Full inversion of a two-level atom with a single-photon pulse in one-dimensional geometries

Eden Rephaeli,[1] Jung-Tsung Shen,[2] and Shanhui Fan[1]
[1]*Ginzton Laboratory, Stanford University, Stanford, California 94305, USA*
[2]*Department of Electrical and Systems Engineering, Washington University in St. Louis, St. Louis, Missouri 63130, USA*

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We analyze a system comprising a one-dimensional single-mode waveguide (a) coupled directly to a two-level atom (b) side-coupled to a cavity containing a two-level atom and show that in both cases it is possible to invert the atom with a single-photon pulse. In contrast to the semiclassical π pulse, the inverting single-photon pulse is unique.

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The interaction between an optical field and two-level atom is one of most important models of light-matter interaction. Here, a fundamental question is whether the optical field can completely invert a two-level atom. In the rate-equation limit, complete inversion is impossible. Beyond the rate-equation limit, in the semiclassical regime, one can consider a pulse with duration far shorter than the spontaneous emission lifetime of the atom, and hence ignore the effect of spontaneous emission. In such a case, an on-resonance pulse will fully invert the atom if it is a "π pulse" with its electric-field envelope $E(\tau)$ at the atom's position satisfying the relation [1–3]

$$(\text{Area}) = \frac{2D}{\hbar}\int_{-\infty}^{\infty} E(\tau)\,d\tau = (2n+1)\pi, \tag{1}$$

where n is an integer, and D denotes the atom's dipole moment. In the semiclassical regime, there are many pulse profiles capable of fully inverting an atom [1].

In recent years, there has been great interest in interactions between atoms or atom-like systems and single-photon or few-photon states. To describe such an interaction properly, a fully quantized treatment of light-matter interaction is necessary. In such a fully quantized treatment, full inversion of a two-level atom was shown possible in free space [4] using a pulse consisting of a single photon. To do so however, requires spatial mode matching between the single-photon incident wave and the emission profile of the atom [4]. Such a spatial mode matching is difficult to accomplish in practice.

In this paper, we consider light-matter interaction when photons are confined to a one-dimensional geometry such as a strongly confining single-mode waveguide [5–19]. We show that full inversion of a two-level atom with a *single-photon pulse* is in general possible in these one-dimensional geometries. In contrast to the three-dimensional case in Ref. [4], the use of a single-mode waveguide geometry provides automatic spatial mode matching, which greatly simplifies practical implementation. Unlike the semiclassical case, here full inversion is related to the spontaneous emission properties of the atom, and requires a specific temporal pulse profile which is *unique*, and differs drastically from a π pulse. This inverting pulse should prove useful in manipulating quantum circuits and accessing strong atom-mediated photon-photon interactions [9]. In addition, our results here complement existing theoretical literature in the study of few-photon transport in one-dimensional systems [5–14]. In contrast to these works, our focus is on pulse behavior, and is in the regime of strong atom excitation. This regime is particularly important because photon-photon interaction is related to atomic excitation. Also, in this regime many standard theoretical techniques, developed for the weak excitation limit [20], do not apply.

We start by outlining a general procedure for determining the single-photon pulse capable of inverting an atom, which we will call the *inverting pulse*. As concrete examples, in this paper we will consider two systems, shown in Fig. 1. Both systems contain a one-dimensional single-mode waveguide. The waveguide is either coupled directly to a two-level atom [Fig. 1(a)] or side-coupled to a cavity containing a two-level atom [Fig. 1(b)].

In these systems, one can rigorously prove both the existence, and the uniqueness of an inverting pulse. An inverting pulse, upon injection into the system at $t_0 \to -\infty$, needs to put the atom completely into the excited state $|+\rangle$ at $t=0$. In the single-excitation Hilbert space, when the atom is completely inverted, there cannot be any photons in the system. Hence at $t=0$ the quantum state of the *composite* photon-atom system is *uniquely* specified to be $|0,+\rangle$, with the photon part in the vacuum state $|0\rangle$. The inverting pulse is therefore

$$|\psi\rangle = e^{-i\hat{H}t_0/\hbar}\,|0,+\rangle\,, \tag{2}$$

where $\hat{H}$ is the Hamiltonian of the system.

To see that the inverting pulse as defined by Eq. (2) is unique, suppose that at t_0 we have another state $|\phi\rangle$ that can also invert the atom at $t=0$. From the discussions above we must have $|0,+\rangle = e^{-i\hat{H}(0-t_0)/\hbar}|\phi\rangle$. Thus $|\phi\rangle = e^{-i\hat{H}t_0/\hbar}|0,+\rangle = |\psi\rangle$. Here, the uniqueness of the inverting pulse is directly related to the uniqueness of the quantum state when the atom is completely inverted. In the following discussion, we will generate the detailed pulse shape using Eq. (2).

Both systems in Fig. 1 are described by a Hamiltonian of the form $\hat{H}/\hbar = H_{\text{wvg}} + H_1$ [5], where

$$H_{\text{wvg}} = \int dx c_R^\dagger(x)\left(\Omega - i v_g \frac{\partial}{\partial x}\right) c_R(x) + \int dx c_L^\dagger(x)\left(\Omega + i v_g \frac{\partial}{\partial x}\right) c_L(x) \tag{3}$$

describes the waveguide. Here, we have linearized the waveguide dispersion relation around the atomic transition frequency Ω, where the group velocity is v_g. $c_R^\dagger(x)[c_R(x)]$ creates [annihilates] a right-moving photon and $c_L^\dagger(x)[c_L(x)]$ creates [annihilates] a left-moving photon. When expressed in terms

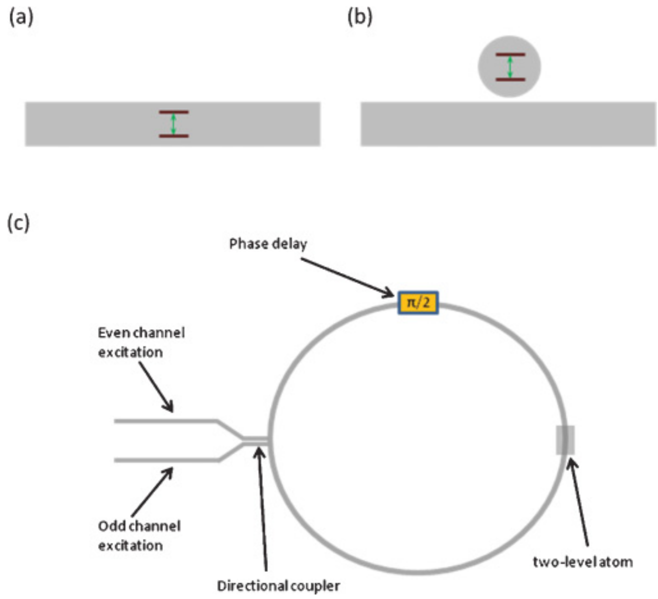

FIG. 1. (Color online) Illustration of setup considered in the paper. (a) Atom placed directly in waveguide; (b) atom placed inside a cavity, side-coupled to a waveguide; (c) a Sagnac interferometer for the purpose of inverting an atom. The gray rectangle region, placed halfway around the ring, symbolizes the system of either (a) or (b). The upper arm of the interferometer includes a $\pi/2$ phase delay element. As a result, the upper input port accesses the even subspace, and the lower input port accesses the odd-subspace of the Hamiltonian.

of even $[c_e(x) \equiv \frac{c_R(x)+c_L(-x)}{\sqrt{2}}]$ and odd $[c_o(x) \equiv \frac{c_R(x)-c_L(-x)}{\sqrt{2}}]$ photon operators, H_{wvg} separates into $H^e_{\text{wvg}} + H^o_{\text{wvg}}$, where $H^{e,o}_{\text{wvg}} = \int dx c^\dagger_{e,o}(x)(\Omega - i v_g \frac{\partial}{\partial x})c_{e,o}(x)$, respectively.

We first consider the system as shown in Fig. 1(a), where an atom located at $x = 0$, is directly coupled to the waveguide. This system has been experimentally realized using a plasmonic nanowire [17], a photonic crystal waveguide [21], and a slot waveguide [22]. In this system, the atom and the atom-photon interaction is described by

$$H_1 = V \int dx \delta(x)[c^\dagger_e(x)\sigma_- + c_e(x)\sigma_+] + \Omega a^\dagger_e a_e. \qquad (4)$$

In Eq. (4), we set the atomic ground-state frequency to zero. a^+_g $[a_g]$ and a^+_e $[a_e]$ are creation [annihilation] operators for the ground and excited state, respectively. $\sigma_+ \equiv a^\dagger_e a_g$ and $\sigma_- \equiv a^\dagger_g a_e$ are atomic raising and lowering operators, respectively. $V \equiv \frac{D}{\hbar}\sqrt{\frac{\hbar\Omega}{\varepsilon_0 A}}$ is the atom-photon coupling strength, where A is the waveguide's cross-sectional area, assuming that the atomic dipole moment is parallel to the electric field of the waveguide mode at the atom's position. Notice from Eq. (4) that the atom-photon interaction, and hence the process of emitting and absorbing a photon, occurs only in the even subspace, as described by the Hamiltonian $H^e = H^e_{\text{wvg}} + H_1$. The odd subspace, as described by $H^o = H^o_{\text{wvg}}$ is interaction free. Thus, only the even subspace contributes to the inversion process.

In the even subspace, at an eigenfrequency ω, we may write a general normalized interacting eigenstate for the one-excitation manifold:

$$|k^+\rangle = \left\{\int dx[\theta(-x) + t_k\theta(x)]\frac{e^{ikx}}{\sqrt{2\pi}}c^\dagger_e(x) + e_a\sigma_+\right\}|0,-\rangle, \qquad (5)$$

where $|0,-\rangle$ denotes the vacuum with zero photons and atom in the ground state, $t_k = (\omega - \Omega - i\Gamma/2)/(\omega - \Omega + i\Gamma/2)$ is the transmission coefficient, $e_a = V/[\sqrt{2\pi}(\omega - \Omega + i\Gamma/2)]$ is the atomic excitation amplitude, and $k = (\omega - \Omega)/v_g$. Here $\Gamma \equiv V^2/v_g$ is the atom's spontaneous-emission rate into the waveguide.

We calculate the inverting pulse using Eq. (2). At $t \to -\infty$, the atom is in the ground state, and the photon wave function is

$$\psi_i(x,t) = \langle 0,-|c_e(x)e^{-iH^e t}|0,+\rangle$$
$$= \int dk e^{-i(kv_g+\Omega)t}\langle 0,-|c_e(x)|k^+\rangle\langle k^+|0,+\rangle. \qquad (6)$$

Using Eq. (5) for $|k^+\rangle$, and noting that in taking the $t \to -\infty$ limit, the term proportional to $\theta(x)$ vanishes, we obtain the inverting one-photon pulse (Fig. 2):

$$\psi_i(x,t) = i\sqrt{\frac{\Gamma}{v_g}}\theta(-x)\theta(x/v_g - t)e^{-i\Omega t - \frac{\Gamma}{2}(x/v_g - t)}. \qquad (7)$$

The inverting pulse grows exponentially in time at the location of the atom, with a characteristic growth rate equal to the atom's spontaneous-emission rate. Propagating $\psi_i(x,t)$ toward the atom found in its ground state at $t \to -\infty$, results in the atomic excitation probability

$$|e_a(t)|^2 = \left|\int dk\langle 0,+|k^+\rangle\langle k^+|e^{-iH^e t}|\psi_i(x,t)\rangle\right|^2 = e^{-\Gamma|t|}. \qquad (8)$$

$|e_a(t)|^2$ indeed reaches unity at $t = 0$. Afterward it decays exponentially due to the spontaneous emission. We now contrast the inverting pulse in Eq. (7) with a π pulse. To quantify the characteristics electric-field strength of the single photon pulse, we examine the quantity

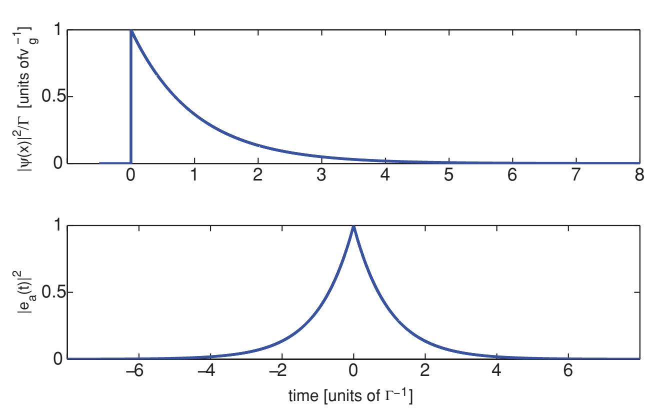

FIG. 2. (Color online) Top: inverting pulse $|\psi(x)|^2/\Gamma$. Bottom: atomic excitation probability $|e_a(t)|^2$ for an atom coupled directly to a 1D waveguide.

$E = \sqrt{\langle E^-(0^-,t)E^+(0^-,t)\rangle}$, where [23]

$$E^-(0^-,t)E^+(0^-,t) = \sum_{k,k'} \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 AL}}\sqrt{\frac{\hbar\omega_{k'}}{2\varepsilon_0 AL}} c_k^\dagger c_{k'}. \quad (9)$$

L is the quantization length (in the longitudinal direction). By transforming to a real-space representation, this quantity simplifies to

$$E = \sqrt{\frac{\hbar\Omega}{2\varepsilon_0 A}}|\psi(0^-,t)|. \quad (10)$$

When applied to the inverting pulse in Eq. (7) we find

$$E = \left(\frac{\hbar\Gamma}{\sqrt{2}D}\right) e^{\frac{\Gamma}{2}t}\theta(-t). \quad (11)$$

Using Eq. (1), this results in a pulse area of $2\sqrt{2}$. Thus, an inverting single-photon pulse has an area comparable to a π pulse. Despite this, the nature of the pulse is dramatically different. In the semiclassical model—in which a π pulse is defined—the inverting pulse's duration is far shorter than the atom's spontaneous-emission lifetime, such that the atom does not decay throughout the inversion process. In contrast, the inverting single-photon pulse' duration is comparable to the spontaneous-emission lifetime. We emphasize that in the semiclassical case, the inversion of the atom is due to stimulated emission, while in the case treated here, the inversion instead arises from spontaneous emission, which is accounted for in our fully quantized treatment.

From an experimental standpoint, an inverting system can be implemented using a Sagnac interferometer setup, as illustrated in Fig. 1(c) [24]. In this setup, a waveguide loop connects the two output ports of a directional coupler. An atom coupled to the waveguide loop is placed at a location such that light propagating in the upper arm acquires an additional $\pi/2$ phase delay as it reaches the atom. A single-photon pulse injected into the upper input port of the directional coupler splits into two amplitudes propagating in the upper and lower arms. These two amplitudes arrive at the atom from both sides with equal phase, ensuring that only the even channel is excited. In this setup, injecting a pulse with a waveform according to Eq. (7) will completely invert the atom.

The procedure outlined above for determining an inverting single-photon pulse is in fact generally applicable to any one-dimensional system. Next, we consider a slightly more complicated example, where single-mode one-dimensional waveguide is side-coupled to a single-mode cavity, which has a resonant frequency ω_c and contains an atom [Fig. 1(b)] [6,7,11]. Understanding this system has direct relevance for experiments in both microwave [15] and optical [16,18,19] frequencies. The Hamiltonian for this system is $H = H_{\rm wvg} + H_1$ [11], where $H_{\rm wvg}$ is given by Eq. (3), and

$$H_1 = V\int \delta(x)\,dx[c_e^\dagger(x)a + c_e(x)a^\dagger] + \Omega a_e^\dagger a_e + \omega_c a^\dagger a + g(a^\dagger\sigma_- + a\sigma_+). \quad (12)$$

Here, $a^\dagger(a)$ creates (annihilates) a cavity photon. V characterizes the strength of cavity-waveguide coupling, and g is the atom-cavity coupling rate. Here too, the atom-photon interaction occurs only in the even channel, as described by the Hamiltonian $H^e = H^e_{\rm wvg} + H_1$.

At an eigenfrequency ω, the Hamitonian H^e has an eigenstate [11]:

$$|k^+\rangle = \left\{\int dx[\theta(-x) + t_k\theta(x)] \times \frac{e^{ikx}}{\sqrt{2\pi}}c_e^\dagger(x) + e_c a^\dagger + e_a\sigma_+\right\}|0,-\rangle, \quad (13)$$

where

$$t_k = \frac{(\omega-\Omega)\left(\omega-\omega_c - i\frac{\Gamma}{2}\right) - g^2}{(\omega-\Omega)\left(\omega-\omega_c + i\frac{\Gamma}{2}\right) - g^2}, \quad (14a)$$

$$e_c = \frac{1}{\sqrt{2\pi}}\frac{V(\omega-\Omega)}{(\omega-\Omega)\left(\omega-\omega_c + i\frac{\Gamma}{2}\right) - g^2}, \quad (14b)$$

$$e_a = \frac{1}{\sqrt{2\pi}}\frac{gV}{(\omega-\Omega)\left(\omega-\omega_c + i\frac{\Gamma}{2}\right) - g^2}, \quad (14c)$$

$$k = \frac{\omega-\Omega}{v_g}. \quad (14d)$$

Here $\Gamma \equiv V^2/v_g$ is the cavity's decay rate into the waveguide. Using the same back-propagating technique, we obtain the inverting pulse:

$$\psi_i(x,t) = \sqrt{\frac{4g^2\Gamma}{v_g}}\frac{e^{-i\Omega t}e^{-\left(\frac{\Gamma}{4}+i\frac{\Delta}{2}\right)\left(\frac{x}{v_g}-t\right)}}{\sqrt{\left(i\frac{\Gamma}{2}-\Delta\right)^2+4g^2}}\theta(-x)\theta(x/v_g - t) \times \sin\left[\left(\frac{x}{v_g}-t\right)\sqrt{\left(\frac{i\Gamma}{4}-\Delta/2\right)^2+g^2}\right]. \quad (15)$$

When this pulse is injected into the system, the resulting atomic inversion probability may be analytically calculated.

In order to better understand the nature of the inverting pulse in this system, we first consider the zero detuning ($\Delta \equiv \Omega - \omega_c = 0$) case (Fig. 3). In the strong-coupling regime ($\Gamma < 4g$), both the inverting pulse and the corresponding atomic excitation probability exhibit oscillatory behavior. In contrast, no oscillatory behavior is present in the weak-coupling regime ($\Gamma > 4g$). Additionally, in the weak coupling regime, the atomic inversion probability has a much shorter duration, defined as the length of time during which the atom has substantial probability to be in its excited state, since in

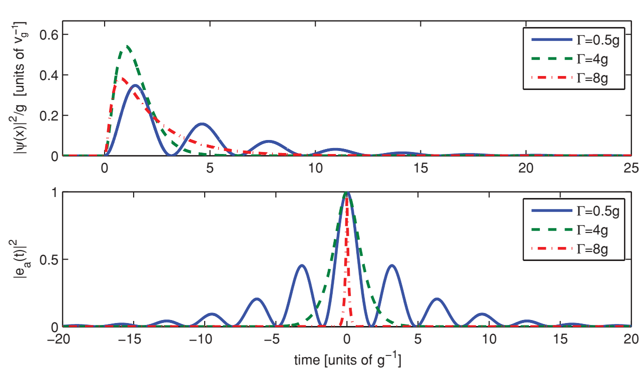

FIG. 3. (Color online) Top: inverting pulse $|\psi(x)|^2/g$. Bottom: atomic excitation probability $|e_a(t)|^2$, for an atom placed inside a side-coupled cavity with $\Delta = 0$.

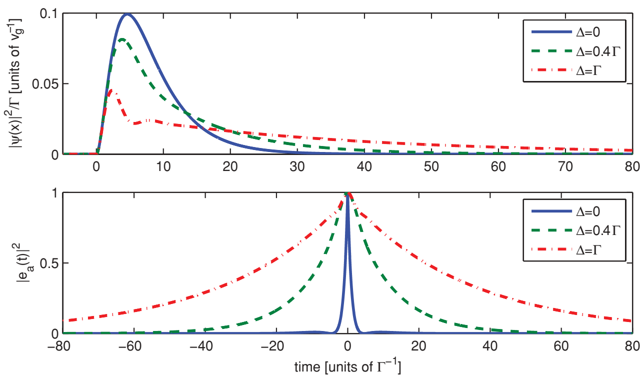

FIG. 4. (Color online) Top: inverting pulse $|\psi(x)|^2/\Gamma$. Bottom: atomic excitation probability $|e_a(t)|^2$, for an atom placed inside a cavity side-coupled to a 1D waveguide. $\Gamma = 5g$.

this regime the cavity photon readily leaks to the waveguide without oscillating back into the atomic excitation.

In general, and perhaps counterintuitively, we emphasize that full inversion of the atom is always possible with any choice of parameters in Eq. (12), as long as the coupling constants are nonzero. In particular, full inversion is possible even in the regime of significant atom-cavity detuning, and does not require the system to be in the strong-coupling regime. To illustrate this, in Fig. 4 we exhibit a case in which $\Gamma = 5g$, and hence the system is in the weak-coupling regime. We see that the larger the detuning, the longer the inverting pulse and the longer the atomic inversion duration.

We now briefly discuss some of the practical aspects for implementing the proposed scheme. Our scheme requires accurate control of the temporal shape of the pulse. Such a capability was experimentally demonstrated in Ref. [25], where one starts with a time-energy entangled pair of Stokes and anti-Stokes photons as generated via spontaneous parametric down-conversion. An electro-optic modulator is then used to shape the temporal profile of the anti-Stokes photon, using a time origin established by the detection of the Stokes photon [25]. Our scheme also requires that the atom predominantly couple to the waveguide. In practice, this requires achieving a high β factor, defined as $\beta \equiv \gamma_{\rm wvg}/\gamma_{\rm total}$, where $\gamma_{\rm wvg}$ is the decay rate of the atom due to atom-waveguide coupling, and $\gamma_{\rm total}$ is the total decay rate that includes nonradiative decay processes as well as emission into nonguided modes. A near-unity β factor has been reported in Refs. [21,22]. In these structures, therefore, near complete inversion of a single quantum emitter should be achievable. Finally, the dispersion relation of the waveguide becomes well defined when the length of the waveguide is several wavelengths long. In an on-chip strongly confined single-mode waveguide system, a waveguide length on the order of 10 μm should already be very well described by our model. On the other hand, the loss of such waveguide is already as low as a few dB/cm experimentally [26]. Therefore, the effect of waveguide loss should be minimal in such a short waveguide structure.

In conclusion, we have shown that a single-photon pulse can give rise to full atomic inversion in various one-dimensional geometries. In the field of quantum information processing there is great interest in demonstrating on-chip quantum circuits, where the flying qubits of photons remain confined to waveguides, as evidenced from recent developments in superconducting transmission line resonators [15] and on-chip photonic circuits [27]. Compared with free-space setups, one-dimensional geometries provide far more control of the photon's behavior, and a viable pathway toward integration. Our work here shows that very strong exchange between the flying photon and the stationary atom qubits may be realized in these quantum circuits. Moreover, in the few-photon limit, photon-photon interaction is directly related to atomic excitation [8,9]. Therefore, the use of an inverting pulse—which maximizes atomic excitation—should be important in achieving strong nonlinearity at the few-photon level.

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