}{\mu_{n-1}^2} \frac{1}{\tau_e \tau_e'} \tag{44}$$

$$G Z_o \longleftrightarrow \frac{\mu_2^2}{\mu_1^2} \frac{\mu_4^2}{\mu_3^2} \cdots \frac{1}{\mu_{n-1}^2} \frac{1}{\tau_e \tau_e'}. \tag{45}$$

It appears that there are too many constraints present for a mapping between P_d and $|s_{-4}|^2$ to exist, but we will see that the mapping enforced by the first equation encompasses the others and the remaining equations are redundant. This is obvious if we consider the special relationship that exists between D_i and D_{i+1} and between Z_i and Y_{i+1}. Specifically, we note that

$$D_1 = j(\omega - \omega_o) + \frac{1}{\tau_e} + \frac{\mu_1^2}{D_2} \tag{46}$$

$$Z_1 = 2j(\omega - \omega_o) L_1 + \frac{1}{Y_2} \tag{47}$$

as is obvious from (30)–(32). It follows that

$$D_1 \tau_e = 1 + j(\omega - \omega_o)\tau_e + \cfrac{1}{D_2 \cfrac{1}{\mu_1^2 \tau_e}} \tag{48}$$

$$1 + \frac{Z_1}{Z_o} = 1 + j(\omega - \omega_o) \frac{2L_1}{Z_o} + \frac{1}{Y_2 Z_o}. \tag{49}$$

Thus, the correspondence in (42) is satisfied

$$Y_2 Z_o \longleftrightarrow D_2 \frac{1}{\mu_1^2 \tau_e}.$$

Similar reasoning can be used to show that the correspondence expressed by (43) is contained in (42). By extension, it follows that all other correspondences are contained in (42). For an equivalence to exist, the form of $D_1 \tau_e$ must be the same as that of $1 + Z_1/Z_o$. We note that (50), shown at the bottom of the page, has a continued fraction form identical to (51), shown at the bottom of the page. In fact, there is a term-by-term correspondence between the two expressions which provides a mapping between the circuit and the optical resonator parameters. This proves that the postulated circuit of Fig. 3 is indeed the equivalent circuit representation of the receiver port of the coupled resonator system. Consider (40); on resonance, $\omega = \omega_o$, it is obvious that

$$|s_{-4}|^2 = \frac{\mu_2^2}{\mu_1^2} \frac{\mu_4^2}{\mu_3^2} \cdots \frac{1}{\mu_{n-1}^2} \frac{1}{\tau_e \tau_e'} |s_{+1}|^2.$$

Complete power transfer is then possible on resonance if

$$\frac{\mu_2^2}{\mu_1^2} \frac{\mu_4^2}{\mu_3^2} \cdots \frac{1}{\mu_{n-1}^2} \frac{1}{\tau_e \tau_e'} = 1 \tag{52}$$

or equivalently if we use the correspondence implied by (45) when $G = Y_o$. This should be obvious if we consider the equivalent circuit. On resonance, the series L–C branches are shorted and the parallel L–C branches are open. The load G is directly connected to the source and perfect transfer is only possible for a matched load. For odd n, the condition for complete power transfer on resonance would be

$$\frac{\mu_2^2}{\mu_1^2} \frac{\mu_4^2}{\mu_3^2} \cdots \frac{\mu_{n-1}^2}{\mu_{n-2}^2} \frac{\tau_e'}{\tau_e} = 1. \tag{53}$$

In the following section, we will design a fourth-order Butterworth filter using mappings provided by the equivalent circuit.

IV. Example: Fourth-Order Filter

As an example of a higher order filter, we consider the coupled resonator system shown in Fig. 4 consisting of four pairs of resonators side-coupled to their nearest neighbors. All the pairs are designed so that their respective symmetric and antisymmetric modes are degenerate at frequency ω_o, and the decay rates τ_e and coupling coefficients μ_i are assumed to be the same for the symmetric and antisymmetric modes, i.e.,

$$\omega_{is,a} \equiv \omega_o$$
$$\tau_{es,a} \equiv \tau_e$$
$$\tau_{es,a}' \equiv \tau_e'$$
$$\mu_{is,a} \equiv \mu_i.$$

For $n = 4$, (50) and (51) yield (54) and (55), shown at the bottom of the next page.

Using the correspondence given by (42), we find the following mapping between the circuit parameters and the optical

$$1 + \frac{Z_1}{Z_o} = 1 + j\Delta\omega \frac{2L_1}{Z_o} + \cfrac{1}{j\Delta\omega 2C_2 Z_o + \cfrac{1}{j\Delta\omega \cfrac{2L_3}{Z_o} + \cdots + \cfrac{1}{j\Delta\omega \cfrac{2L_{n-1}}{Z_o} + \cfrac{1}{2j\Delta\omega C_n Z_o + G Z_o}}}} \tag{50}$$

$$D_1 \tau_e = 1 + j(\omega - \omega_o)\tau_e + \cfrac{1}{\cfrac{j(\omega - \omega_o)}{\mu_1^2 \tau_e} + \cfrac{1}{j(\omega - \omega_o) \cfrac{\mu_1^2 \tau_e}{\mu_2^2} + \cdots \cfrac{1}{\left(j(\omega - \omega_o) + \cfrac{1}{\tau_e'}\right)\left[\cfrac{1}{\mu_{n-1}^2} \cdots \cfrac{\mu_4^2}{\mu_3^2} \cfrac{\mu_2^2}{\mu_1^2} \cfrac{1}{\tau_e}\right]}}} \tag{51}$$

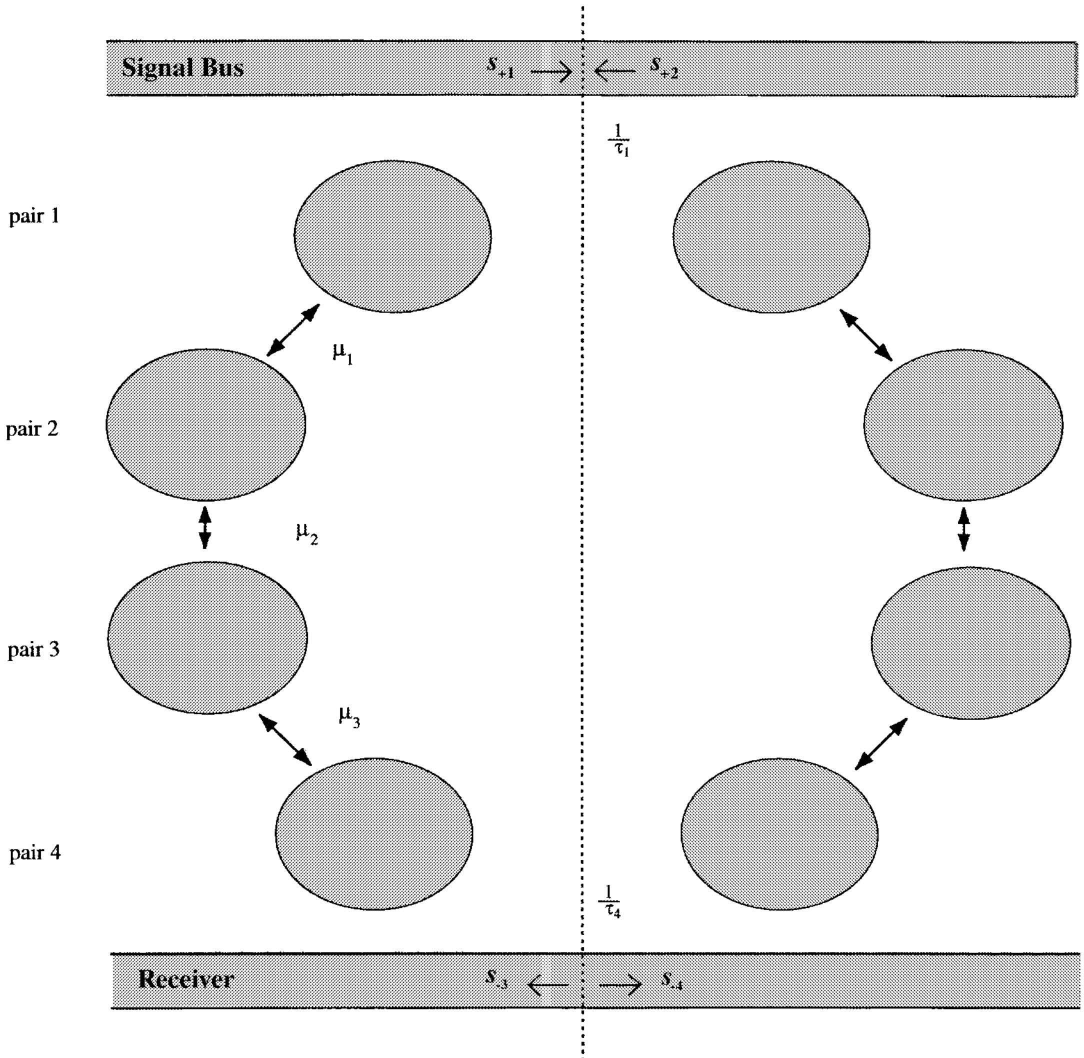


Fig. 4. Fourth-order filter.

parameters:

$$\tau_e \longleftrightarrow \frac{2L_1}{Z_o} \tag{56}$$

$$\frac{1}{\mu_1^2 \tau_e} \longleftrightarrow 2C_2 Z_o \tag{57}$$

$$\frac{\mu_1^2 \tau_e}{\mu_2^2} \longleftrightarrow \frac{2L_3}{Z_o} \tag{58}$$

$$\frac{\mu_2^2}{\mu_1^2 \mu_3^2} \frac{1}{\tau_e} \longleftrightarrow 2C_4 Z_o \tag{59}$$

$$\frac{\mu_2^2}{\mu_1^2 \mu_3^2} \frac{1}{\tau_e \tau_e'} \longleftrightarrow G Z_o. \tag{60}$$

These mappings can be rewritten as

$$\tau_e \longleftrightarrow \frac{2L_1}{Z_o} \tag{61}$$

$$D_1 \tau_e = 1 + j(\omega - \omega_o)\tau_e + \cfrac{1}{j(\omega - \omega_o)\cfrac{1}{\mu_1^2 \tau_e} + \cfrac{1}{j(\omega - \omega_o)\cfrac{\mu_1^2 \tau_e}{\mu_2^2} + \cfrac{1}{\left(j(\omega - \omega_o) + \cfrac{1}{\tau_e'}\right)\left[\cfrac{1}{\mu_3^2}\cfrac{\mu_2^2}{\mu_1^2}\cfrac{1}{\tau_e}\right]}}} \tag{54}$$

$$1 + \frac{Z_1}{Z_o} = 1 + j(\omega - \omega_o)\frac{2L_1}{Z_o} + \cfrac{1}{j(\omega - \omega_o)2C_2 Z_o + \cfrac{1}{j(\omega - \omega_o)\frac{2L_3}{Z_o} + \cfrac{1}{j(\omega - \omega_o)2C_n Z_o + G Z_o}}} \tag{55}$$

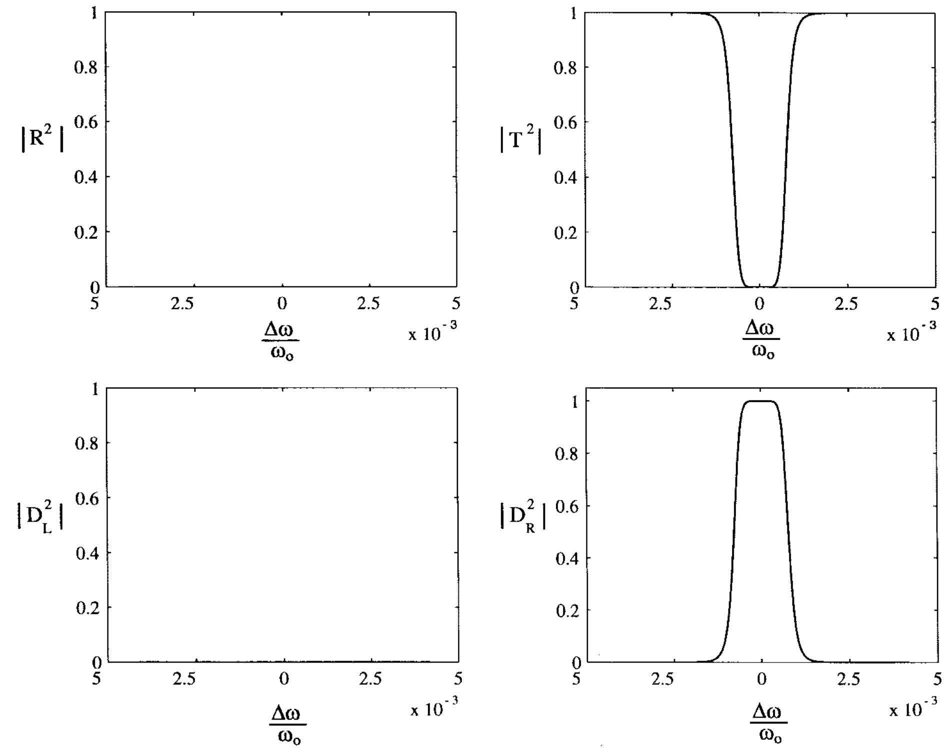


Fig. 5. Response of the fourth-order Butterworth filter shown in Fig. 4.

$$\mu_1 \longleftrightarrow \frac{1}{2\sqrt{L_1C_2}} \tag{62}$$

$$\mu_2 \longleftrightarrow \frac{1}{2\sqrt{C_2L_3}} \tag{63}$$

$$\mu_3 \longleftrightarrow \frac{1}{2\sqrt{L_3C_4}} \tag{64}$$

$$\tau_e' \longleftrightarrow \frac{2C_4}{G}. \tag{65}$$

For the nth-order filter, the couplings between the resonators are given by

$$\mu_i \longleftrightarrow \frac{1}{2\sqrt{L_iC_{i+1}}}, \qquad i \text{ odd} \tag{66}$$

$$\mu_i \longleftrightarrow \frac{1}{2\sqrt{C_iL_{i+1}}}, \qquad i \text{ even} \tag{67}$$

with $i = 1, 2, \cdots, n$. To design a higher order filter, we look up filter design tables which give the values of inductances and capacitances needed to obtain the desired spectral response. Using the above mappings, we obtain the coupling and decay parameters needed. Note that the coupling coefficients can be found by inspection from the equivalent circuit once its inductances and capacitances have been chosen. This technique was used to design a fourth-order Butterworth filter. The response at the various output ports of the coupled resonator system is shown in Fig. 5. The receiver port is maximally flat as expected for a Butterworth filter.

V. Conclusion

Using coupling of modes in time, we have extended the analysis of [4] to higher order filters produced by coupling among a number of resonator pairs. The advantage of this approach is that the filter response can be brought to a one-to-one correspondence with a standard L–C filter design, providing a rough layout of the structure and the optical parameters needed to achieve desirable transfer characteristics.

References

[1] H. A. Haus and Y. Lai, "Narrow-band optical channel-dropping filter," *J. Lightwave Technol.*, vol. 10, pp. 57–62, Jan. 1992.

[2] B. E. Little, S. T. Chu, H. A. Haus, J. Foresi, and J.-P. Laine, "Microring resonator channel dropping filters," *J. Lightwave Technol.*, vol. 15, pp. 998–1005, June 1997.

[3] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, "Channel drop tunneling through localized states," *Phys. Rev. Lett.*, vol. 80, p. 960, 1998.

[4] C. Manolatou, M. J. Khan, S. Fan, P. R. Villeneuve, H. A. Haus, and J. D. Joannopoulos, "Coupling of modes analysis of resonant channel dropping filters," *IEEE J. Quantum Electron.*, vol. 35, pp. 1322–1331, Sept. 1999.

[5] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, "Channel drop filters in photonic crystals," *Opt. Express*, vol. 3, p. 4, 1998.

[6] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, C. Manolatou, M. J. Khan, and H. A. Haus, "Theoretical investigation of channel drop tunneling processes," *Phys. Rev. B*, vol. 59, no. 24, pp. 15 882–15 892, June 1999.

[7] H. A. Haus and Y. Lai, "Theory of cascaded quarter wave shifted distributed feedback resonators," *IEEE J. Quantum Electron.*, vol. 28, pp. 205–13, Jan. 1992.

[8] A. B. Williams, *Electronic Filter Design Handbook.* New York: McGraw-Hill, 1981.

[9] E. A. Guillemin, *Synthesis of Passive Networks.* New York: Wiley, 1957.

[10] H. Y.-F Lam, *Analog and Digital Filters: Design and Realization.* Englewood Cliffs, NJ: Prentice-Hall, 1979.

[11] H. A. Haus, *Waves and Fields in Optoelectronics.* Englewood Cliffs, NJ: Prentice-Hall, 1984.

M. J. Khan, photograph and biography not available at the time of publication.

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