Dynamically tuned coupled-resonator delay lines can be nearly dispersion free

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We investigate dispersion effects in dynamically tuned, coupled-resonator delay lines. Provided that the system is tuned to a zero-bandwidth state, a signal can be delayed indefinitely with almost no dispersion. We present a theoretical analysis of such a light-stopping system and verify the results using numerical simulations. © 2006 Optical Society of America

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Controlling the group velocity of light is of great interest in classical and quantum information processing.[1,2] For this purpose, static delay lines based on cascaded optical resonators have been widely studied.[3–5] In general, such structures are fundamentally limited by the delay–bandwidth product constraint.[5,6] The maximum delay for a structure of given length is inversely proportional to the system bandwidth. Moreover, it has been pointed out that the use of sharp resonances can lead to strong group-velocity dispersion.[7–9] As a result, the spread of the pulse width increases with propagation distance and hence, delay. Recently, it was shown that the delay-bandwidth constraint of resonator delay lines can be overcome with a dynamic process.[10,11] The key idea is to compress the system bandwidth through refractive index modulation while the signal is in the system. Compared with a static delay line with the same length, and hence similar storage capacity, the result is a much larger signal delay. In this Letter, we show that by further requiring the system bandwidth to be compressed to zero,[11] dispersion can be strongly suppressed. In such a light-stopping process, the delay time can be increased arbitrarily without increasing dispersion. Thus, a properly designed, dynamically tuned, coupled-resonator delay line can be nearly dispersion free.

We first examine the performance of a static coupled-resonator delay line. The dispersion relation $\omega(k)$ can be expanded around a wave vector k_c as

$$\omega(k) \approx \omega(k_c) + \omega_{k_c}^{(1)}(k-k_c) + \frac{\omega_{k_c}^{(2)}}{2!}(k-k_c)^2, \qquad (1)$$

where $\omega_{k_c}^{(n)} \equiv \mathrm{d}^n\omega(k)/\mathrm{d}k^n|_{k=k_c}$. Consider a Gaussian pulse with frequency centered at $\omega(k_c)$, with an initial width in time Δt_{in}. (The width is defined as the amplitude standard deviation of the pulse.) The delay time τ is related to the system length L by $\tau = L/v_g$, where $v_g = \omega_{k_c}^{(1)}$ is the group velocity. The final width at the output Δt_{out} is given by

$$\Delta t_{\mathrm{out}}^2 = \Delta t_{\mathrm{in}}^2 + \left[\frac{\tau\omega_{k_c}^{(2)}}{v_g^2 \Delta t_{\mathrm{in}}}\right]^2 \qquad (2)$$

and increases with delay time. As a result, the maximum achievable delay in a static lossless delay line is dispersion limited.

In contrast, a properly designed, dynamic delay line can overcome such dispersion limitations. The line is described by a time-varying dispersion relation $\omega(k,t)$. We assume that $\omega(k,t)$ results from a translationally invariant tuning process, so that each signal wave vector component is preserved.[10,11] Expanding around k_c,

$$\omega(k,t) \approx \omega(k_c,t) + \omega_{k_c}^{(1)}(t)(k-k_c) + \frac{\omega_{k_c}^{(2)}(t)}{2!}(k-k_c)^2, \qquad (3)$$

where $\omega_{k_c}^{(n)}(t) = \partial^n\omega(k,t)/\partial k^n|_{k=k_c}$. $v_g(t) \equiv \omega_k^{(1)}(t)$ is the group velocity. The spatial profile of the signal can be written as

$$E(z,t) = \int \mathrm{d}k \exp\left[j\int_0^t \omega(k,t')\mathrm{d}t'\right]E(k,t=0)\exp(-jkz)$$
$$= E_o \exp\left[-\frac{(z-z_t)^2}{2\sigma_t^2} - j(k_c z - \theta_t)\right], \qquad (4)$$

where

$$z_t \equiv \int_0^t v_g(t')\mathrm{d}t', \qquad (5)$$

$$\theta_t \equiv \int_0^t \omega_{k_c}(t')\mathrm{d}t', \qquad (6)$$

$$\sigma_t^2 \equiv [v_g(0)\Delta t_{\mathrm{in}}]^2 + i\int_0^t \omega_{k_c}^{(2)}(t')\mathrm{d}t'. \qquad (7)$$

The output width in space, Δz_t, is

$$\Delta z_t^2 \equiv \frac{1}{\mathrm{Re}(1/\sigma_t^2)} = [v_g(0)\Delta t_{\mathrm{in}}]^2 + \left[\frac{\int_0^t \omega_{k_c}^{(2)}(t')\mathrm{d}t'}{v_g(0)\Delta t_{\mathrm{in}}}\right]^2. \qquad (8)$$

We now specialize to a light-stopping process, in which the system bandwidth is adiabatically and reversibly compressed to zero.[11] In the zero-bandwidth state, the group velocity vanishes. Assume that the band is compressed during a time T, held for a time τ', and then decompressed during an additional time T. The total delay $\tau = 2T + \tau'$ can be made arbitrarily long by increasing the holding time, τ'. Because τ' is independent of the initial signal bandwidth, the conventional delay–bandwidth constraint for static resonator delay lines does not apply.[10,11]

The evolution of the pulse during the light-stopping process can be inferred from Eqs. (4)–(8). During bandwidth compression, the pulse slows down as $v_g(t)$ is reduced to zero, and the pulse width slightly increases. During the holding time, $v_g(t)$ is identically zero, and the signal pulse remains stopped. Moreover, since $\omega_{k_c}^{(2)}(t)$ as well as all higher-order derivatives are zero, no pulse spreading occurs. During bandwidth decompression, the pulse speeds up, returning to its original group velocity and spreading slightly. Using Eq. (8), the output signal width in time is

$$\Delta t_{\text{out}}^2 = \Delta t_{\text{in}}^2 + \left[\frac{\int_0^T \omega_{k_c}^{(2)}(t')\mathrm{d}t' + \int_{\tau-T}^{\tau} \omega_{k_c}^{(2)}(t')\mathrm{d}t'}{v_g^2(0)\Delta t_{\text{in}}} \right]^2 . \tag{9}$$

In the limit that $2T \ll \tau$,

$$\Delta t_{\text{out}}^2 \ll \Delta t_{\text{in}}^2 + \left[\frac{\tau \omega_{k_c}^{(2)}(0)}{v_g^2(0)\Delta t_{\text{in}}} \right]^2 . \tag{10}$$

This result is in contrast to the static resonator delay line result [Eq. (2)]. In the static delay line, the signal width increases throughout the entire propagation process. In the dynamically tuned light-stopping system, the signal width increases only while the pulse is being slowed down (bandwidth compression) or sped up (bandwidth decompression). For the majority of the delay time, the pulse does not spread at all.

Below, we verify these results using the example light-stopping system shown in Fig. 1. The unit cell consists of two main resonators with mode amplitudes, $q_i(t)$ and $r_i(t)$, and one side resonator with mode amplitude $s_i(t)$. Each main resonator has a static resonance frequency ω_0, while the side resonator has a dynamic resonance frequency $\omega_s(t)$. The coupling rate between neighboring main resonators is κ, while the coupling rate between neighboring main and side resonators is κ_s; non-adjacent resonators do not couple directly. The system can be described by the following equations, which have been

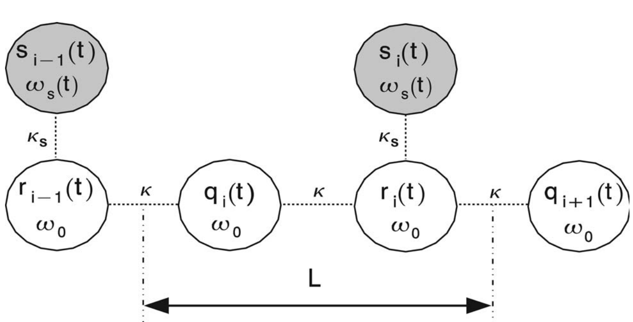

Fig. 1. A section of the example coupled-resonator light-stopping system. The unit cell has length L.

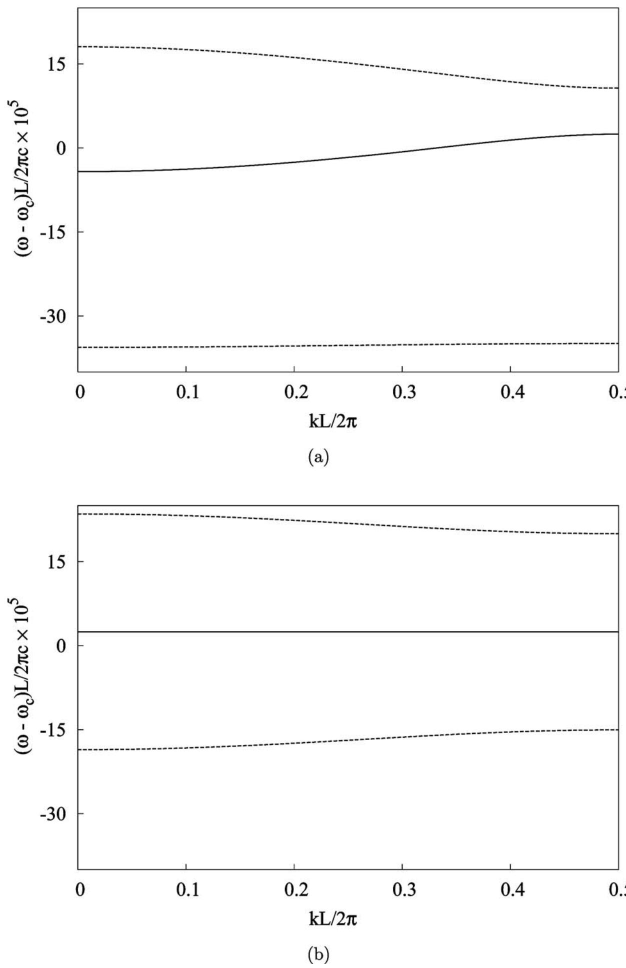

Fig. 2. Band structure for the dynamic system shown in Fig. 1. (a) Detuning $\Delta = -9.43 \times 10^{-5}\omega_c$. (b) Detuning $\Delta = 0$.

shown to accurately describe light propagation in dynamic photonic-crystal resonator systems,[10]

$$\frac{\mathrm{d}q_i(t)}{\mathrm{d}t} = j\omega_0 q_i(t) + j\kappa r_i(t) + j\kappa r_{i-1}(t), \tag{11}$$

$$\frac{\mathrm{d}r_i(t)}{\mathrm{d}t} = j\omega_0 r_i(t) + j\kappa q_{i+1}(t) + j\kappa q_i(t) + j\kappa_s s_i(t), \tag{12}$$

$$\frac{\mathrm{d}s_i(t)}{\mathrm{d}t} = j\omega_s(t) s_i(t) + j\kappa_s r_i(t). \tag{13}$$

Using the discrete translational invariance, the dispersion relation can be derived:

$$\cos(kL) = \frac{1}{2\kappa^2}(\omega - \omega_0)\left[\omega - \omega_0 - \frac{\kappa_s^2}{\omega - \omega_s(t)} \right] - 1. \tag{14}$$

Figure 2 shows the band structure for two different detuning conditions. (Similar band structures have been calculated for quantum well Bragg structures.[12]) In Fig. 2(a), the upper two bands describe propagating modes that are concentrated in the main resonators and have relatively large bandwidths. ω_0 was chosen to place $\omega_c = 2\pi c/(1.55\ \mu\text{m})$ in the linear region of the second band (indicated by a solid line); $(\omega_0 - \omega_c)/\omega_c = 8 \times 10^{-6}$. The detuning of the side resonator is $\Delta \equiv \omega_s - \omega_0 = -9.43 \times 10^{-5}\omega_c$. For this value, the side resonator is detuned far from the main resonators and gives rise to a nearly flat band in the band structure at ω_s. We assume a lattice constant $L = 4.8\ \mu\text{m}$, a reasonable value for photonic crystal implementations. The coupling constant $\kappa = 1.89 \times 10^{-5}\omega_c$ was chosen such that the second band would accomodate a signal of nanosecond time width. Figure 2(b) shows the band structure for a detuning $\Delta = 0$. (Tuning to $\Delta = 0$ requires a refractive index modulation of $\sim 10^{-4}$, which can be achieved by free carrier injection.[13]) Following the procedure in Ref. 11, one can prove that the band at ω_s is completely flat. Furthermore, it can be shown that the separation between the flat band and either of the two other bands is $\geqslant \kappa_s$. $\kappa_s = 3\kappa$ was chosen large enough to prevent energy loss to other bands during tuning.

By dynamically varying the system from Fig. 2(a) to Fig. 2(b), one can stop a light pulse. To study pulse propagation, Eqs. (11)–(13) were solved numerically for a system of $N = 43$ unit cells. A Gaussian input signal with $\Delta t_{\text{in}} = 0.5$ ns was used [Fig. 3(a)]. The detuning was reduced from its initial to final value on a time scale of $8/\kappa$. (To maintain the adiabatic condition, the modulation time needs to be[14] $\gg 1/\kappa$.) After a finite holding time, the dynamic process was applied in reverse to retrieve the signal. The solid curve in Fig. 3(b) shows the pulse output. Notice that the pulse is still Gaussian. For comparison, the circles in Fig. 3(b) are obtained from the analytical model of Eqs. (3) and (4), in which $\omega(k)$ was expanded up to second order. Figure 3(c) shows the output of a static coupled-resonator delay line corresponding to the system of Fig. 2(a). Note that the delay time is far shorter. While a longer static structure [$N = 13286$ in Fig. 3(d)] achieves a comparable delay to the dynamic structure, the pulse disperses and becomes distorted. Figure 4(a) shows the ratio of output- to input-signal width for both the dynamic and static systems. In the

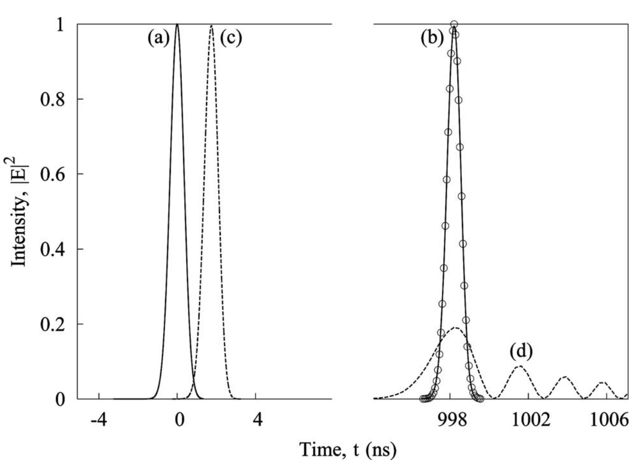

Fig. 3. Signal profiles: (a) input signal, (b) signal at output of dynamic light-stopping system with $N = 43$, (c) signal at output of static line with $N = 43$, and (d) output pulse for a static line with $N = 13286$.

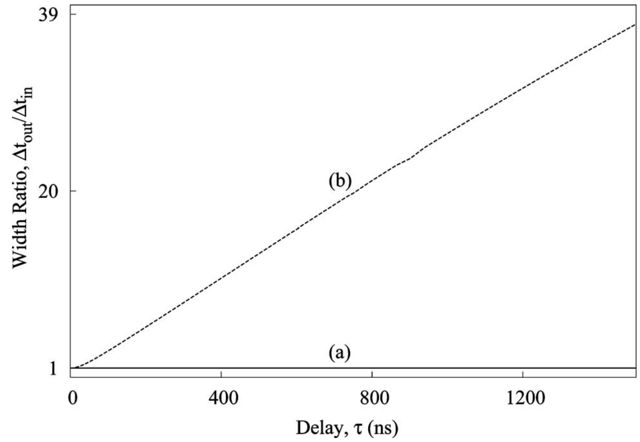

Fig. 4. Ratio of output- to input-signal width: (a) dynamic light-stopping system with $N = 43$. Delay is increased by increasing the holding time. (b) Static line. Delay is increased by increasing the length N.

light-stopping system, the ratio is constant with increasing delay, since pulse spreading occurs only during bandwidth compression and decompression. In contrast, for the static line, the pulse width increases arbitrarily with increasing delay [Fig. 4(b)].

In conclusion, a light-stopping delay line made from dynamically tuned coupled resonators strongly suppresses dispersion effects when a zero-bandwidth state is used. In contrast, significant dispersion effects have been observed in a dynamic system lacking a zero-bandwidth state.[15] Thus, the correct design of the system is crucial for realizing the potential of dynamic resonator systems.

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