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Quantum critical coupling conditions for zero single-photon transmission through a coupled atom-resonator-waveguide system

Jung-Tsung Shen (沈榮聰)[1,*] and Shanhui Fan (范汕洄)[2]
[1]*Department of Electrical and Systems Engineering, Washington University in St. Louis, St. Louis, Missouri 63130, USA*
[2]*Ginzton Lab and Department of Electrical Engineering, Stanford University, Stanford, California 94305, USA*
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The quantum critical coupling conditions, where single-photon transmission goes to zero, for systems of a coupled ring resonator and atom in a one-dimensional single-mode waveguide are discussed. The quantum critical coupling conditions could be achieved even with nonvanishing intrinsic losses of the resonator and the atom, with intermode backscattering between the whispering-gallery modes of the ring resonator, and with time-reversal symmetry breaking. The energy transfer efficiency from the waveguide to the resonator and the atom is also discussed.

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For a system of coupled waveguides and resonators in general, the critical coupling is defined as when the transmission of the *classical* input signal at resonance goes to zero at the output port [1–3]. The notion of critical coupling is fundamental to any systems of coupled waveguides and resonators, and consequently, has important applications in both engineering [4–11] and fundamental science [12–19]. With recent advances in efficient on-demand single-photon sources [20–22], and proposals of using single photons for quantum information processing, a practical question naturally arises: When a quantum system includes atomic degrees of freedom that couple to single photons, what are the criteria for the *single-photon* transmission to be zero? Such quantum critical coupling conditions would be of crucial importance, for example, in the designs of single-photon devices that regulate single-photon transport for photon-based quantum information processing, and in quantum optics experiments probing photon-photon correlations [18,23–25], where one often desires to completely eliminate single-photon transmission so that the signatures of two-photon correlated transmission is most prominent. In this paper, we investigate this problem and present the quantum critical coupling conditions for single-photon transmission.

The critical coupling conditions for a general system of coupled waveguides and resonators involves the intricate balance between the characteristics of the constituents: The internal modes and the intrinsic dissipations of the resonators, the interresonator couplings, the resonator-waveguide coupling, and the frequency detuning between resonators and the photon. It should be pointed out that it is not always possible to achieve the critical coupling conditions. To illustrate this point, consider the simplest case of a standing-wave cavity side-coupled to a single-mode waveguide. When the intrinsic loss of the cavity is present, zero transmission is not possible, regardless if an atom is coupled to the cavity or not [26]; in contrast, for a ring resonator, without an atom, coupled to a waveguide, the critical coupling can be achieved when the intrinsic dissipation of the resonator matches the coupling to the waveguide [1–3].

Here we show that, in a general system involving a waveguide side-coupled to a ring resonator that couples to an atom (Fig. 1), there exists a wide range in the parameter space wherein critical coupling conditions can be satisfied by continuously varying the experimental parameters. In particular, the critical coupling can be achieved with or without the intrinsic dissipations of the resonators. Moreover, we show that the maximal energy transfer from the waveguide to the resonator in general does not occur simultaneously with the quantum critical coupling conditions; the energy transfer is maximal at critical coupling only for special cases, such as a lossy ring resonator with small backscattering and with the atom decoupled.

The system considered in this paper is shown in Fig. 1: A two-level atom interacts with a whispering-gallery-type resonator which is side-coupled to a single-mode waveguide. The two-level atom can be implemented using either a real atom or a quantum dot with appropriate energy levels; the whispering-gallery type resonator possesses azimuthal rotational symmetry and supports a pair of degenerate counterpropagating whispering-gallery modes (WGM's), such as a ring resonator, microtoroid, microdisk, microsphere, or a two-dimensional dielectric cylinder. In the following we will simply use "atom" and "ring resonator" for brevity. Such a configuration has been extensively studied both theoretically and experimentally [17,18,24,27–32] in the context of quantum optics, cavity quantum electrodynamics, single-atom manipulation and detection, and biochemical sensing. Moreover, such a system of coupled oscillators could have potential applications in future chip-scale integrated photonic circuits such as modulators, optical filters, sensors, optical switches, and dispersion engineering [3,33].

The single-photon transmission amplitude $t(\omega)$ of the coupled system in Fig. 1 is given in Ref. [34], where the possibility of zero transmission for the special on-resonance case (i.e., the ring resonator and the atom are in-tuned) is briefly mentioned. In this paper, we provide a comprehensive discussion of the underlying physics that results in zero transmission.

*jushen@ese.wustl.edu

The explicit form of $t(\omega)$ is [34]

$$t(\omega) = \frac{\left(\omega-\omega_c+i\frac{1}{\tau_c}\right)\left[\left(\omega-\Omega+i\frac{1}{\tau_q}\right)\left(\omega-\omega_c+i\frac{1}{\tau_c}\right)-G_+^2\right]+\left(\omega-\Omega+i\frac{1}{\tau_q}\right)\Gamma^2-g_a^*g_bh-g_ag_b^*h^*-|h|^2\left(\omega-\Omega+i\frac{1}{\tau_q}\right)+iG_-^2\Gamma}{\left(\omega-\omega_c+i\frac{1}{\tau_c}+i\Gamma\right)\left[\left(\omega-\Omega+i\frac{1}{\tau_q}\right)\left(\omega-\omega_c+i\frac{1}{\tau_c}+i\Gamma\right)-G_+^2\right]-g_a^*g_bh-g_ag_b^*h^*-|h|^2\left(\omega-\Omega+i\frac{1}{\tau_q}\right)}, \tag{1}$$

where ω_c is the is the resonance frequency of the WGM's, Ω is the atomic transition frequency, and $1/\tau_c \equiv \gamma_c$ and $1/\tau_q \equiv \gamma_q$ are the intrinsic dissipation rates of each WGM of the ring resonator and of the atom, respectively. g_a and g_b are the resonator-atom coupling strength for WGM a and b modes, respectively (see Fig. 1). Γ is the external linewidth of each WGM due to waveguide-cavity coupling. h is the intermode backscattering strength between the two degenerate WGM's, due to, for example, the imperfection of the resonator. g_a, g_b, and h, in general, are complex numbers. $G_+^2 \equiv |g_a|^2 + |g_b|^2$. $G_-^2 \equiv |g_a|^2 - |g_b|^2$. $G_-^2 \neq 0$ signifies the breaking of the time-reversal symmetry in the atomic degrees of freedom [34], which occurs, for example, when the degeneracy of the atomic energy levels (in fine or hyperfine manifolds) is lifted by an external magnetic field, such that the transition between the two energy levels of interest couples only to σ^+ or σ^- circularly polarized light [35]. We note that the reflection amplitude r is directly proportional to the excitation amplitude of WGM b mode which is induced by h and by coupling to the atom [34], and therefore is generally nonzero at critical coupling conditions when the transmission is zero.

The quantum critical coupling conditions are given by solving $t(\omega) = 0$. Setting both the real and imaginary parts to zero yield the following conditions

$$\gamma_q\Gamma^2 + G_-^2\Gamma = \gamma_q\left(|h|^2+\gamma_c^2-\Delta_c^2\right) + \gamma_c G_+^2 - 2\gamma_c\Delta_c\Delta_q, \tag{2a}$$

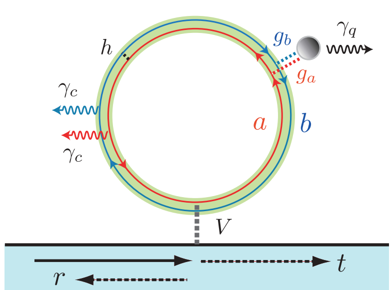

FIG. 1. (Color online) Schematics of the system. A two-level atom (gray sphere) is coupled to a ring resonator (green ring) which is side-coupled to a single-mode waveguide (blue channel). The two degenerate whispering-gallery modes are labeled by a (counterclockwise) and b (clockwise), respectively. The coupling between the atom and each WGM is g_a and g_b, respectively. The coupling strength of each WGM to the waveguide is V. The strength of the intermode scattering between two WGM's is h. The intrinsic loss of the atom and of each WGM is γ_q and γ_c, respectively. t and r are the transmission and reflection amplitude, respectively.

$$\Delta_q\Gamma^2 = \Delta_q\left(|h|^2+\gamma_c^2-\Delta_c^2\right) + \Delta_c G_+^2 + 2\gamma_c\gamma_q\Delta_c + g_a^*g_bh + g_ag_b^*h^*, \tag{2b}$$

where $\Delta_c \equiv \omega - \omega_c$ and $\Delta_q \equiv \omega - \Omega$ are the frequency detuning with the resonator and with the atom, respectively. Equations (2a) and (2b) are the most general form of the quantum critical coupling conditions for the system. Depending upon the values of the parameters, the set of equations may or may not have real solutions for ω.

To have a better understanding of the rather complicated critical coupling conditions Eqs. (2a) and (2b) and to gain deeper insights of the properties of the system at such conditions, we first examine some special cases.

(1) *Lossless resonator, atom decoupled, with backscattering* ($\gamma_c = 0$, $g_a = g_b = 0$, $h \neq 0$): In this case, Eqs. (2a) and (2b) reduce to the following single condition:

$$\Delta_c^2 = |h|^2 - \Gamma^2. \tag{3}$$

For $|h| > \Gamma$, there exists two solutions for Δ_c and thus, in general, there are two dips in the transmission spectrum, with a spectral separation $2|\Delta_c| = 2\sqrt{|h|^2 - \Gamma^2}$; the two dips merge when $|h| = \Gamma$. The critical coupling conditions cannot be satisfied when $|h| < \Gamma$. The intermode backscattering h induces the clockwise WGM b mode; together with the counterclockwise WGM a mode, they form a stationary and nonrotating cavity-like mode that gives rise to the zero transmission [26]. The reflection amplitude is proportional to the amplitude of the WGM b mode (which is $\propto h$) and is, in general, nonzero.

(2) *Lossy resonator, atom decoupled, with backscattering* ($\gamma_c \neq 0$, $g_a = g_b = 0$, $h \neq 0$): When the resonator is lossy, Eqs. (2a) and (2b) reduce to a pair of conditions

$$\Gamma^2 = \gamma_c^2 + |h|^2, \tag{4a}$$

$$\Delta_c = 0. \tag{4b}$$

Thus, when the ring resonator becomes lossy, the critical coupling could be achieved only at $\omega = \omega_c$, the resonance frequency of the resonator.

Figure 2 plots a phase diagram on the $\gamma_c - |h|$ plane that summarizes the ring resonator-waveguide case. The black thick line denotes the "critical coupling curve" on which the critical coupling conditions are satisfied: On the circular arc part ($\gamma_c \neq 0$) of the critical coupling curve, defined by Eq. (4a), there is only one critical frequency at $\omega = \omega_c$ at which the single-photon transmission is zero; on the vertical part of the curve ($\gamma_c = 0$ and $|h| > \Gamma$), there are two critical frequencies given by Eq. (3), as shown by the transmission spectra plotted at several different values of γ_c and $|h|$.

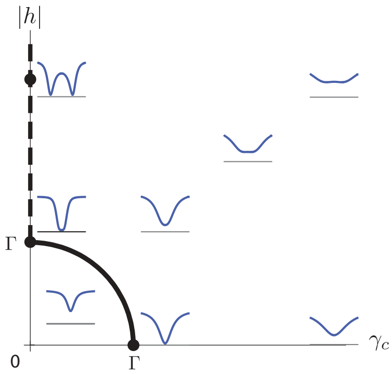

FIG. 2. (Color online) The phase diagram for the system of coupled ring resonator-waveguide. The black thick line denotes the critical coupling curve where the critical coupling conditions are satisfied [dashed: Eq. (3); solid: Eq. (4a)]. The transmission spectra for several different values of parameters are plotted.

Another important quantity of practical interest is the energy transfer from the waveguide to the resonator [36], which is proportional to $\mathcal{W}(\omega) \equiv |e_a(\omega)|^2 + |e_b(\omega)|^2$, where e_a and e_b are the excitation amplitude of WGM a and b modes, respectively. On the circular arc part, when the critical coupling conditions of Eqs. (4a) and (4b) are satisfied, $\mathcal{W}(\omega_c) = v_g/(\gamma_c + \Gamma)$. This energy, however, in general is *not* the maximum energy that could be carried by the resonator. On the circular arc part, for example, the condition for the resonator to carry the maximum energy at $\omega = \omega_c$ is $3|h|^2 < (\gamma_c + \Gamma)^2$, given by $\partial^2 \mathcal{W}(\omega_c)/\partial\omega^2 < 0$, which is different from the critical coupling condition of Eq. (4a). Only at the special cases when $h = 0$, the resonator carries the maximum energy at critical frequency [1–3,36], with $\mathcal{W}(\omega_c) = 2v_g\Gamma/(\gamma_c + \Gamma)^2 > \mathcal{W}(\omega \neq \omega_c)$. When $3|h|^2 > (\gamma_c + \Gamma)^2$, the frequency of maximum power transfer occurs at $\Delta_c^2 = 2|h|\sqrt{|h|^2 + (\gamma_c + \Gamma)^2} - |h|^2 - (\gamma_c + \Gamma)^2$. At these two frequencies, $\mathcal{W}(\omega = \omega_c \pm |\Delta_c|) = 1/4[|h|\sqrt{|h|^2 + (\gamma_c + \Gamma)^2} - |h|^2]$. A classical analysis has been discussed in Ref. [15]. Note that cases (1) and (2) also apply to the case where the input is a weak cw laser.

(3) *Lossless resonator, lossless atom, no backscattering* ($\gamma_c = 0$, $\gamma_q = 0$, $h = 0$): When a lossless atom is coupled, the critical coupling conditions Eqs. (2a) and (2b) reduce to

$$G_-^2 = 0, \tag{5a}$$

$$\Delta_q\left(\Gamma^2 + \Delta_c^2\right) = \Delta_c G_+^2. \tag{5b}$$

Equation (5a) indicates that the time-reversal symmetry breaking term G_-^2 must be zero for a lossless system at critical coupling. Equation (5b) can be rewritten as a cubic polynomial for Δ_c (using $\Delta_q = \Delta_c + \omega_c - \Omega$):

$$\Delta_c^3 + (\omega_c - \Omega)\Delta_c^2 + (\Gamma^2 - G_+^2)\Delta_c + \Gamma^2(\omega_c - \Omega) = 0, \tag{6}$$

of which the criterion for the number of solutions could be easily determined from the standard discriminant criterion [37]. For the special case of $\omega_c = \Omega$, Eq. (6) yields the simple criterion that Eq. (6) has three different solutions for ω when $G_+^2 > \Gamma^2$, and thus the transmission spectrum has three dips; the three dips merge when $G_+^2 = \Gamma^2$. In the latter case, the atom induces a clockwise WGM b mode which, together with a counterclockwise WGM a mode, form a stationary, nonrotating cavity-like mode that gives rise to the zero transmission.

(4) *Lossless resonator, lossless atom, with backscattering* ($\gamma_c = 0$, $\gamma_q = 0$, $h \neq 0$): When $h \neq 0$, Eqs. (2a) and (2b) reduce to

$$G_-^2 = 0, \tag{7a}$$

$$\Delta_q\left(\Gamma^2 - |h|^2 + \Delta_c^2\right) = \Delta_c G_+^2 + g_a^* g_b h + g_a g_b^* h^*, \tag{7b}$$

which also yield a cubic polynomial for Δ_c and can be analyzed in the same fashion.

(5) *Lossy resonator, lossy atom, with backscattering* ($\gamma_c \neq 0$, $\gamma_q \neq 0$, $h \neq 0$): For general cases with lossy resonator and atom, quantum critical coupling conditions can be attained if the set of equations Eqs. (2a) and (2b) yield *real* values for Δ_c and Δ_q, which in turn determines either Ω or ω_c, when the other is fixed. For example, if the atomic resonance frequency Ω can be fine-tuned, then by adjusting the atomic resonance frequency to $\Omega = \Delta_c - \Delta_q + \omega_c$ would yield zero single-photon transmission at frequency $\omega = \Delta_c + \omega_c (= \Delta_q + \Omega)$. Figure 3 plots the transmission spectra at quantum critical coupling for the general cases. In Fig. 3(a), the intrinsic atomic dissipation γ_q is increased from 0.5Γ to 3Γ, while $G_-^2 = 0$. For the parameters used, γ_q has a critical value at 4Γ, with $(\Delta_q, \Delta_c) = (0,0)$. Beyond the critical value $\gamma_q > 4\Gamma$, quantum critical coupling cannot be reached. Figure 3(b) shows the cases when $G_-^2 \neq 0$. For a system without time-reversal symmetry, quantum critical coupling is possible only

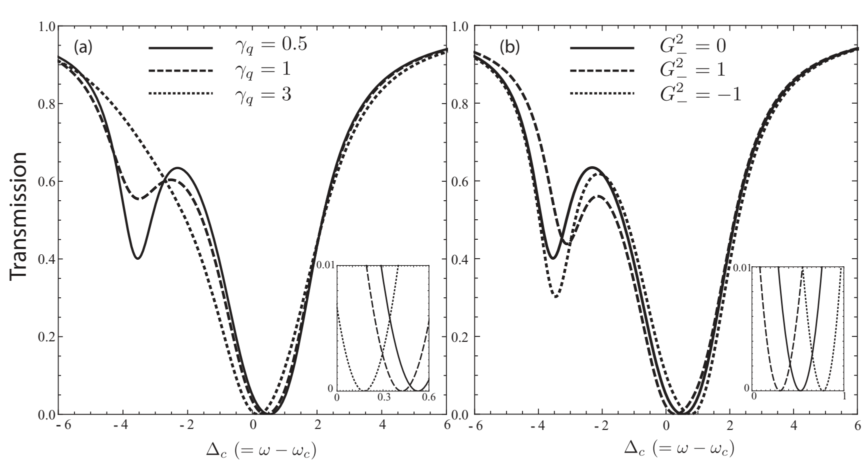

FIG. 3. Transmission spectra at quantum critical coupling. (a) The cases for increasing the intrinsic atom dissipation rate γ_q. $(\Delta_q, \Delta_c, \gamma_q) = (3.05081, 0.527939, 0.5)$ for thick line, $(3.11734, 0.423615, 1)$ for dashed line, $(2.31748, 0.175762, 3)$ for dotted line. $G_-^2 = 0$ for all three curves; (b) The cases for time-reversal symmetry breaking $G_-^2 \neq 0$. $(\Delta_q, \Delta_c, G_-^2) = (3.05081, 0.527939, 0)$ for thick line as a reference, $(2.31142, 0.304428, 1)$ for dashed line, $(3.16926, 0.773354, -1)$ for dotted line. $\gamma_q = 0.5$ for all three curves. $(\gamma_c, G_+^2, |h|, g_a^* g_b h + g_a g_b^* h^*) = (0.5, 4, 0.5, 0)$ is used for all curves in (a) and (b). Γ is the frequency unit. The insets show the details around the dips.

if dissipations are present in the system. With all other parameters fixed in this case, G_-^2 has an upper critical value 1.75Γ, beyond which quantum critical coupling cannot be reached. The critical values, if they exist, in general have to be found by numerically scanning through Eqs. (2a) and (2b). Moreover, we have also numerically confirmed that, when the atom is coupled, at quantum critical coupling, the energy carried by the resonator at critical frequencies in general is not the maximum energy.

We now discuss the experimental feasibility. Each of the set of two critical coupling equations, Eqs. (2a) and (2b), respectively, defines a hyper-surface in the multidimensional parameter space; the critical coupling is achieved when the two hyper-surfaces intersect. In experiments, some parameters are tunable so the system response can be fine-tuned to meet the critical coupling conditions. For example, the waveguide-resonator coupling Γ could be varied by changing the distance between the waveguide and the ring resonator [12,14,17]; the resonator frequency ω_c could be thermally tuned by varying the temperature of the ring resonator [17], while the loss of the resonator γ_c could also be tuned via, for example, carrier injection [38,39] or laser pumping [7], depending upon the realization; the atomic transition frequency Ω could be tuned via external magnetic field or electric field; the phase of g_a and g_b depend upon the angular position of the atom relative to the ring resonator [34]. For example, the tuning capability for resonator resonance frequency ω_c reported in Ref. [17] is 3 MHz per mK of temperature change, while $\Gamma/2\pi$ is approximately 6.9 MHz [17,34]. The explicit form of the critical coupling conditions makes it possible to systematically adjust as few parameters as possible to achieve the critical coupling conditions, and moreover, at a frequency where the resonance is of atomic nature. The latter capability is crucially important for observing strong photon-photon correlations [34].

In summary, we presented the quantum critical coupling conditions for the systems of coupled ring resonator and atom. With the capability of tuning the experimental conditions over a finite range, and the capability of precise control of them, zero single-photon transmission can be achieved and modulated even for complicated coupled systems. As a final remark, we note that the quantum critical coupling conditions could also be derived for more complicated cases consisting of cascaded multiresonators and multiatoms, which would be useful for future photonic integrated circuits.

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