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Transmission Through a Scalar Wave Three-Dimensional Electromagnetic Metamaterial and the Implication for Polarization Control[⋆]

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An interweaving-conductor metamaterial (ICM) is a metamaterial composed of multiple, interlocking, conducting networks. It exhibits unusual optical properties in the low-frequency linear-dispersion regime. In particular, two-network ICM supports only one, non-dispersive mode in the low frequency range, and is best described as an effective medium supporting a scalar wave in full three dimensions. We explore the light transmission properties of such a metamaterial, and the implications of a scalar wave medium for polarization control. Polarizers and polarization rotators with subwavelength sizes are numerically demonstrated.

Keywords: Metamaterial, Scalar Wave, Polarization Control.

1. INTRODUCTION

There have been numerous recent activities in trying to design metamaterials for the control of electromagnetic waves.[1–4] While most proposed metamaterial systems can be described by an effective electric permittivity tensor and magnetic permeability tensor in the low frequency limit, it has been recently recognized that there are exceptions.[5,6] In particular, it has been pointed out in Ref. [5] that interweaving conductor metamaterial (ICM), composed of multiple, interlocking, disjoint conducting networks,[7] is best described by a non-Maxwellian effective medium in the low frequency range. Moreover, a two-interweaving-conductor metamaterial (2-ICM) supports only one mode in the low frequency regime, which is non-dispersive, and is best described as an effective medium supporting a scalar wave in full three dimensions.[5]

In this article, we explore the properties of 2-ICM in terms of transmission of electromagnetic waves, and the implication of such a medium for polarization control. Specifically, we show that, due to the quasi-static nature, the low-frequency scalar mode inside a truncated, finite 2-ICM structure is decoupled from external plane waves. With the use of a coupling layer of a subwavelength thickness (i.e., antenna), on the other hand, one can selectively couple this scalar mode to one of the two external polarization modes of the outside. Such coupling then automatically implies a complete rejection of the other polarization, as explained with a scattering matrix analysis and confirmed by numerical simulations. Moreover, it is demonstrated that, by properly designing the coupling layers on both the input and output sides of a 2-ICM crystal, a subwavelength-size polarization rotator can be realized.

2. THEORY AND NUMERICAL ANALYSIS

As a concrete example, we consider the structure depicted in Figure 1. It has a body centered cubic (BCC) lattice, and consists of two interweaving networks of identical shape. The dispersion relation is plotted in Figure 1(b), which shows only one band in the low frequency range. The singly degenerate nature of this band can be directly inferred from the electric field profile of the mode, which has the same symmetry as the structure (Fig. 1(c)). As has been pointed out in Ref. [5], the number of low frequency bands of such interweaving metamaterials is directly controlled by the number of metal networks. For an N-network structure, there are exactly $N-1$ internal modes. For a 2-ICM, its modal properties are thus best explained in terms of a scalar effective field.

The scalar nature of such a mode has important consequences for the transmission of electromagnetic waves. Intuitively, in the low frequency regime, the wavelength

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[⋆] This is an invited paper.

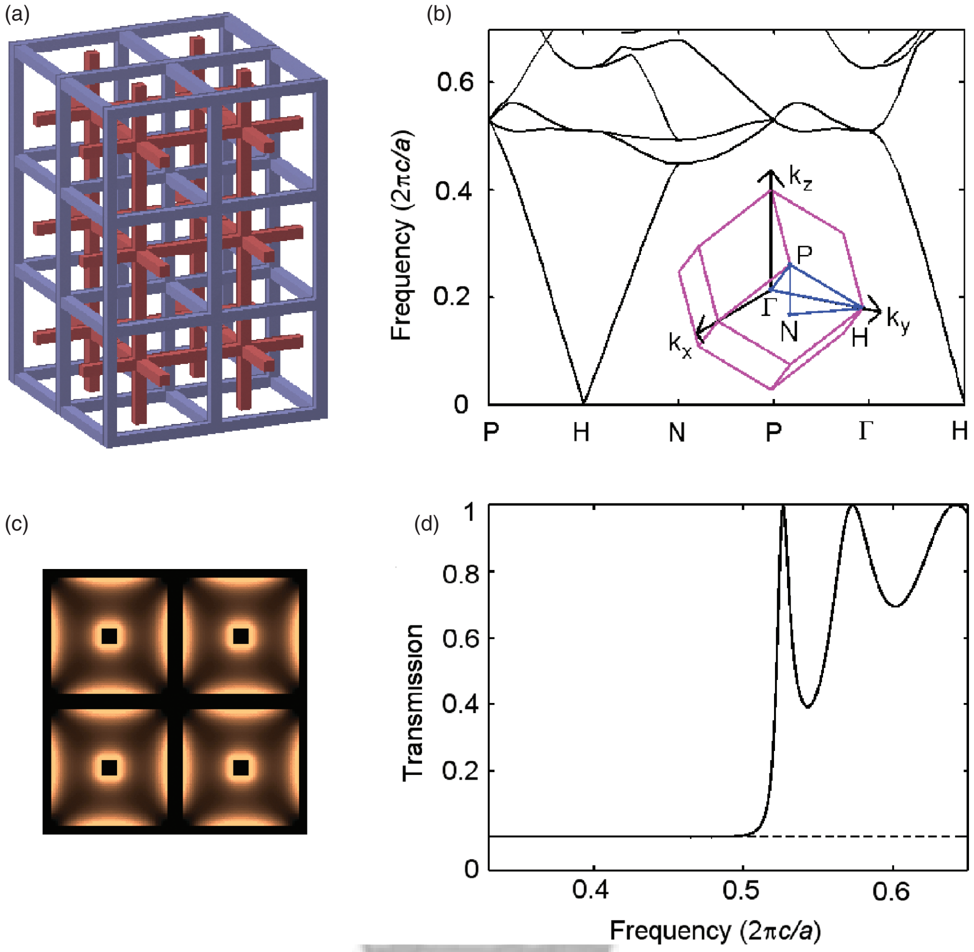

Fig. 1. Two-interweaving-conductor metamaterial structure. (a) A 2-ICM structure with $2 \times 2 \times 3$ unit cells are shown. Blue and red colors indicate separate metal networks. (b) Band structure of the 2-ICM crystal. (c) The distribution of electric field strength, for a mode propagating along the [100] direction, at a frequency of 0.2 c/a. Shown here is the distribution on a (001) surface. The black region denotes the metallic network. (d) Normalized transmission spectrum for an incident plane wave into the slab. The plane wave is polarized along the x-direction. The solid line is for the transmitted x-polarized wave. The dashed line is for the transmitted y-polarized wave.

is much larger than the structural period and the scalar mode has field distributions that closely resemble the electrostatic and magnetostatic field configurations supported by the structure. Hence, the transverse electric field, averaged over the unit cell face, is zero, and the mode does not couple to the incident plane waves. This intuition is validated by a finite-difference time-domain (FDTD) simulation,[8] where we consider a slab of 2-ICM crystal of three-unit-cells thick (Fig. 1(d)). A plane wave that is linearly polarized in the x-direction is incident from air. At low frequencies, the incident wave is almost completely reflected. Significant transmission occurs above a cut-off frequency, when the deviation from quasi-static becomes significant and the modes with different symmetries start to appear in the dispersion relation. The simulation also indicates that, when substantial transmission occurs, the polarization is preserved due to the mirror symmetry of the structure: the measured electric field in y-direction of the transmitted wave is always zero.

To couple the scalar mode to an external plane wave in the low frequency range, we place a 2-D array of micro-antennas at the metamaterial-air interface. Each micro-antenna consists of a two-terminal loop that connects the ends of the two networks (Fig. 2). The direction of this connection (shown in Fig. 2 is an antenna pointing in the x-direction) defines the directionality of the antenna. The antenna alone, by design, transmits both polarizations of the incident waves. When the antenna is connected with the 2-ICM, however, if the electric field of the incident wave is aligned with the direction of the antenna, an electric current is generated and propagates down the two networks (with opposite polarities). If the incident field polarization is normal to the direction of the antenna, the two terminal of the antenna is of the same electric potential and thus no current propagates.

Based on this antenna design, we perform FDTD simulations, in which x-directional antennas are present on both ends of an ICM slab that is ten-unit-cell thick. In the entire

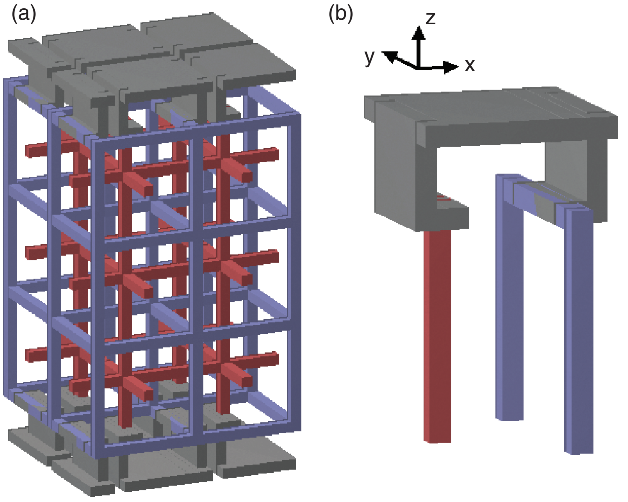

Fig. 2. 2-ICM structure with coupling antennas. (a) A three-unit-cell-thick slab. (b) Magnified view of one antenna and its connection to the two networks of 2-ICM.

low frequency range, it is shown that the y-polarized wave is always completely reflected, while only the x-polarized wave can transmit (Fig. 3(b)). With the use of a layer of anti-reflection dielectric layer on top of the antenna to match the impedance between air and the structure, the transmission for the x-polarized wave becomes close to 1 over a broad frequency range (Fig. 3(c)).

A very unusual property of a scalar wave medium is that a complete coupling of one external polarization to the internal scalar mode automatically implies a complete rejection of the other external polarization. This can be understood using an S-matrix description as follows: At the air-ICM interface, the scattering matrix in general is a 3-by-3 matrix:

$$\begin{pmatrix} b_x \\ b_y \\ b_s \end{pmatrix} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_s \end{pmatrix}$$

where the subscripts x and y denote the two linear polarization states of light in air and the subscript s refers to the scalar wave mode inside the ICM. The a's and b's represent the incoming and outgoing wave amplitudes respectively. Assuming, for example, the x-polarization can couple perfectly into the scalar mode, i.e., $S_{xs} = 1$. Then, from the reciprocity and energy conservation, the entire scattering matrix is determined (up to phase constants):

$$\begin{pmatrix} b_x \\ b_y \\ b_s \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_s \end{pmatrix}$$

Consequently, the other polarization, y-polarization, is completely reflected ($S_{yy} = 1$). Therefore, as long as the structure is designed to admit one polarization completely, the other polarization is then automatically rejected. This exclusivity is solely due to the existence of only a single internal mode. For systems with two internal modes, the knowledge of the transmission of one outside polarization does not determine the transmission of the other polarization.

The directionality of the antenna determines to which external polarization the internal scalar wave couples. Once the external energy is coupled into the structure,

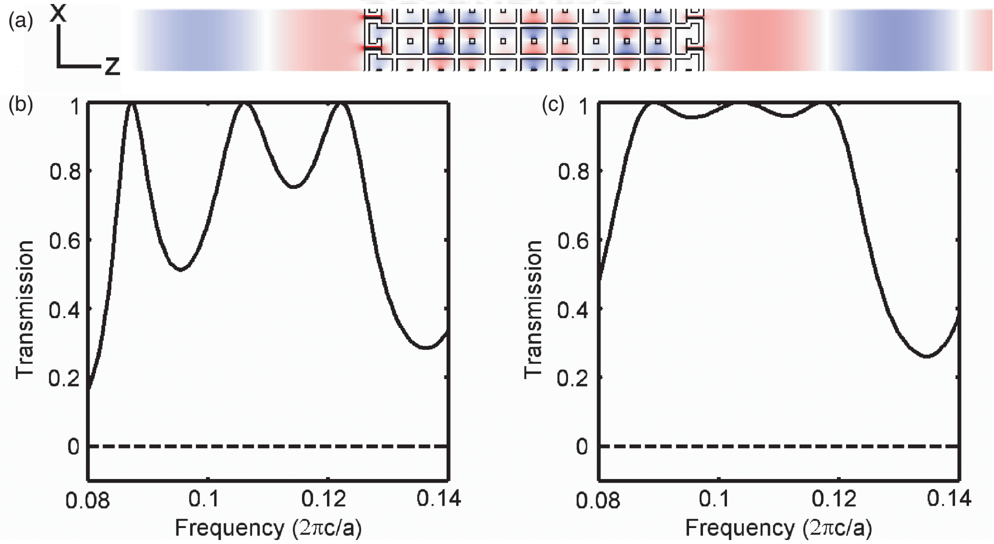

Fig. 3. Transmission through the 2-ICM structure with coupling mechanisms. (a) Longitudinal cross-section of the electric field profile for the E_x component. The structure is overlaid in black lines. (b) Transmission spectrum when a layer of antenna is connected to the 2-ICM. The solid line is for x-polarized incident wave; the dashed line is for y-polarized incident wave. (c) Transmission spectrum when an additional dielectric layer, with refractive index 1.72 and thickness 2.2a, is added on top of the antenna array.

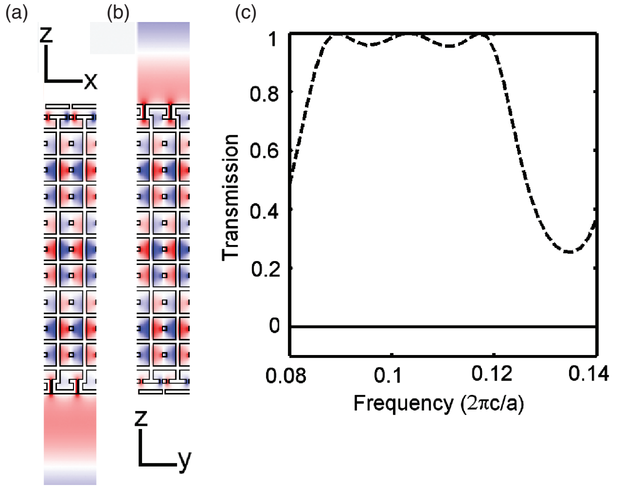

Fig. 4. Transmission through the 2-ICM structure with coupling antennas on opposite sides of the slab oriented in orthogonal directions. The antennas are covered with an additional dielectric layer, with refractive index 1.72 and thickness 2.2a. (a) E_x field profile on an x–z cross-section. The structure is overlaid in black lines. (b) E_y field profile on a y–z cross-section. (c) Transmission spectrum. The solid line is for the same polarization as the incident wave; the dashed line is for the orthogonal polarization. A complete conversion of the polarization occurs at resonant frequencies.

the scalar mode inside the 2-ICM always has the same field profile regardless of the incident polarization. The two half-spaces of the input and output ports separated by the scalar wave medium do not share polarization state information. The scalar wave property therefore enables a new opportunity for polarization control: if the antennas at both interfaces are in the x-direction, the slab will function as an ideal polarizer as we have demonstrated above: the transmitted wave is polarized in the same direction as the incident wave with perfect transmission. On the other hand, if the output antennas are rotated to a different direction from that of the input antennas, the polarization is accordingly rotated by the same amount. This polarization rotation can be achieved independent of frequency over a very broad frequency range, as seen in the FDTD simulations shown in Figure 4.

3. CONCLUDING REMARKS

Finally, we note that the decay length of the evanescent wave is approximately of that of one unit cell. Our FDTD simulations show that for a thin slab of only one unit cell, when operating as a polarizer, the contrast ratio between the two polarizations is more than 30 dB. Thus, the structures presented in this paper can be much smaller than the wavelength of light, providing a much more compact, subwavelength-scale polarization control device that operates over a much broader bandwidth compared to other proposed schemes.[9–10]

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