

# Detailed balance analysis of nanophotonic solar cells

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**Abstract:** We present a detailed balance based approach for performing current density-voltage characteristic modeling of nanophotonic solar cells. This approach takes into account the intrinsic material non-idealities, and is useful for determining the theoretical limit of solar cell efficiency for a given structure. Our approach only requires the cell's absorption spectra over all angles, which can be readily calculated using available simulation tools. Using this approach, we elucidate the physics of open-circuit voltage enhancement over bulk cells in nanoscale thin film structures, by showing that the enhancement is related to the absorption *suppression* in the immediate spectral region above the bandgap. We also show that with proper design, the use of a grating on a nanoscale thin film can increase its short-circuit current, while preserving its voltage-enhancing capabilities.

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## 1. Introduction

The performance of any solar cell is characterized by its current density-voltage (J-V) relationship. Therefore, it is useful to calculate this J-V relation theoretically. For bulk structures, to calculate the J-V relations, there have been two complimentary approaches. In the first approach [1], one performs a combined electrical and optical modeling of a particular device

structure, including the detailed carrier dynamics of the device. In the second approach [2, 3], one instead calculates the limiting efficiency based on the principle of detailed balance and taking into account only the intrinsic material non-idealities. Both these approaches are important. The first approach is crucial in understanding the detailed performance of a particular device, whereas the second approach provides a fundamental limit in terms of efficiency for a given class of structures or materials.

In recent years there have been strong interests in the study of nanophotonic solar cells as a way to improve solar cell efficiency and reduce cost [4, 5]. Most of the simulations of these nanophotonic cells have been focussed on calculating the absorption properties of the device in order to determine the short-circuit current enhancement [6–17]. There has also been work towards combining the optical and electrical simulations of these devices [18, 19]. Complementary to these works, it is of interest to pursue calculations based on the principle of detailed balance in order to determine the limiting efficiency of a given structure. Reference [20] performed one such study on a GaAs thin film cell with nanoscale film thickness, and showed that the open-circuit voltage in such a thin film can be significantly enhanced beyond that of bulk cells. This obviously has important implications in improving solar cell efficiency, and in reducing material usage and solar cell cost. However, Ref. [20] only considered a flat film and, consequently, the absorption and short-circuit current of their structure suffers significantly as compared to bulk cells. A natural approach to enhance the short-circuit current would be to introduce light trapping in these nanophotonic structures [21]. An important open question, therefore, is whether the physics of voltage enhancement can be preserved in a nanoscale thin film after light trapping is introduced.

To address this question, one will need to do a detailed balance analysis of nanophotonic solar cells. In Ref. [20], this analysis was performed by a direct calculation of the thermal emission of thin film cells using the fluctuation-dissipation theorem [22]. It is well known that such a direct calculation is quite involved for nanophotonic structures [23].

In this paper we introduce an alternative approach for the detailed balance analysis of a nanophotonic solar cell. We show that the absorption spectrum of a solar cell at all angles, as one routinely calculates in solar cell optical modeling, is in fact sufficient for determining the limiting open-circuit voltage, leading to a far easier way to perform solar cell J-V characteristics modeling as compared to the direct emission calculation in Ref. [20]. Our approach, moreover, can be readily generalized to include intrinsic material non-idealities. Using our approach, we numerically demonstrate that a grating structure on a nanoscale thin film can indeed enhance the short-circuit current while preserving the open-circuit voltage enhancement of the thin film, as shown in Fig. 1. We also elucidate the underlying physics of open-circuit voltage enhancement by showing that it is related to the *suppression* of absorption and, hence, thermal emission, in the frequency range immediately above the bandgap.

## 2. Computational procedure

We first outline the procedure that we use to calculate the J-V characteristics. We start with the detailed balance equation in Ref. [24], which is applicable to solar cells in general:

$$F_s - F_c(V) + R(0) - R(V) - J/q = 0. \quad (1)$$

In Eq. (1),  $V$  is the voltage across the cell,  $J$  is the current density generated by the cell, and  $q$  is the electron charge.  $F_s$  is the radiative generation rate per unit area of hole-electron pairs by the incident sunlight, while  $F_c(V)$  is the corresponding radiative recombination rate per unit area.  $R(0)$  and  $R(V)$  are the rates of non-radiative recombination and generation, respectively, of hole-electron pairs per unit area [24]. These non-radiative rates are equal at the thermal equilibrium condition  $V = 0$ .

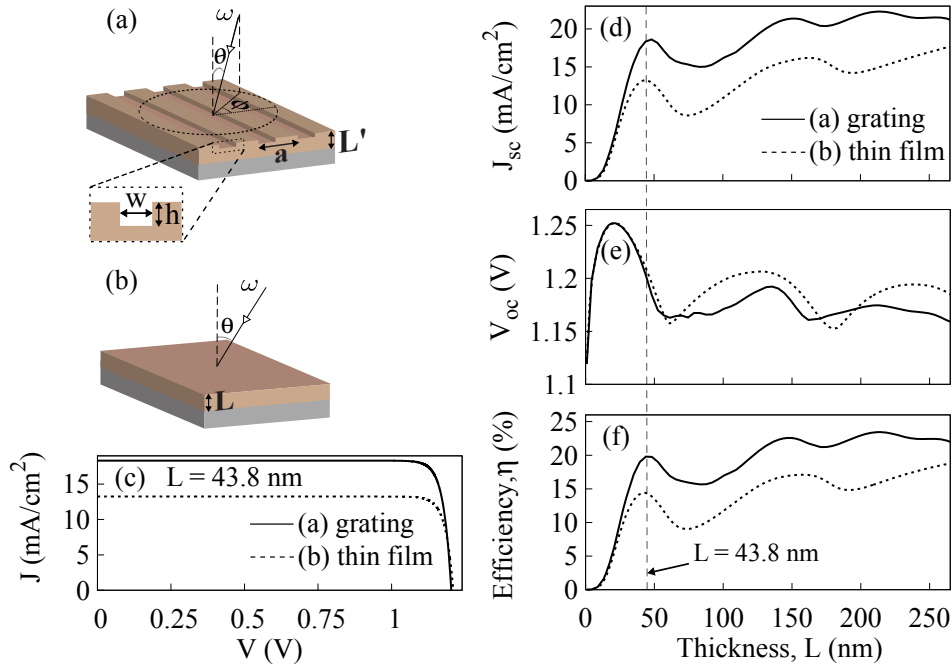


Fig. 1. (a) GaAs grating nanostructure (gold color) with effective thickness  $L = \left(1 - \frac{wh}{L'a}\right)L'$  where  $L'$  is its actual thickness,  $a$  is its periodicity, and each air groove has width  $w$  and depth  $h$ , (b) GaAs thin film (gold color) with thickness  $L$ . Both structures in (a) and (b) have a perfect reflecting back surface (grey color). Also shown in (a) and (b) is the propagation direction of an incident plane wave with frequency  $\omega$ , polar angle  $\theta$ , and azimuthal angle  $\phi$  [ $\phi$  is omitted for the azimuthally symmetric thin film in (b)]. (c) shows the J-V curves for the structures in (a) and (b) with thickness  $L = 43.8$  nm. (d)-(f) shows the following characteristics versus thickness  $L$  for the structures in (a) and (b): (d) short-circuit current density  $J_{sc}$ , (e) open-circuit voltage  $V_{oc}$ , and (f) efficiency  $\eta$ . In (c)-(f), the grating structure has periodicity  $a = 456$  nm, and air-groove dimensions ( $w = 0.24 a, h = 0.52 L'$ ).

We consider the scenario where the solar cell is under direct sunlight with an incident solar photon rate  $S(\omega)$  per unit bandwidth per unit area at a frequency  $\omega$ . For all our calculations, we used the AM 1.5 global spectrum standard [25] for  $S(\omega)$ . We also assume the cell has an absorption spectra of  $A(\omega, \theta, \phi)$  where  $\theta$  and  $\phi$  are the incident polar and azimuthal angles, respectively [Fig. 1(a)].  $A(\omega, \theta, \phi)$  here is the sum of the absorbing spectras for both the transverse electric and transverse magnetic incident polarizations. The radiative generation rate  $F_s$  in Eq. (1) can be calculated as follows [24]:

$$F_s = \int_{\omega_g}^{\infty} d\omega S(\omega) A(\omega, \theta = 0, \phi = 0) \quad (2)$$

where the integration is taken over all frequencies  $\omega$  starting from the cell's material bandgap frequency  $\omega_g$ .

The radiative recombination rate  $F_c(V)$  in Eq. (1) relates to the voltage  $V$  across the cell as follows [24]:

$$F_c(V) = F_{co} \exp\left(\frac{qV}{kT_c}\right) \quad (3)$$

where  $F_{\text{co}}$  is the radiative recombination rate when the cell is in thermal equilibrium with a surrounding blackbody,  $k$  is the Boltzman constant, and  $T_c$  is the temperature of the cell. Assuming that our cell has a perfectly reflecting back surface as in the case of the nanostructures in Figs. 1(a)-1(b),  $F_{\text{co}}$  is then just the thermal emission through the  $2\pi$  solid angle of the front surface of the cell [26]:

$$F_{\text{co}} = \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} d\theta \int_{\omega_g}^{\infty} d\omega \Theta(\omega) A(\omega, \theta, \phi) \cos(\theta) \sin(\theta). \quad (4)$$

where  $\Theta(\omega) = \frac{\omega^2}{4c^2\pi^3} \left[ \exp\left(\frac{\hbar\omega}{kT_c}\right) - 1 \right]^{-1}$  is *Planck's law* [26] for the incident spectral photon flux density from a surrounding blackbody at a temperature  $T_c$ ,  $c$  is the speed of light in vacuum, and  $\hbar$  is the reduced Planck constant. In Eq. (4), we have used *Kirchoff's law* [26] to relate the thermal emission rate of the cell to its absorption spectra  $A(\omega, \theta, \phi)$ .

The short-circuit current  $J_{\text{sc}}$  of the cell can be obtained by setting  $V = 0$  in Eq. (1) [24]:

$$J_{\text{sc}} = q(F_s - F_{\text{co}}). \quad (5)$$

And by setting  $J = 0$  in Eq. (1) we get the following equation from which we can solve for the open-circuit voltage  $V_{\text{oc}}$  across the cell [24]:

$$F_s + R(0) = F_c(V_{\text{oc}}) + R(V_{\text{oc}}). \quad (6)$$

In the case of solar cells where the contribution from non-radiative processes is small as compared to radiative processes, the cell's  $V_{\text{oc}}$  in Eq. (6) can be approximated as follows:

$$V_{\text{oc}} \approx \frac{kT_c}{q} \log\left(\frac{F_s}{F_{\text{co}}}\right). \quad (7)$$

From Eqs. (1)-(7), we see that having the absorption spectra  $A(\omega, \theta, \phi)$  over all angles is sufficient for the detailed balance analysis of nanophotonic solar cells. Such spectra controls both the absorption and the emission properties of the cell that enters into *Shockley-Queisser's analysis* [24].

The absorption spectra  $A(\omega, \theta, \phi)$  can be readily calculated using available simulation tools such as the Rigorous Coupled Wave Analysis (RCWA) [27]. RCWA is a frequency-domain computation tool that can perform a full electromagnetic analysis of layered structures. All absorption spectras  $A(\omega, \theta, \phi)$  for the cells discussed in this paper were calculated using the freely available RCWA package in Ref. [27]. The only material related quantity required by these calculations is the solar cell's complex permittivity as a function of frequency. For the GaAs cells discussed in the following sections, this permittivity data was obtained from Ref. [28].

### 3. Results on a GaAs thin film with and without the grating

In the following, we will first present the results of the J-V characteristics associated with the calculated absorption spectra for the nanostructures shown in Figs. 1(a)-1(b). We specialize to a GaAs cell with a perfect back reflector. For such a cell, the important non-radiative recombination mechanisms in Eq. (1) are Auger recombination, the defect mediated Shockley-Read-Hall effect, and surface recombination [19, 29–35]. Furthermore, since we want to establish an upper bound in the solar cell's performance, we idealize to the case of a defect free GaAs cell with perfect surface passivation. Under these conditions, the only important non-radiative mechanism in our detailed balance analysis is Auger recombination which is given by [36, 37]:

$$R(V) = C_n L n^2 p + C_p L p^2 n \quad (8)$$

where  $n$  ( $p$ ) is the electron (hole) concentration,  $C_n$  ( $C_p$ ) is the conduction-band (valence-band) Auger coefficient, and  $L$  is the thickness of the cell. In addition, we also assume that the GaAs cell is approximately intrinsic under illumination (i.e.  $n \approx p$ ). In this intrinsic approximation, the Auger recombination rate [Eq. (8)] is minimized and given by [2, 3]:

$$R(V) = (C_n + C_p) L n_i^3 \exp\left(\frac{3qV}{2kT_c}\right) \quad (9)$$

where  $n_i$  is the intrinsic carrier concentration. In all our calculations, we consider GaAs cells operating at a temperature of  $T_c = 300\text{K}$ , where  $C_n + C_p = 7 \times 10^{-30} \text{cm}^6 \cdot \text{s}^{-1}$  [36] and  $n_i = 2 \times 10^6 \text{cm}^{-3}$  [38].

Although we do include Auger recombination [Eq. (9)] in all our calculations below, we note that for the GaAs solar cells we consider in this paper, this non-radiative rate is small when compared to the radiative rate. Therefore, Eq. (7), which ignores non-radiative recombination, is relevant for these solar cells. The validity of this approximation [Eq. (7)] has also been experimentally verified for micron thick GaAs solar cells in Ref. [39]. We emphasize that one cannot, in general, ignore non-radiative recombination for most solar cell materials. Hence, one should not use Eq. (7) in general. For example, in the case of Si solar cells, the contribution from Auger recombination in Eq. (6) is significant when compared to the radiative recombination rate [2].

The dashed line in Fig. 1(c) plots the J-V curve of a flat GaAs thin film structure [Fig. 1(b)] with thickness  $L = 43.8 \text{ nm}$ . The use of the thin film results in a  $V_{oc}$  of 1.21V, which is significantly higher than the  $V_{oc}$  of 1.12V for bulk GaAs cells [20]. This result indicates a very important potential of nanoscale thin film structures for enhancing solar cell performance. However, the  $J_{sc}$  of the thin film suffers significantly as compared to that of GaAs bulk cells [3]. One natural way to enhance the  $J_{sc}$  of the thin film is by introducing light trapping [21]. We therefore consider the grating structure shown in Fig. 1(a). The solid line in Fig. 1(c) shows the J-V curve associated with an optimized grating structure with an effective thickness of  $L = \left(1 - \frac{wh}{La}\right) L' = 43.8 \text{ nm}$  where  $L' = 50 \text{ nm}$  is the actual grating structure thickness,  $a = 456 \text{ nm}$  is the grating periodicity, and ( $w = 0.24 a, h = 0.52 L'$ ) are the (width, height) of the grating's air groove [Fig. 1(a)]. [The definition of the effective thickness here is such that different structures with the same effective thickness have the same amount of absorbing material.] Figure 1(c) shows a large enhancement in the  $J_{sc}$  of the grating structure relative to that of the thin film. Moreover, the  $V_{oc}$  of the grating structure is maintained at approximately the same enhanced value as that of the thin film.

Figures 1(d)-1(f) compares the  $J_{sc}$ ,  $V_{oc}$  and efficiency ( $\eta$ ) of the thin film and the grating structure for a wide range of thicknesses  $L$ . The efficiency here is defined as  $\eta = \text{FF} \frac{J_{sc} V_{oc}}{P_{inc}} \times 100\%$  where  $P_{inc}$  is the total incident sun radiation power per unit cell area, and FF is the cell's fill-factor [24]. The grating structures in Figs. 1(d)-1(f) have a periodicity  $a = 456 \text{ nm}$ , and air groove dimensions ( $w = 0.24 a, h = 0.52 L'$ ) [Fig. 1(a)]. We see that, in general, the grating structures maintain the same  $V_{oc}$  enhancement as in the thin films while still drastically improving the  $J_{sc}$  and, hence, the efficiency of such thin solar cell structures.

#### 4. Physics of voltage enhancement

To illustrate the physics of  $V_{oc}$  enhancement in nanophotonic structures, we first examine the thermal emission properties of an  $L = 10 \mu\text{m}$  thick bulk structure with multi-layer anti-reflection coating on its front surface. Figure 2(a) shows its absorptivity spectra as a function of incident polar angle  $\theta$ . We see that the absorptivity of the bulk structure is  $\sim 100\%$  for almost all polar angles and wavelengths up to the GaAs band edge at  $\sim 870 \text{ nm}$  [38]. This strong absorptivity results in a strong radiative generation rate  $F_s$  [Eq. (2)] and, consequently, a  $J_{sc}$  of 31.6mA,

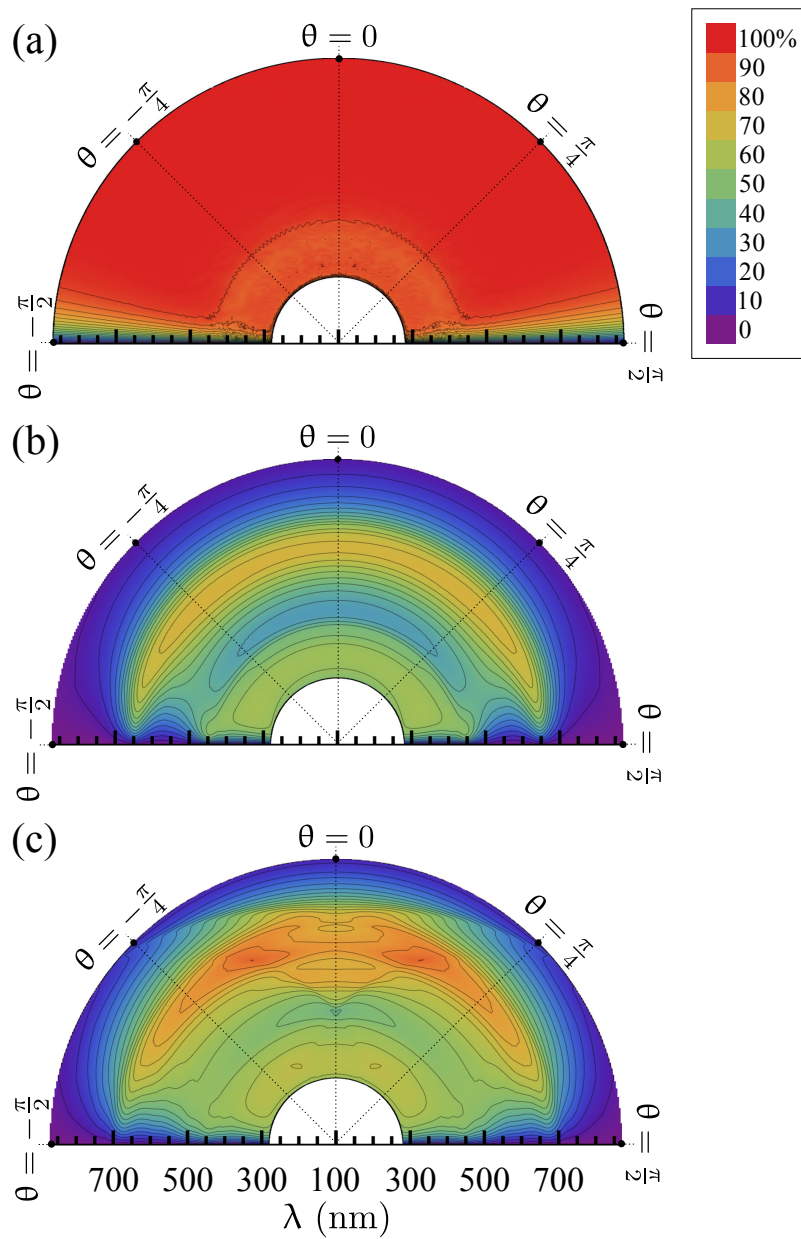


Fig. 2. Contour-density plots of absorptivity spectra for different polar angles  $\theta$  of: (a) an  $L = 10 \mu\text{m}$  thick bulk GaAs structure with a multi-layer anti-reflection coating on its front surface, (b) thin film structure associated with dashed J-V curve in Fig. 1(c), and (c) grating structure associated with solid J-V curve in Fig. 1(c). The plot in (c) includes an integration over all azimuthal angles  $\phi$  for each polar angle  $\theta$ . The absorptivity in all plots is the mean absorptivity of the transverse electric and transverse magnetic incident polarizations.

which is close to the maximum possible  $J_{sc}$  for a GaAs solar cell [3]. However, this strong absorptivity also results in a thermal equilibrium recombination rate  $F_{co}$  [Eq. (4)] that is close to the maximum value for GaAs. Therefore, the contrast between  $F_s$  and  $F_{co}$  is low [Eq. (7)], resulting in a  $V_{oc}$  of 1.16V, which is significantly lower than the enhanced  $V_{oc}$  of the nanostructures associated with the J-V curves in Fig. 1(c).

Figure 2(b) shows the absorptivity spectra for the thin film associated with the dashed J-V curve in Fig. 1(c). The key difference from the bulk structure's spectra in Fig. 2(a) is the thin film's strong absorption suppression in the region of the GaAs band edge at  $\sim 870$  nm for all polar angles  $\theta$ . Such absorption suppression in the immediate frequency range above the bandgap in Fig. 2(b) results in a large reduction of the thin film's thermal equilibrium recombination rate  $F_{co}$ . This can be seen by examining the spectral integration above the bandgap frequency  $\omega_g$  in Eq. (4). Since the cell is operating at a temperature  $T_c$  that satisfies  $kT_c \ll \hbar\omega_g$ , the thermal emission photon flux spectrum  $\Theta(\omega)$  in Eq. (4) can be approximated as:

$$\Theta(\omega) \approx \frac{\omega^2}{4c^2\pi^3} \exp\left(-\frac{\hbar\omega}{kT_c}\right) H(\omega - \omega_g) \quad (10)$$

where  $H(\cdot)$  is the Heaviside step function. Therefore, the thermal emission spectrum of the cell is located immediately above the bandgap and has a relatively narrow width of  $kT_c$ . Accordingly, reducing the absorption of the cell in this frequency range will have a very strong influence on the cell's thermal emission and, hence, strongly influences  $F_{co}$ . On the other hand, the thermal emission spectral width corresponds only to a very small fraction of the total absorption bandwidth of the cell. Moreover, since the solar radiation has a much wider bandwidth, reducing the absorption in the immediate frequency range above the bandgap has far less influence on the radiative generation rate  $F_s$ . Therefore, it follows that reducing the absorption in the immediate frequency range above the bandgap increases the contrast between  $F_s$  and  $F_{co}$  [Eq. (7)] and, consequently, leads to the enhancement of the structure's  $V_{oc}$ .

For the thin film's absorptivity spectra in Fig. 2(b), we see that there is also a strong reduction of absorption at normal incidence  $\theta = 0$  over the entire frequency range above the bandgap as compared to the bulk structure's spectra in Fig. 2(a). This leads to a significant reduction of the  $J_{sc}$  in the thin film case. However, as illustrated above, the  $V_{oc}$  and  $J_{sc}$  are really controlled by different parts of the absorptivity spectra. This observation motivates incorporating light trapping [21] into the thin film so as to possibly: (i) enhance the solar radiation absorption away from the bandgap region in order to enhance the thin film's  $J_{sc}$ , and (ii) still preserve the absorption suppression in the  $kT_c$  region immediately above the bandgap in order to maintain the  $V_{oc}$  enhancement over the bulk structure. The optimized grating structure associated with the solid J-V curve in Fig. 1(c) indeed has such characteristics in its absorptivity spectra [Fig. 2(c)]. In particular, the spectra in Fig. 2(c) preserves the absorption suppression in the spectral region immediately above the GaAs bandgap. Consequently, the grating structure has a  $V_{oc}$  of 1.2V, which is a large enhancement over the bulk structure, and similar to the  $V_{oc}$  of the thin film with equivalent thickness  $L$  [Fig. 1(c)]. In addition, the grating's enhanced absorption, away from the GaAs bandgap region, at the normal incidence angle  $\theta = 0$  [Fig. 2(c)] results in a large  $J_{sc}$  enhancement over the thin film as shown in Fig. 1(c).

## 5. Conclusion

In summary, we presented a computational tool for the detailed balance analysis of solar cell efficiency, and elucidated the physics of open-circuit voltage enhancement by analyzing the angular and spectral distribution of the cell's absorption spectrum. We showed that the open-circuit voltage ( $V_{oc}$ ) and the short-circuit current ( $J_{sc}$ ) are controlled by different parts of the cell's absorption spectrum, and that the  $V_{oc}$  can be enhanced by suppressing the absorption in



the immediate vicinity of the material's bandgap. Our analysis here, moreover, indicates other opportunities for creating a cell with high  $V_{oc}$  and  $J_{sc}$ . For example, in the angular integral within Eq. (4), the cell's absorption strength is modulated by a  $\sin(2\theta)$  factor. Therefore, the absorption strength in the immediate vicinity of  $\theta = 0$  and  $\theta = \frac{\pi}{2}$  has a negligible contribution to the thermal equilibrium recombination rate  $F_{co}$ . However, strengthening the absorption at the normal incidence angle  $\theta = 0$  does enhance the cell's radiative generation rate  $F_s$  [Eq. (2)]. These observations together with Eq. (5) and Eq. (7) indicate that: (i) significant  $V_{oc}$  enhancement due to absorption suppression comes only from the angular region away from  $\theta = 0$  and  $\theta = \frac{\pi}{2}$  with spectral width  $kT_c$  immediately above the bandgap, and (ii) strengthening the absorption at  $\theta = 0$  leads to both  $V_{oc}$  and  $J_{sc}$  enhancement. We anticipate that our analysis tool will prove useful to understand the ultimate performance of nanophotonic solar cells in general.

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