High Transmission through Sharp Bends in Photonic Crystal Waveguides

Attila Mekis, J. C. Chen, I. Kurland, Shanhui Fan, Pierre R. Villeneuve, and J. D. Joannopoulos
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 3 June 1996)

We demonstrate highly efficient transmission of light around sharp corners in photonic band-gap waveguides. Numerical simulations reveal *complete* transmission at certain frequencies, and very high transmission (>95%) over wide frequency ranges. High transmission is observed even for 90° bends with zero radius of curvature, with a maximum transmission of 98% as opposed to 30% for analogous conventional dielectric waveguides. We propose a simple one-dimensional scattering theory model with a dynamic frequency-dependent well depth to describe the transmission properties. [S0031-9007(96)01522-0]

PACS numbers: 42.79.Gn, 41.20.Jb, 03.80.+r

Photonic crystals have inspired great interest recently because of their potential ability to control the propagation of light. They can modify and even eliminate the density of electromagnetic states inside the crystal [1,2]. Such periodic dielectric structures with complete band gaps can find many applications, including the fabrication of lossless dielectric mirrors and resonant cavities for optical light [3]. In this Letter, we demonstrate a novel method for guiding light around sharp corners, using photonic crystal waveguides.

Two main designs are commonly employed to guide electromagnetic waves along a line: metallic pipe waveguides which provide lossless transmission only for microwaves, and dielectric guides for infrared and visible light. Although metallic waveguides can be used to steer light around tight corners, the operation of conventional dielectric guides, based on the principle of total internal reflection, is restricted by radiation losses to moderate curvature bends. In fact, when light is steered around a corner in such a guide, the radius of curvature must well exceed the wavelength of the light even for high dielectric contrasts to avoid large losses at the corners [4]. In a recent article, Meqade *et al.* showed that a linear defect in a photonic band-gap (PBG) material can support a linearly localized mode when the mode frequency falls inside the gap [5]. Such a defect can act as a waveguide for electromagnetic (EM) waves, without relying on total internal reflection. In this Letter we shall further show that a PBG waveguide can efficiently guide light around corners. The losses are very low for a wide range of frequencies, and vanish for specific frequencies, even if the radius of curvature of the bend is on the order of one wavelength.

For simplicity, we choose to study a 2D photonic crystal of dielectric rods in air on a square array with lattice constant a. Choosing the refractive index of the rods to be 3.4 (which corresponds to GaAs at the canonical wavelength of 1.55 μm) and their radius to be $0.18a$, we find that the crystal has a TM [6] gap which extends from frequency $\omega = 0.302 \times 2\pi c/a$ to $\omega = 0.443 \times 2\pi c/a$. One can create a single nondegenerate guided TM mode inside the gap by removing a row of rods. Since the waveguide has translational symmetry, a mode can be characterized by its reciprocal space wave vector k along the direction of the guide. The band appears at a frequency $\omega = 0.312 \times 2\pi c/a$ when $k = 0$ and reaches the top of the gap when $k = 0.38 \times 2\pi/a$. Since the characteristics of a PBG material remain unchanged under rescaling, we can easily assure that the guided light will be in the infrared or visible region. For example, if we choose a lattice constant a of 0.58 μm, the wavelength corresponding to the midgap frequency will be 1.55 μm.

If a bend is introduced into such a waveguide, no power will be radiated out of the guide as light travels around the bend, since there are no extended modes into which the propagating mode can couple. Light will either be transmitted or reflected; only back reflection will hinder perfect transmission. We study the transmission and reflection properties of waveguide bends using a vector finite-difference time-domain program with quartic perfectly matched layer boundaries [7]. In our simulation, a dipole located at the entrance of the waveguide creates a pulse with a Gaussian envelope in time. The field amplitude is monitored inside the guide at two points, one before the bend (point *A*) and one after (point *B*) as indicated in the top panel of Fig. 1. Although most of the light that reaches the edge of the computational cell is absorbed by the boundaries, some light gets reflected back from the ends of the waveguide. By using a sizable computational cell of 100×120 lattice constants and by positioning each monitor point appropriately, we can distinguish and separate all the different pulses propagating in the cell; the useful pulses, such as the input pulse and the pulses reflected by and transmitted through the bend, and the parasite pulses which are reflected from the edges of the cell. These pulses are clearly shown in the bottom panel of Fig. 1.

In the specific case shown in Fig. 1, six pulses are sent down the guide, covering different ranges of frequencies [8]. The pulses are then Fourier transformed to obtain the reflection and the transmission coefficients for each

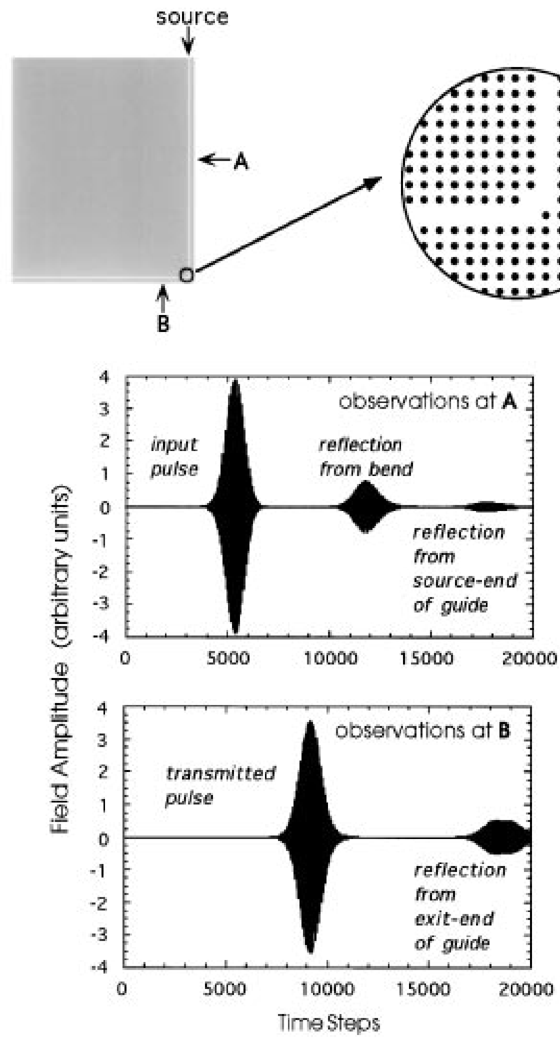

FIG. 1. Top panel: Schematic view of the $100a \times 120a$ computational cell. The field amplitude is monitored at points A and B. The guide is located five lattice constants from the edge of the cell. Bottom panel: Field amplitude recorded at points A and B, as a function of time. The pulses reflected by and transmitted through the bend, as well as the pulses reflected from the edges of the cell, are easily discernible.

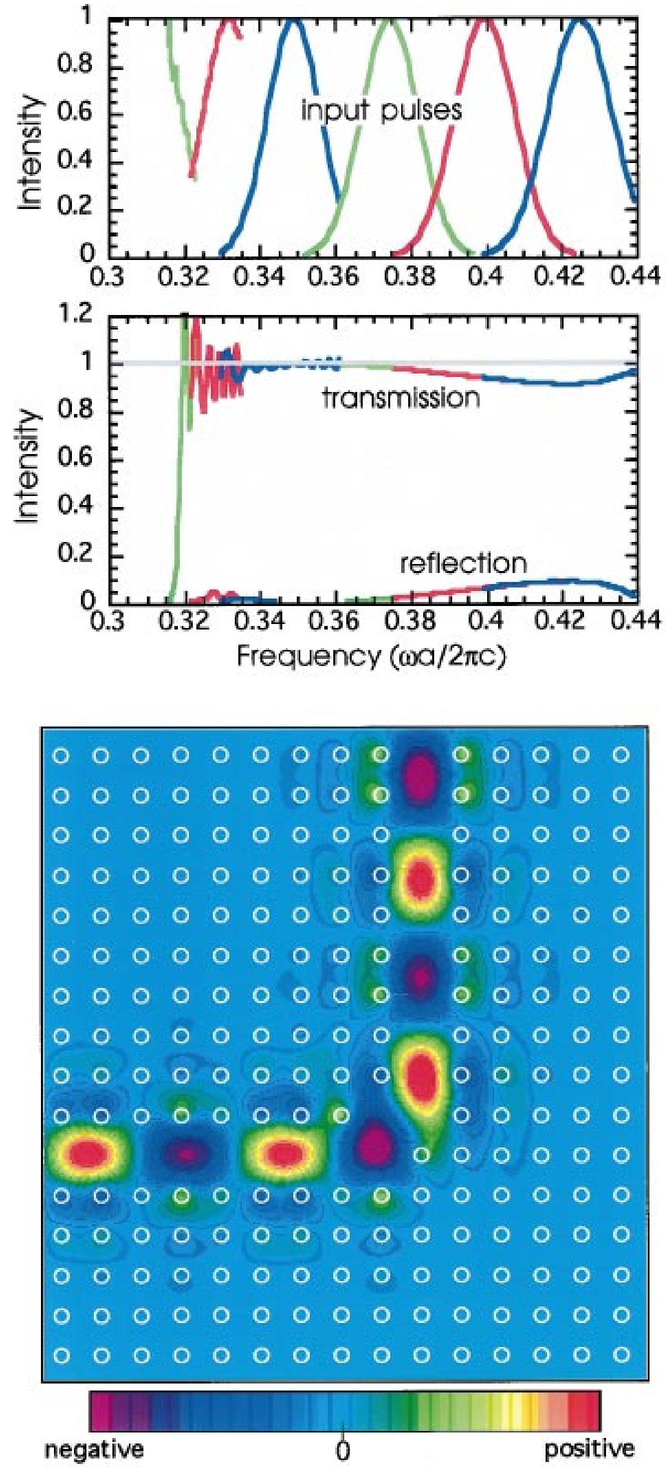

FIG. 2(color). Top two panels: Spectral profile of six input pulses. Computed transmission and reflection coefficients for each input pulse. The oscillations in transmission at low frequencies are numerical artifacts and are discussed in Ref. [9]. Bottom panel: Electric field pattern in the vicinity of the bend for frequency $\omega = 0.353 \times 2\pi c/a$. The electric field is polarized along the axis of the dielectric columns.

frequency. The results are shown in the top two panels in Fig. 2. The excellent agreement between the transmission and reflection coefficients obtained from the different pulses demonstrates the consistency of our approach [9]. The transmission and reflection coefficients do add up to unity for every frequency in the gap, which confirms that there is no observable radiation loss, in spite of the close proximity of the waveguide to the edge of the computational cell. The transmission drops sharply to zero below the cutoff frequency of the guided mode. The transmission for frequencies $\omega = 0.392 \times 2\pi c/a$ is larger than 95%, and reaches 100% when $\omega = 0.353 \times 2\pi c/a$. The field pattern of the propagating mode can be observed by a cw excitation of the guided mode. We show in the bottom panel of Fig. 2 the electric field pattern for the case where $\omega = 0.353 \times 2\pi c/a$. The mode is completely confined inside the guide, and the light wave travels smoothly around the sharp bend, even though the radius of curvature of the bend is on the order of the wavelength of the light. For comparison purposes, we have calculated the transmission through a traditional rib dielectric waveguide of refractive index 3.5 with a similar radius of curvature. The radius of curvature R was taken to be equal to the width of the guide and the transmission was measured for a wide range of frequencies centered around $\omega = 0.143 \times 2\pi c/R$. The power transmission was found not to exceed 80%.

We now propose a simple model to explain both the high transmission through the bends and the oscillatory behavior of the transmission spectrum. Our PBG waveguide structure can be viewed as separate waveguide sections, one in the (01) direction and one in the (10) direction, connected by a short waveguide section in the (11) direction. For any given frequency ω, there is a single wave vector $k(\omega)$ for the guided modes in any particular waveguide section. We label these wave vectors $k_1(\omega)$ for propagation along the (01) or (10) direction, and $k_2(\omega)$ for the (11) direction. These wave vectors are given by the dispersion relations [10] shown in Fig. 3. From this figure, we can define a frequency-dependent effective refractive index $n(\omega) = ck(\omega)/\omega$ governing the wave propagation in each of the waveguide sections.

We model the transmission through the bend as a simple one-dimensional scattering process in which the mode propagating with wave vector k_1 is scattered into the mode with wave vector k_2, then back into the mode with wave vector k_1. At the interface, we require continuity of the field and of its derivative, as we would in the case of a plane EM wave normally incident on a boundary between materials with different refractive indices. By complete analogy with the one-dimensional Schrödinger equation, we can map this problem onto that of a wave propagating in a "dielectric potential." This potential consists of three constant pieces, corresponding to the (01), (11), and (10) propagation directions, respectively, as shown in the inset of Fig. 3. Our model differs from the usual one-dimensional scattering problem in that the depth of the well, determined by the difference $k_1^2(\omega) - k_2^2(\omega)$, now depends on the frequency of the traveling wave.

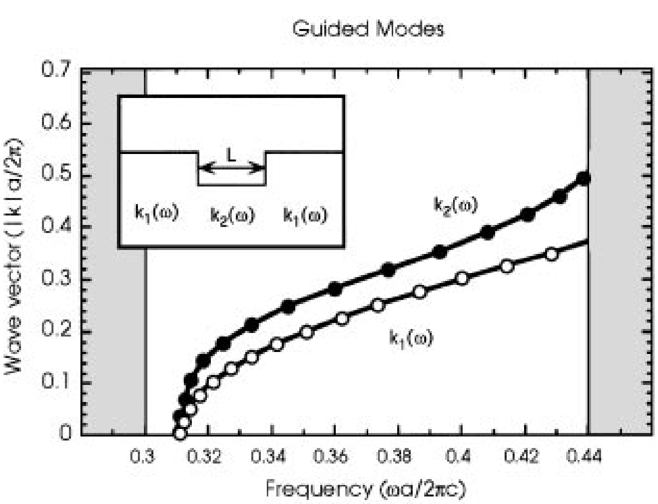

FIG. 3. Dispersion relation $k_1(\omega)$ for propagation along the (01) or (10) direction, and $k_2(\omega)$ for the (11) direction. The gray regions correspond to the edges of the band gap. The "potential" associated with the bend is shown in the inset.

The reflection coefficient is then given by

$$R(\omega) = \left[1 + \left(\frac{2k_1(\omega)k_2(\omega)}{[k_1^2(\omega) - k_2^2(\omega)]\sin[k_2(\omega)L]} \right)^2 \right]^{-1}. \tag{1}$$

The sole parameter in Eq. (1) is the length L of the well (or of the bend). To set this parameter, we select a single point from the computational results shown in Fig. 2. We choose the point at $\omega = 0.353 \times 2\pi c/a$, where the reflection coefficient is zero. Our choice of solution is $L = 1.33\sqrt{2}\,a$, which is the one closest to the physical length of the (11) portion of the waveguide.

To test the validity of this model, we vary the length of the (11) waveguide section and compare the reflection coefficients computed from the numerical simulations to those obtained from Eq. (1). The value $L = 1.33\sqrt{2}\,a$ found above is used to set the parameter L in each case. As we vary the bend length by integer multiples of $\sqrt{2}\,a$, the effective length L should also change by the same amount, giving $L = 0.33\sqrt{2}\,a$, $1.33\sqrt{2}\,a$, $2.33\sqrt{2}\,a$, and $3.33\sqrt{2}\,a$ for the four bends shown in Fig. 4. The reflection coefficients are plotted in Fig. 4. We find good agreement between the one-dimensional scattering model (solid line) and the numerical simulations (diamonds). Our model correctly predicts the frequencies where the reflection coefficient vanishes, as well as the general quantitative features of the transmission spectrum. The apparent disagreement at low frequencies arises from numerical limitations. Our simulations cannot accurately determine reflection in this frequency regime [9].

We note that the 90° bend with zero radius of curvature, as shown in the top panel of Fig. 4, is not described in this model by a uniformly constant potential, but by the potential shown in the inset of Fig. 3 with an effective length $L = 0.33\sqrt{2}\,a$. This length is extrapolated from the bends with longer (11) sections. Our model accurately predicts the existence of reflection from the bend, with transmission exceeding 95% for guided modes below $\omega = 0.403 \times 2\pi c/a$. This behavior is in marked contrast to that of a conventional dielectric waveguide with a sharp 90° bend. Power transmission reaches at most 30% even for a guide with a refractive index contrast of 3.5 to 1 with its surroundings, due to large radiation losses at the corner.

The one-dimensional scattering analysis presented above relies on the existence of a band gap along every direction in the plane of the 2D crystal. Therefore, our analysis should also hold for 3D photonic crystals

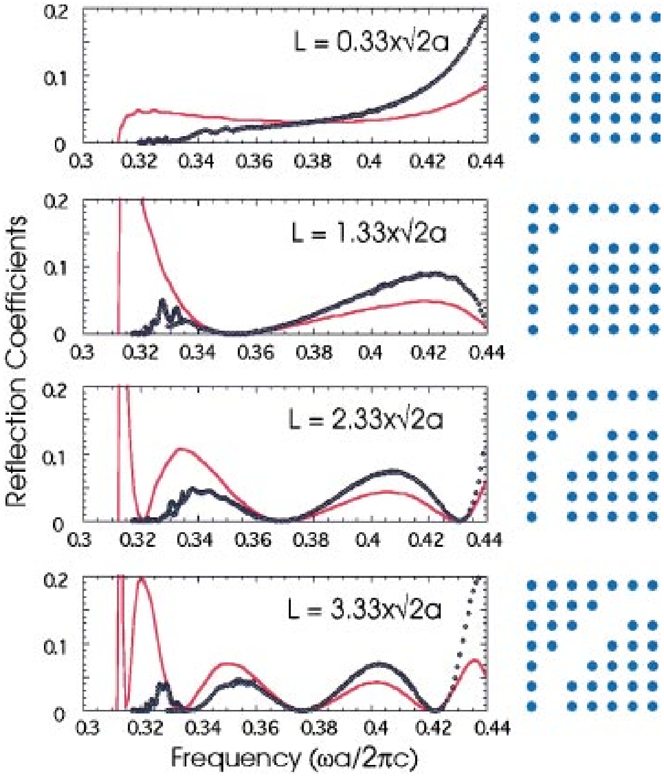

FIG. 4(color). Reflection coefficients computed from numerical simulations (diamonds) and from one-dimensional scattering theory (solid line), for four different bend geometries.

with complete omnidirectional band gaps. By adjusting the length of the bend section, we should be able to achieve 100% transmission through sharp bends for several frequencies. Furthermore, transmission should remain high as long as the dispersion relations in the two different waveguides making up the bend do not differ considerably, that is, as long as the depth of the "dielectric potential well" remains small.

Finally, a natural question to pose about these photonic crystal waveguides concerns the possible existence of bound states localized in the vicinity of the corner. These bound states are known to exist in other similar structures such as quantum wires [11]. In the case of photonic crystals, bound states may appear in a frequency range where guided modes exist inside the bend section while being forbidden in the other sections of the guide. Although this particular condition does not hold in the waveguide structures investigated above, it is possible to alter the waveguide geometry in order to change the dispersion relations, thereby creating a configuration which would support bound states.

The authors would like to acknowledge helpful discussions with Hermann Haus and Erich Ippen. This work was supported in part by the MRSEC Program of the NSF under Award No. DMR-9400334.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[3] See, for instance, J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton, New York, 1995), and references therein.
[4] E. A. J. Marcatili, Bell Syst. Tech. J. **48**, 2103 (1969).
[5] R. D. Meade, A. Devenyi, J. D. Joannopoulos, O. L. Alerhand, D. A. Smith, and K. Kash, J. Appl. Phys. **75**, 4753 (1994).
[6] TM modes are defined as the modes for which the magnetic field is transverse to the normal direction of the plane of translational symmetry; or, equally, TM modes are the modes for which the electric field is parallel to the axis of the dielectric columns.
[7] J. C. Chen, and K. Li, Microwave Opt. Tech. Lett. **10**, 319 (1995).
[8] It would be possible, in principle, to use only one pulse with a spectrum covering the entire gap to determine the transmission properties of the bend. However, the spatial width of such a pulse—initially small—would increase rapidly in time, resulting in an overlap between the useful and parasite pulses propagating in the computational cell.
[9] It is somewhat difficult to determine the reflection and transmission coefficients near the cutoff frequency $\omega = 0.312 \times 2\pi c/a$. At these frequencies, the pulse comprises long-wavelength components and spreads out as it propagates inside the guide. This widening results in overlaps among the useful and the parasite pulses. The separation of the overlapping pulses introduces Gibbs oscillations.
[10] The dispersion relations are determined using a frequency-domain band-structure computation code. For reference, see R. D. Meade, A. M. Rappe, K. D. Brommer, and J. D. Joannopoulos, Phys. Rev. B **48**, 8434 (1993).
[11] J. Goldstone and R. L. Jaffe, Phys. Rev. B **45**, 100 (1992); K. Lin and R. L. Jaffe, "Bound States and Threshold Resonances in Quantum Wires with Circular Bends", Phys. Rev. B (to be published).