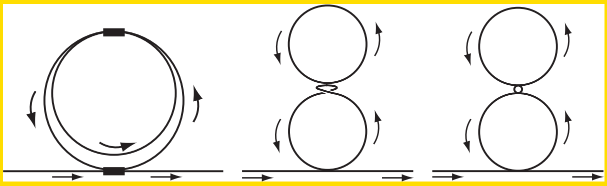
Abstract We investigate the connection between group velocity and rotation sensitivity in a number of resonant gyroscope designs. Two key comparisons are made. First, we compare two conventional sensors, namely a resonant fiber optic gyroscope (RFOG) and an interferometric fiber optic gyroscope (FOG). Second, we compare the RFOG to several recently proposed coupled-resonator optical waveguide (CROW) gyroscopes. We show that the relationship between loss and maximum rotation sensitivity is the same for both conventional and CROW gyroscopes. Thus, coupling multiple resonators together cannot enhance rotation sensitivity. While CROW gyroscopes offer the potential for large group indices, this increase of group index does not provide a corresponding increase in the maximum sensitivity to rotation. For a given footprint and a given total loss, the highest sensitivity is shown to be achieved either in a conventional RFOG utilizing a single resonator, or a conventional FOG.



Unidirectional CROWs: Folded configuration, twisted configuration and planar configuration with alternating large and small rings.

Performance comparison of slow-light coupled-resonator optical gyroscopes

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1. Introduction

A lot of progress has been made in recent years towards the generation of slow light using various mechanisms in different kinds of physical structures, including fiber Bragg gratings [1], coupled resonators [2–4], photonic-bandgap planar structures [5], Bragg fibers [6], and stimulated Brillouin scattering in optical fibers [7]. Because the sensitivity of many of these structures to external parameters (e.g., temperature or strain) is generally proportional to the reciprocal of the group velocity of the light, this new capability offers great promises for developing sensors far more sensitive than conventional optical sensors. As a result, an increasing number of slow-light sensor studies have appeared in print. However, there is one parameter that should be treated very cautiously in the context of slow-light sensing, and that is rotation rate. Several recent publications have described various gyroscope schemes utilizing slow light to purportedly enhance the sensitivity to small rotation rates, but further analysis of each of these schemes has revealed that there is no slow-light enhancement. In [8], a standard Sagnac-based fiber optic gyroscope (FOG) was described in which slow light was implemented to improve the sensitivity. Later publications [9, 10] pointed out that this scheme is flawed, as was expected from the well-known pioneering publication of Arditty and Lefèvre [11], who elegantly demonstrated using relativistic arguments that the sensitivity of a Sagnac-based gyro is independent of the index of the medium in which light travels.

More recently, a pair of papers by Matsko et al. [12, 13] investigated the possibility of slow-light enhancement in an interferometric gyroscope with side-coupled high-Q res-

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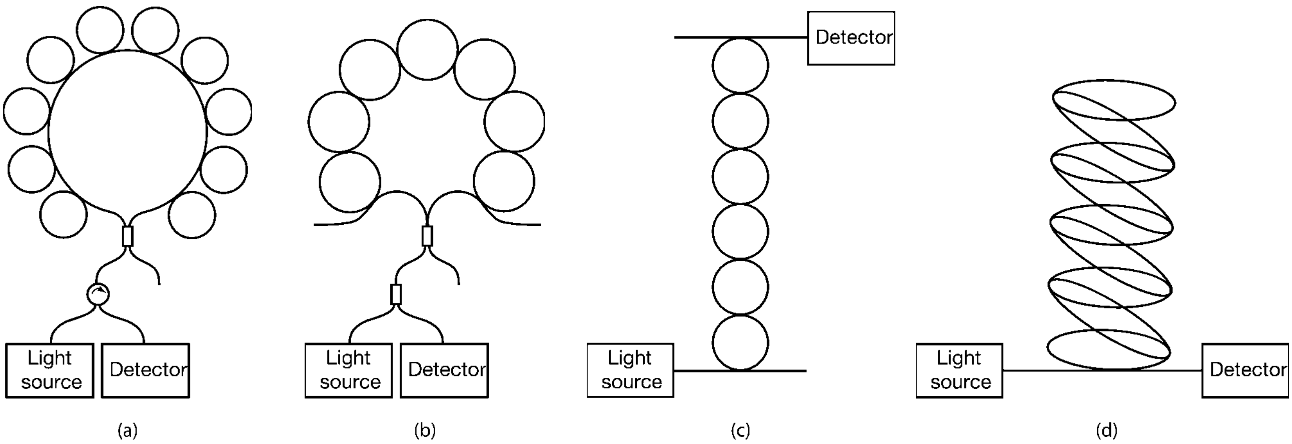

Figure 1 Several proposed slow-light-enhanced gyroscope setups.

onators (Fig. 1a). In the second of these papers, the authors cleared up some errors in the first paper and arrived at two important conclusions. First, the apparent dependence of the group index on the rotation sensitivity is coincidental, not fundamental. Second, rotation sensitivity cannot be enhanced by replacing a single side-coupled resonator of area A with N smaller resonators ($N = 10$ in Fig. 1a), each having an area A/N. Shahriar [10] pointed out that in Matsko's system the important physical quantity is the resonator finesse, not the group index.

Two years later, another publication by Scheuer et al. [14] claimed enhanced sensitivity as a result of the use of slow light in high-finesse coupled resonator optical waveguides (CROW) that are now coupled to each other, and arranged in the form a Sagnac loop (Fig. 1b). We have recently provided physical and mathematical arguments demonstrating that for equal footprint and total loss, this gyroscope is not any more sensitive than a conventional FOG, and that the reduced group velocity of light traveling through the CROW is only coincidentally related to its sensitivity [15]. In this configuration, as in the gyro considered by Matsko, the resonator finesse is the important factor, not the group index.

A more recent publication by Steinberg et al. [16] describes a different CROW gyroscope in which the resonators are not arranged in a Sagnac loop but in a straight line, as shown in Fig. 1c. The authors observed that this device exhibits a rotation-induced bandgap in its transmission, resulting in a transmitted power that is asymptotically a decaying exponential function of rotation rate. While this effect is novel, it has little bearing on the gyroscope's maximum rotation sensitivity, since the exponential dependence of transmitted power on rotation rate only holds when the power transmission is very small. Again, for equal loss and area, this CROW gyro has no better sensitivity than an RFOG.

Another recent series of publications by Peng et al. [17, 18] point out that by reversing the direction of propagation of light in adjacent rings, so that now light propagates in the same direction in all rings (see Fig. 1d), the sensitivity can be further enhanced. This argument is correct. However, as we will show, even when taking this enhancement into account, this unidirectional CROW gyro still does not perform any better than an RFOG once loss is taken into account.

All of these recent proposals share a common characteristic: the slow light is due to *waveguide* properties, not material properties. It has been conclusively established [10, 11, 17] that slow light arising from material properties (i.e., material dispersion) cannot enhance the rotation sensitivity of an optical gyroscope. Consequently, slow-light enhancement of rotation sensitivity, if any, *must* be due to waveguide properties only, not the material group or phase indices. Thus, without any loss of generality, we restrict our analysis in this paper to gyroscopes made from nondispersive materials.

While slow light due to material properties cannot enhance the sensitivity of an optical gyroscope, CROW gyroscopes with low group velocity also have good rotation sensitivity [14, 16–18]. In a CROW, light circulates around each individual high-Q resonator many times before moving on to an adjacent resonator, so the apparent group velocity is small if the coupling between resonators is weak. The rotation-induced Sagnac phase shift is then enhanced, since a signal circulates many times around each resonator. Thus, weak coupling gives rise to both high rotation sensitivity and low group velocity in CROWs. However, we shall show that loss limits the maximum sensitivity of a CROW gyroscope, and CROW gyroscope cannot outperform a conventional resonant gyroscope with the same footprint and loss. Again, group velocity is only coincidental, and does not affect the overall sensitivity.

The main purpose of this article is to review these various coupled-resonator gyroscope schemes and bring clarity to the misunderstandings associated with them in the literature by going back to basic principles. First, we compare resonant and interferometric fiber gyroscopes. Next, we consider non-fiber resonant gyroscopes, which may offer the possibility of high sensitivity in a small footprint. We then investigate whether coupled-resonant gyroscopes offer a sensitivity enhancement over conventional single-resonator configurations. To do so, we take the intuitive and tutorial approach of comparing a conventional resonant gyroscope to increasingly complex CROW gyros, beginning with a simple conventional resonant gyro, then moving on to two-ring coupled resonant gyros. These gyroscopes are all simple enough to be modeled with analytical expressions. We then develop arguments for CROW gyros utilizing an arbitrarily large number of rings. Finally, we extend this discussion to Sagnac-based CROW gyros, which offer the additional unrelated advantage of reciprocity.

This study shows unambiguously that there is no benefit to coupling several resonators together in a gyroscope: for equal footprint and waveguide loss, CROW gyros never exceed the sensitivity of a simple resonant gyro, which is the CROW gyro with $N = 1$. This is because loss ultimately limits the maximum sensitivity of all passive resonant optical gyroscopes, whether they consist of a single resonator or multiple resonators coupled together. Since coupling multiple resonators together does not offer the potential for sensitivity enhancement, it is better to instead use a single resonator with as little loss as possible. While compact non-fiber gyroscopes cannot currently outperform a conventional RFOG, future improvements in high-Q optical resonators may make small resonant gyroscopes with high rotation sensitivity possible.

2. Gyroscope classification

Although all the optical gyroscopes considered in this work are based on the same Sagnac effect, they differ markedly first in the role optical resonances play in their sensing principle, and second in the type of optical waveguide they use. Since these differences have a major impact on performance, for fair comparison it is important to classify gyroscopes in a manner that recognizes these distinctions. For the purpose of this work, we therefore divide optical gyroscopes in three categories: (1) non-resonant optical gyroscopes, of which the only representative is the FOG; (2) resonant optical gyroscopes (ROGs), defined as any gyroscope utilizing a single resonant loop, such as a resonant fiber optic gyroscope (RFOG); and (3) gyros utilizing multiple coupled resonant optical waveguides (CROWs), such as the gyros of Fig. 1.

It is useful to further divide CROW gyros into two subclasses: bidirectional and unidirectional. In bidirectional CROW gyros (for example, the configuration in Figs. 1b and c), a clockwise mode in one resonator couples to a counterclockwise mode in its neighbor, and vice versa. In unidirectional CROW gyros (for example, the configuration in Fig. 1d), light travels in the same direction (e.g., clockwise) in all resonators. Unidirectional CROWs satisfy the so-called “direction requirement” discussed by Peng [17].

In each of the three gyroscope categories, the waveguide may be provided by one of several technologies, for example optical fiber [19], microspheres [12], or photonic-crystal waveguides [20]. These last two technologies have several key impacts. First, because these waveguides can be made with considerably tighter bend radii than a fiber without suffering dominant bending loss (e.g., 100 µm in a microsphere versus ~ 1 cm in a fiber), a gyro utilizing such a waveguide can have a much smaller footprint than its fiber counterpart. For applications requiring extremely small gyroscopes ($\sim$ mm^3), these technologies may constitute the only available solution. Second, these waveguides can be fabricated in materials exhibiting a much lower loss than silica, e.g., CaF_2 [21]. Therefore they can have even greater Q’s than a fiber-based gyro, and hence, for equal footprint, a higher sensitivity. Importantly, it is not currently possible to make optical fibers, and thus FOGs, from these ultra-low-loss materials. Likewise, it is not currently possible to make large (~ 10 cm size), high-Q resonators from these ultra-low-loss materials. We must keep in mind these size and loss factors when comparing these various types of gyroscopes.

Based on the foregoing, in this review we make two broad comparisons. In the first one, we compare resonant and non-resonant gyroscopes, assuming that they are both made of the same low-loss 1.5- µm fiber. In the second comparison, we investigate whether coupled-resonant gyros offer the potential for slow-light sensitivity enhancement over conventional resonant gyroscopes, for any material loss and footprint. If such enhancement were possible, this would allow for ultra-sensitive and compact gyros – the best of all worlds.

3. FOGs

The basic configuration of a standard interferometric fiber-optic gyroscope is shown in Fig. 2a. The sensing portion consists of a loop of fiber of length L typically coiled in N turns onto a mandrel of diameter R. This loop is closed upon itself with a 3-dB coupler. A light source sends an input signal of power P_0 at frequency ω to the coupler, which divides it equally into a clockwise and a counter-clockwise signal that counter-propagate through the coil. When the coil is not rotating and in the absence of nonreciprocal effects, the two signals travel the same distance at exactly the same velocity, and they recombine in phase at the power detector, which is at the reciprocal port [22]. When the coil is rotated about its axis of symmetry, when viewed from a non-rotating reference frame the two signals travel different distances (due to the moving coupler) and at different velocities (due to the Fresnel drag on light in a moving material). This results in a phase shift between the two signals, proportional to the rotation rate. This is known as the Sagnac effect. After propagating through the coil, the signals are recombined at the coupler, and this Sagnac phase shift alters the power returning from the interferometer. This phase difference is given by [22] $\Delta\phi = 4\pi N R^2 \omega \Omega / c^2$. It depends on the area of the coil, the number of turns, and the rotation rate Ω, but it is independent of refractive index. The photodetector measures the power P resulting from the interference of the two signals, which is a function of the

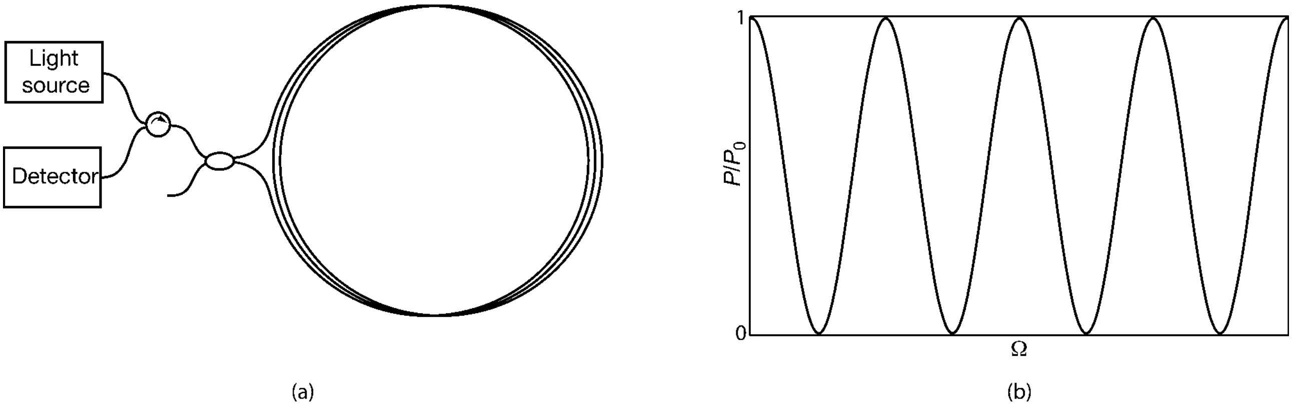

Figure 2 a) Diagram of a basic FOG. b) Rotation response of the FOG.

Sagnac-induced phase difference $\Delta\phi$. The response of the FOG is that of a two-wave interferometer, namely, a raised cosine (Fig. 2b). A phase shifter placed asymmetrically in the loop adds a nonreciprocal differential phase bias of $\pi/4$ between the two signals, which makes the response depend linearly on small rotation rates and maximizes the sensitivity. If the coil fiber loss coefficient is α so that the power attenuation through the coil is $\exp(-\alpha L)$, then the sensitivity to small rotations is:

$$S = \frac{1}{P_0}\frac{dP}{d\Omega} = \frac{2\pi N R^2 \omega}{c^2} e^{-2\pi R N \alpha}\,. \tag{1}$$

Eq. (1) states that for a given fiber length, the sensitivity is maximum for $N = 1$. In practice, however, a single-turn gyro would require an impractically large loop radius to exhibit a high sensitivity (for example, the ability to detect 1/1000th of Earth rate). In practical FOGs, a multi-turn coil is used to keep the sensitivity high while maintaining a reasonable footprint. For a more in-depth look at the FOG, refer to [19].

4. Resonant optical gyroscopes

As defined in Sect. 2, resonant optical gyroscopes include any gyroscope utilizing a single resonant circuit, such as a loop. They include of course the historic RFOG [23], whose waveguide is made of optical fiber, but also gyros utilizing a microsphere [12] or a photonic-crystal circuit [20] as a waveguide. Since all these sensors are based on the same broad principle as the RFOG, in this section we concern ourselves only with the latter. Several configurations and signal-processing methods have been tested for the RFOG (see [24]), but the fundamental physics of all configurations is essentially the same. To understand their basic physics, we therefore consider the simpler configuration shown in Fig. 3a. A fiber ring resonator consisting of N turns of radius R, effective index n and loss coefficient α is coupled to a single input/output fiber by a coupler with a power coupling ratio κ. A laser outputs a power P_0 at frequency ω into the waveguide. When ω is matched to a resonant frequency of the coil, a significant portion of the input power is coupled into the coil, where it is dissipated by the coil loss. When the coil is rotated, the Sagnac effect changes the phase accumulated by the signal as it propagates around the coil by $2\pi N R^2 \omega \Omega / c^2$, which shifts the resonant frequency of the coil. This shift results in a rotation-induced change in the power P measured at the output of the resonator. In this configuration, the source frequency is held constant, and the rotation rate is inferred from the change in transmitted power. The transmission function of this RFOG is given by:

$$T = \frac{P}{P_0} = 1 - \frac{A}{1 + B\sin^2(\phi/2)} \tag{2}$$

where

$$A = \kappa \frac{1 - e^{-2\pi R N \alpha}}{\left(1 - e^{-\pi R N \alpha}\sqrt{1-\kappa}\right)^2}\,, \tag{3}$$

$$B = 4\frac{e^{-\pi R N \alpha}\sqrt{1-\kappa}}{\left(1 - e^{-\pi R N \alpha}\sqrt{1-\kappa}\right)^2}\,, \tag{4}$$

$$\phi = 2\pi R N n \omega / c + 2\pi R^2 N \omega \Omega / c^2\,. \tag{5}$$

Here, ϕ is the phase that a signal accumulates in one round-trip through the N-turn coil, B is the coefficient of finesse of the RFOG, and A is the fraction of incident power that is absorbed by the coil on resonance. The transmission spectrum of this RFOG consists of a series of narrow resonances where transmission is low, corresponding to frequencies where $\phi = 2\pi m$ for integer m (see Fig. 3b). Eq. (2) shows that the transmission of a given RFOG depends only on ϕ, which makes it easy to see the connection between the rotation dependence and the frequency dependence of the transmission function. The response to a rotation-induced change in phase is exactly the same as the response to a change in phase caused by a change in the source frequency. When the RFOG is rotated, the transmission spectrum is simply shifted by $\Delta\omega = R\omega\Omega/nc$. This shift is independent of N and can be easily derived from Eq. (5).

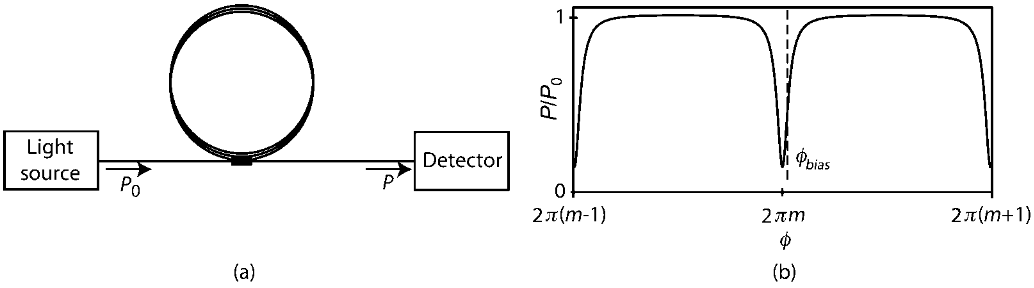

Figure 3 a) Diagram of a basic RFOG. b) Transmission function of the RFOG.

Several things must be done to optimize the sensitivity of a resonant gyroscope. First, the loss must be minimized – a smaller loss allows for a higher finesse and thus a greater sensitivity. Second, the resonator must be biased for maximum response. This means that in the non-rotating gyro, ϕ should be set to the steepest point of the transmission spectrum (dashed line in Fig. 3b), so that a rotation-induced change in ϕ results in the largest possible change in detected power. Mathematically, this phase bias condition is obtained by selecting the value of ϕ that maximizes the sensitivity to small rotations $S = \left(\frac{dT}{d\Omega}\right)_{\Omega\to 0} = \frac{\partial T}{\partial \phi}\left(\frac{\partial \phi}{\partial \Omega}\right)_{\Omega\to 0}$. Since $d\phi/d\Omega$ is constant, one simply needs to maximize $dT/d\phi$. From Eq. (2), the optimum bias phase is then (modulo 2π):

$$\phi_{\text{bias}} = \cos^{-1}\left(\frac{\sqrt{4+4B+9B^2}-2-B}{2B}\right) . \tag{6}$$

Finally, the coupling ratio κ must be optimized for maximum sensitivity. Changing κ changes the shape of the resonances. It can be shown that the value of κ that gives the steepest slope at the phase bias point is $\kappa_{\text{opt}} \approx \pi N R\alpha$. This approximation is accurate when $\pi N R\alpha$ is small (i.e., small round-trip loss). The optimum coupling ratio is therefore half the critical coupling ratio (the coupling that maximizes the power circulating in the resonator on resonance).

The rotation response of this resonant gyro is qualitatively very similar to the rotation-induced bandgap [16] observed in a gyro consisting of a linear CROW coupled to separate input and output waveguides (see Fig. 1c). In the resonant gyro considered here, the transmission is high except around the resonances, which occur when the round-trip phase is a multiple of 2π. In the linear CROW gyro, transmission through the entire system is low, except near resonances. While the exact functional form of the transmission of an RFOG and a linear CROW differ, their basic physics is the same. Both the high transmission of the linear CROW near resonance and the low transmission of the RFOG near resonance are manifestations of the same well-known physical effect; on resonance, a significant fraction of power is transferred from the input waveguide into the resonator, so the circulating power in the resonator is large. In the RFOG, this circulating power is dissipated by material loss, resulting in a transmission minimum on resonance. In contrast, in the linear CROW gyro the circulating power is transferred from one ring to the next, until the power circulating in the final ring is transferred to the output waveguide. Hence, the transmission is maximum on resonance.

5. Comparison between FOG and RFOGs

Since the fiber loss is so small, in a FOG one can increase the number of turns to a very large number before the signal attenuation starts reducing the sensitivity. To prove this point, Fig. 4 shows the dependence of the maximum sensitivity of a FOG on the number of turns computed using Eq. (1). We considered a 5-cm radius FOG made from a fiber with a loss of 0.2 dB/km and operated with a 1550-nm source. As N is increased from small values, the sensitivity increases linearly. It is not until N is very large (69000 in our numerical example) that the loss of the coil becomes so large (73%) that adding more loops decreases the sensitivity. The FOG's maximum sensitivity $S_{\text{max,FOG}}$ is obtained by taking the derivative of S (Eq. (1)) with respect to N, which also yields the optimum number of loops in the coil $N_{\text{opt,FOG}}$:

$$N_{\text{opt,FOG}} = \frac{1}{2\pi R\alpha} , \tag{7}$$

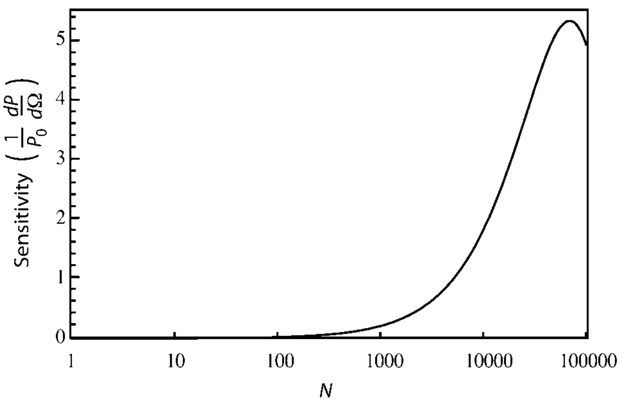

Figure 4 Maximum sensitivity of an N-loop FOG.

$$S_{\max,\mathrm{FOG}} = \frac{1}{e} \cdot \frac{R\omega}{\alpha c^2}\,. \tag{8}$$

The same argument does not apply to an RFOG. This can be understood by comparing an RFOG with a single loop to an RFOG with an N-turn coil. If we assume that the total loss in the coil is small, (so that $\exp(-N\pi R\alpha) \approx 1 - N\pi R\alpha$) the comparison is straightforward. Increasing the number of turns then multiplies the total round-trip loss by a factor of approximately N, which in turn multiplies the optimum value of κ by this same factor. This divides the finesse, and hence the maximum slope of the transmission curve, by a factor of approximately N. On the other hand, for a given rotation rate the Sagnac phase shift is also increased N-fold. These two effects very nearly cancel each other, and there is no net increase in sensitivity. Physically, the N-turn RFOG has an effective area that is N times larger than the single-loop RFOG. But a signal in the N-loop RFOG makes $1/N$ times fewer round-trips through the coil than a signal in the single-loop RFOG. The rotation sensitivity for both devices scales like the effective coil area times the number of round-trips, and this product is the same for both gyros.

This argument holds until N is so large that the loss in a single pass through the coil is significant and the approximation $\exp(-N\pi R\alpha) \approx 1 - N\pi R\alpha$ is no longer valid. A more careful analysis (valid for all values of N) shows that in fact, the increase in loss wins over the increased Sagnac phase for all values of N and the sensitivity actually decreases (albeit very slowly for small N) with increasing N. To wit: we show on Fig. 5 the exact calculated dependence of the sensitivity of an RFOG on N. For small values of N, it slowly degrades as N increases. Once N is so large ($\sim$ 20000 in our example) that the loss of the coil is significant ($\sim$ 25%), the sensitivity decreases rapidly. Such long coils simply have too much loss to make good resonant gyros. The maximum sensitivity of an RFOG $S_{\max,\mathrm{RFOG}}$ is therefore achieved when $N = 1$. Eq. (9) gives an approximate expression (valid when $\pi R\alpha$ is small) for $S_{\max,\mathrm{RFOG}}$,

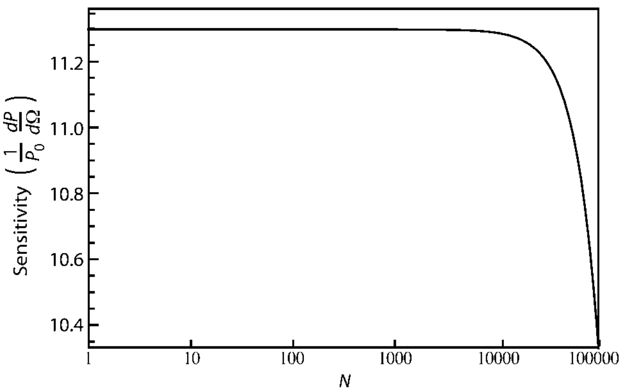

Figure 5 Maximum sensitivity of an N-loop RFOG.

which occurs when $\phi = \phi_{\mathrm{bias}}$ and $\kappa = \pi R\alpha$:

$$S_{\max,\mathrm{RFOG}} = \frac{4}{3\sqrt{3}} \cdot \frac{R\omega}{\alpha c^2}\,. \tag{9}$$

Comparison between Eq. (9) and Eq. (8) shows that the maximum possible sensitivity of an RFOG is $4e/3^{3/2} \approx 2.09$ times greater than that of an equivalent FOG. It is not surprising that the maximum possible sensitivities of the RFOG and FOG are so similar; both rely on the same Sagnac effect, and both suffer the same material loss per unit length. The slight superiority of the RFOG is in fact due mostly to an incidental difference in the signal processing used in these two gyros. If a second detector was placed at the nonreciprocal port of the FOG and the difference between the powers detected at the two ports was measured, the sensitivity of the FOG would be doubled. This is not done in practice because the increase in sensitivity is not worth the loss of reciprocity arising from the use of the nonreciprocal port.

It is important to be careful when interpreting the above analysis of the multi-loop RFOG. While absent coupler loss, and for a given radius and waveguide loss, the RFOG has its highest sensitivity with only one loop ($N = 1$), a single-loop RFOG may not be the best configuration under all practical experimental conditions. If the coupler has significantly more loss than a single loop of fiber, then a signal propagating through an N-turn coil achieves the same Sagnac phase with less loss than a signal propagating N times through a single-loop coil does. In the single-loop coil the signal incurs the coupler loss with each pass, so the effective loss per unit length is greater. Consequently, with a lossy coupler the optimum sensitivity is achieved for a value of N greater than 1. Although coupler loss changes the optimum value of N in an RFOG, it does not change the main advantage of an RFOG over an FOG. Even when realistic coupler loss is accounted for, the sensitivity of an RFOG is at best $\sim$ 2.09 times greater than that of a FOG with the same loss and footprint.

The bottom line is that the fair metric from a *practical* standpoint is to compare an N-turn FOG of radius R with a one-turn RFOG of same radius (in which κ is optimized). Although the RFOG requires less fiber and has a greater maximum sensitivity, the FOG is a better sensor in most practical applications. This is because the very challenging stabilization requirements of the RFOG, combined with the low cost of fiber, outweigh the slightly higher sensitivity of the RFOG. Rather than tackling the difficult task of stabilizing an RFOG, commercial gyro designers instead opted long ago to increase the sensitivity of the FOG by using a long fiber in a multi-turn coil. Increasing the number of turns does not increase the footprint of the gyro, which is primarily determined by the coil radius. Other components (couplers, modulator, source, polarizer, etc.) all take space, so to first order they impose the thickness of the gyro package. Given this second constraint, piling up more fiber loops on top of the first loop takes up only a small additional volume. The cost of fiber is also relatively low, so adding fiber has little cost implication. The superiority of the FOG over the RFOG for practical applications is best illustrated by the fact that while RFOGs have never been commercialized, hundreds of thousands of FOGs have been sold to date.

6. Two-ring bidirectional CROW gyro

In our analysis of the RFOG above, we saw that the RFOG is a sensitive and compact gyroscope. This high sensitivity raises a natural question: If a single resonator yields such a good sensitivity, can the maximum sensitivity be increased by adding more resonators? To answer this question, in this section we look first at the simpler case of a CROW gyro consisting of two coupled resonators and a single input/output waveguide (see Fig. 6). To keep our analysis straightforward, and without loss of generality, we make the simplifying assumptions that the couplers are all lossless and have the same coupling ratio κ, and that both rings have the same radius R.

When this CROW gyro is rotated, the signal undergoes a Sagnac phase shift in two rings instead of one. When the rotation is in the particular direction shown in Fig. 6, i.e., in the direction light circulates around the first ring (counterclockwise), the signal picks up a positive Sagnac phase shift every time it goes around the first ring. But every time the signal goes around the second ring, where it travels in the opposite direction (clockwise), it picks up a negative Sagnac phase shift. Because the two rings have identical radii and are subjected to the same rotation rate, these two contributions to the Sagnac phase shift have the exact same magnitude but opposite signs. Furthermore, we expect the signal to resonate approximately the same number of times around the first and around the second ring. The reason is as follows. First, the first ring contains two couplers, whereas the second ring contains only one. Consequently, the first ring has a higher round-trip loss, and this effect alone causes the signal to resonate fewer times around it than around the second ring. Second, the signal entering the second ring has already resonated around the first ring, and it is therefore weaker. This effect causes the signal to resonate fewer times around the second ring. Although one cannot conclude qualitatively without a detailed model as to which of these two effects dominates, we intuitively expect that the signal travels about the same number of times around each of ring. Therefore, we expect that it accumulates about the same total Sagnac phase shift around both rings, except that one is positive, and the other is negative. To first order, the two Sagnac contributions cancel out. This intuitive argument leads to the anticipated conclusion that the two-ring bidirectional CROW not only does not perform better than the RFOG, but that it has a much poorer sensitivity than an RFOG.

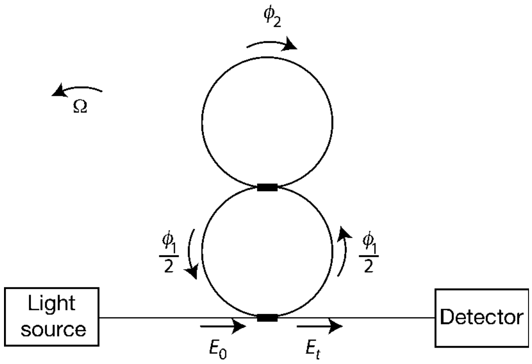

Figure 6 A two-ring bidirectional coupled-resonator gyro.

To confirm this important prediction, we carried out a detailed numerical simulation of the sensitivity of the two-ring CROW gyro of Fig. 6. The total complex phase shift experienced by the signal as it travels once around the first ring is given by:

$$\phi_1 = 2n\pi R\omega/c + 2\pi R^2\omega\Omega/c^2 + i\pi R\alpha\,. \qquad (10)$$

The first term is the propagation-induced phase, the second term is the rotation-induced Sagnac phase shift, and the third term accounts for the waveguide loss. Similarly, the total complex phase shift experienced by the signal as it travels once around the second ring is:

$$\phi_2 = 2n\pi R\omega/c - 2\pi R^2\omega\Omega/c^2 + i\pi R\alpha\,. \qquad (11)$$

Because the rings have identical radii, the first and third term in Eq. (11) are the same as in Eq. (10). But because light travels in opposite direction in the two rings, the second (Sagnac) term has the opposite sign. It is straightforward to derive an analytical expression for the electric field at the output of this two-ring CROW gyro as a function of these two phases ϕ_1 and ϕ_2:

$$\frac{E_t}{E_0} = \frac{e^{i(\phi_1+\phi_2)}+(1-e^{i\phi_1})\sqrt{1-\kappa}-e^{i\phi_2}(1-\kappa)}{1+(e^{i(\phi_1+\phi_2)}-e^{i\phi_2})\sqrt{1-\kappa}-e^{i\phi_1}(1-\kappa)}\,. \qquad (12)$$

The rotation-dependent transmitted power P measured by the detector is proportional to $|E_t|^2$. Since the Sagnac components in ϕ_1 and ϕ_2 have opposite signs and equal magnitude (see Eqs. (10) and (11)), in Eq. (12) $\phi_1+\phi_2$ is rotation-independent. Therefore the terms in $\phi_1+\phi_2$ do not contribute at all to the rotation sensitivity. It is unfortunately not straightforward to manipulate Eq. (12) analytically to show that this phase cancellation results in a significant reduction in rotation sensitivity compared to a one-ring CROW gyro (i.e., and RFOG). However, in the limit of small κ prevailing in the high-finesse resonators of most interest, in Eq. (12) $(1-\kappa)$and $\sqrt{1-\kappa}$ both approach unity, and the output field is symmetric in ϕ_1 and ϕ_2: as hinted in our physical argument, the two rings contribute about equally to the output of this gyro. Since their Sagnac components have opposite signs, we expect that this symmetry will lead to a near cancellation of the two Sagnac phase shifts, and hence of the rotation sensitivity.

To confirm this result in the more general case of an arbitrary value of κ, we resorted to exploiting Eq. (12) numerically. To maximize the sensitivity of this CROW gyroscope, it is important to find the optimum phase bias and

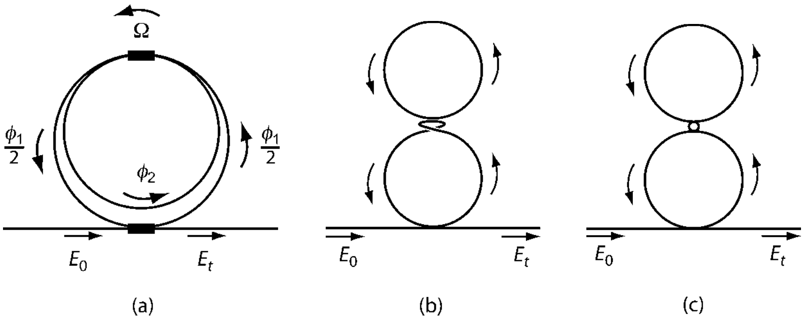

Figure 7 Unidirectional CROWs. a) Folded configuration. For clarity, the second ring is not drawn to scale. b) Twisted configuration. c) Planar configuration with alternating large and small rings.

coupling ratio, just as in the case of the RFOG. To this end, we proceeded in essentially the same way as for the RFOG. The propagation-induced phase $\phi_{\text{prop}} = 2n\pi R\omega/c$ was biased by adjusting the source frequency. For each test value of ϕ_{prop}, we calculated the sensitivity to small rotations for a large number of coupling ratios κ until we identified the combination $(\kappa, \phi_{\text{prop}})$ that gave the maximum sensitivity. As was expected both intuitively and from the form of Eq. (12), we found that even after optimizing both the phase bias and coupling ratio, the sensitivity is very poor. Specifically, for the particular sensor parameters used earlier ($R = 5$ cm, $\lambda = 1550$ nm, and 0.2 dB/km of fiber loss), the sensitivity is less than 1% of the RFOG's sensitivity. As expected on physical grounds, this bidirectional sensor performs very poorly, and it does so because the Sagnac phase accumulated in the two rings almost exactly cancel each other.

As an aside, it is interesting to note that if the size of one of the rings is slightly increased or decreased, and the same bias and coupling optimization is applied again, the sensitivity increases dramatically. The reason is of course that when the rings are no longer identical, the Sagnac phases in the two rings no longer have exactly the same amplitude, and they no longer cancel as strongly. However, the sensitivity still remains below that of an optimized RFOG of same dimension and loss.

7. Two-ring unidirectional CROW gyro

The result of the previous section clearly hints that if in a two-ring CROW gyro the light could be forced to travel around both rings in the *same* direction, the gyro sensitivity would be significantly increased. This direction reversal can be implemented in a number of simple schemes, illustrated in Fig. 7, which we refer to as unidirectional. The simplest one is to fold the second ring on top of the first ring (see Fig. 7a). In this folded configuration, the Sagnac phase shifts in the two rings now have the same sign. A second scheme consists of twisting the first ring (see Fig. 7b). A third one is inserting a ring of negligibly small radius between adjacent rings, as shown in Fig. 7c. The smaller ring acts to reverse the direction of travel in the larger rings, without adding any new Sagnac phase shift since its area is negligible. This last configuration has the significant advantage that it can be fabricated using the same technology as other planar-waveguide CROWs. If need be, all of these configurations can be extended to an arbitrary number of rings.

These unidirectional two-ring CROW are expected to have a much greater sensitivity than the bidirectional configuration, since the effects of the Sagnac phase shifts in the two ring resonators now reinforce one another instead of cancelling out. To find the exact degree to which the sensitivity is improved over the bidirectional case, we numerically modeled the gyroscope of Fig. 7a using the same methods as in the previous section. Analytically, the output field of this gyro has the same dependence on the round-trip phases ϕ_1 and ϕ_2 as for the bidirectional configuration (Eq. (12)). The only difference with the bidirectional configuration is that the Sagnac phase shift in the two rings now have the same sign, hence the two round-trip phases are now equal:

$$\phi_1 = \phi_2 = 2n\pi R\omega/c + 2\pi R^2\omega\Omega/c^2 + i\pi R\alpha\,. \quad (13)$$

Since the signals now circulate the same direction in both rings, Eq. (13) holds even when the gyro is rotating, and the transmission now depends on only one phase variable, namely $\phi_{\text{ring}} = \phi_1 = \phi_2$. The output field (Eq. (12)) can therefore be simplified to:

$$\frac{E_t}{E_0} = \frac{\sqrt{1-\kappa}+e^{2i\phi_{\text{ring}}}+e^{i\phi_{\text{ring}}}(\kappa-1-\sqrt{1-\kappa})}{1+e^{2i\phi_{\text{ring}}}\sqrt{1-\kappa}+e^{i\phi_{\text{ring}}}(\kappa-1-\sqrt{1-\kappa})}\,. \quad (14)$$

It is interesting to compare the transmission spectrum of this two-ring CROW gyro to that of the RFOG. The RFOG transmission spectrum consists of a series of periodic resonant dips occurring wherever the total round-trip phase is a multiple of 2π. In the two-ring unidirectional CROW gyro, the transmission spectrum has the same periodicity in ϕ_{ring}, but the resonances come in pairs, with each pair centered on $\phi_{\text{ring}} = 2\pi m$. Fig. 8 illustrates this behavior with the transmission spectrum for the two-ring unidirectional CROW gyro calculated from Eq. (14) for $\alpha = 0.03\,\text{m}^{-1}$, $\lambda = 1550$ nm, and $\kappa = 0.038$. The spacing and sharpness of these resonances depend on the ring loss and coupling ratio.

Using Eq. (14), we numerically optimized the coupling and bias frequency for this resonant gyro using the same method as for the bidirectional gyro. The optimum coupling ratio is $\kappa_{\text{opt}} \approx 2\pi R\alpha$. As expected, the sensitivity

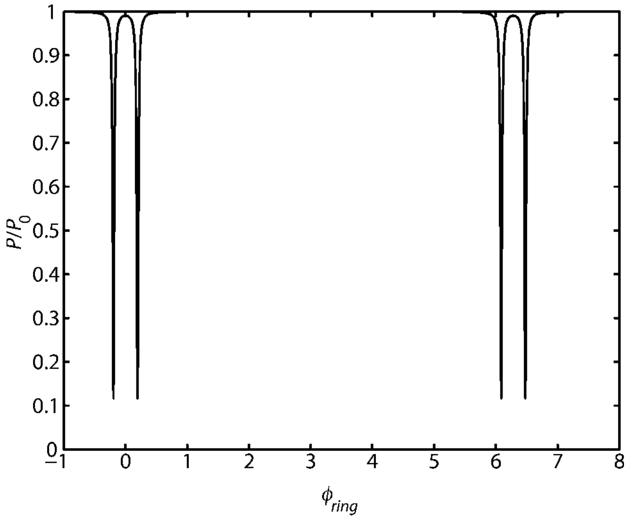

Figure 8 Transmission spectrum of a two-ring unidirectional CROW gyro.

of the optimized two-ring unidirectional gyro of Fig. 7a is much higher than that of the corresponding bidirectional gyro. The optimized two-ring unidirectional gyro is only very slightly ($< 0.01\%$) less sensitive than an RFOG with equal material loss and footprint. The main conclusion is that while the unidirectional two-ring CROW gyro performs much better than the bidirectional configuration, it is still not more sensitive than an RFOG. Fig. 9 shows the transmission spectrum of the optimized RFOG and a single resonance of the optimized two-ring bidirectional CROW, with the same ring radius and loss. The two plots are nearly identical except for the bias phase.

A previous analysis of unidirectional CROW gyros by Peng et al. [17] concluded that a unidirectional CROW gyro could have significantly greater rotation sensitivity than a conventional optical gyro. However, their analysis was restricted to lossless waveguides ,and considered only the phase sensitivity of the unidirectional CROW gyroscope, while ignoring the effect of signal attenuation on sensitivity. While the authors correctly pointed out that in a unidirectional CROW the total Sagnac phase shift of the output signal $\Delta\Phi$ is proportional to the group delay τ_g (see Fig. 10b), this relationship does not mean that the rotation sensitivity is enhanced. Our own analysis of CROWs made from lossy waveguides has shown that any signal that accumulates a large group delay in the unidirectional CROW also experiences significant attenuation, which degrades the rotation sensitivity.

Furthermore, we note that the relationship between Sagnac phase shift and group delay is not unique to unidirectional CROWs. To illustrate this point, we compare a unidirectional CROW (see Fig. 10b) to the N-loop fiber coil from a FOG (see Fig. 10a), both of radius R. Straightforward manipulation of Peng's result (Eq. (22) in [17]) shows that for the unidirectional CROW, the total rotation-induced phase shift in the output signal is related to the group delay by:

$$\Delta\Phi = \frac{\tau_g R\omega\Omega}{nc}\,. \tag{15}$$

For a coiled fiber, the group delay is $\tau_g = 2\pi RNn/c$, and the rotation-induced phase shift is $\Delta\Phi = 2\pi R^2 N\Omega\omega/c^2$ [22]. Combining these last two expressions, we find that Eq. (15), originally derived for CROWs, is also satisfied by the fiber coil from a FOG.

8. General comments and predictions

Every resonant gyroscope that we have looked at so far has shared a common characteristic: adding resonators does not increase the sensitivity. Adding loops to the resonant coil in the RFOG only decreased the sensitivity. Similarly, in Sects. 6 and 7 we showed that even after optimization, a two-ring CROW gyro has a lower sensitivity than a single-loop RFOG. Other authors [10, 13, 15] have arrived at similar conclusions about other resonant gyros. A simple physical argument gives a good intuition as to why all other coupled-resonator gyroscopes should have the same performance limitation as the structures mentioned above. Consider the Sagnac effect on a signal propagating once around a circle of radius R in a material with loss coefficient α. After

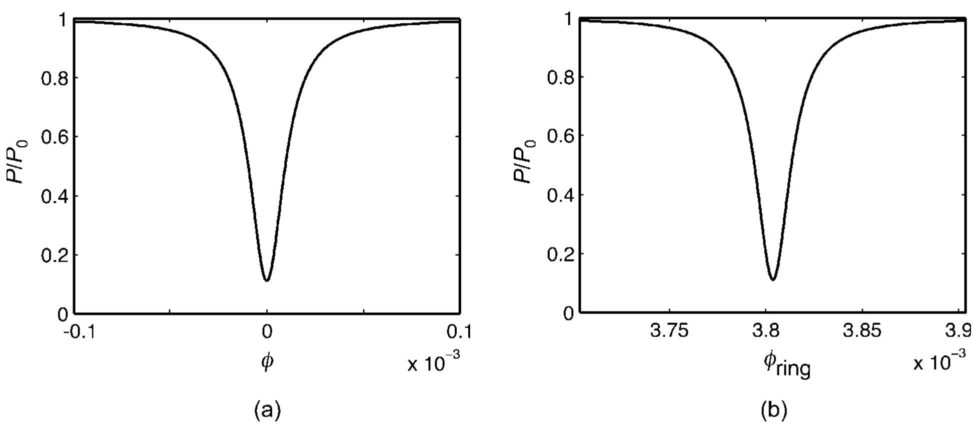

Figure 9 a) Transmission spectrum of optimized RFOG. b) Transmission spectrum of optimized two-ring unidirectional CROW gyro.

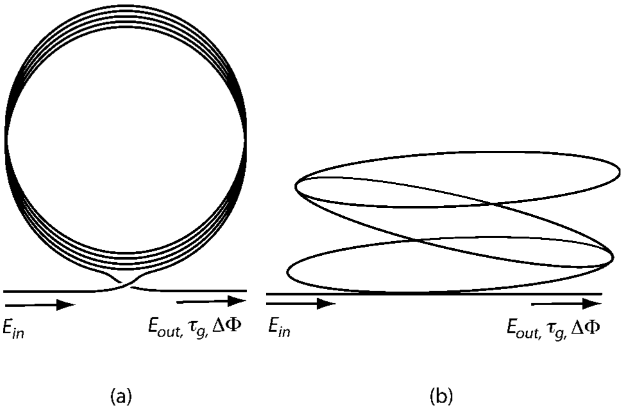

Figure 10 a) The fiber coil from a FOG. b) A unidirectional CROW. Both satisfy Eq. (15).

traveling once around this circular path, the signal has accumulated a Sagnac phase shift $\phi_s = \pm 2\pi\omega R^2 \Omega / c^2$, and the amplitude is multiplied by $\exp(-\pi R\alpha)$. The key point is that regardless of the gyro configuration, Sagnac phase and attenuation always accumulate in the same fixed ratio: in order to accumulate a Sagnac phase of ϕ_s, the signal must suffer an attenuation of $\exp(-|\phi_s|\alpha c^2/2\omega R\Omega)$.

This connection between rotation-induced phase and loss is important because every optical gyroscope, regardless of configuration and signal processing method, is responsive to the same Sagnac phase shift. Hence, in order to obtain maximum sensitivity, the optimization of every optical gyroscope is done in essentially the same way. All gyros are optimized when the marginal sensitivity enhancement of an increase in Sagnac phase is exactly balanced by the marginal increase in loss (and decrease in sensitivity) that such an increase in Sagnac phase would require. Since the amount of loss associated with a given Sagnac phase shift depends only on the radius and loss of the path traversed by the signal, all gyros with the same resonator radius and material loss will have similar sensitivity after they are optimized.

9. N-ring CROW gyroscopes

We now verify the general predictions made in Sects. 6 and 7 for two-ring CROW gyros by numerically modeling coupled-resonator gyroscopes with $N > 2$. The algebraic methods we have used so far become increasingly cumbersome as N is increased, so to analyze coupled-resonator structures with arbitrary N we use the more general transfer-matrix method [3,4,14]. The transfer-matrix method keeps track of the loss, propagation-induced phase, and rotation-induced phase shift [25] that a signal accumulates as it propagates from one coupler to the next. The E-field transmission through each coupler and waveguide section of the CROW is represented by a matrix, which depends on the CROW parameters and the rotation rate. The total matrix of the CROW M_{tot} is calculated by taking

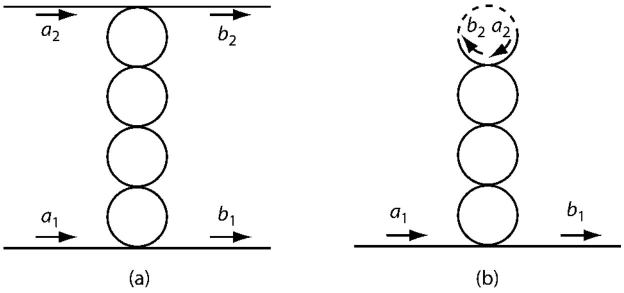

Figure 11 a) Fields used in the transfer-matrix method for a CROW with two leads. b) Fields for a CROW with a single lead.

the product of all the individual component matrices. This total matrix relates the incoming and outgoing fields at one end of the CROW to the incoming and outgoing fields at the other end of the CROW (see Fig. 11a):

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = M_{\text{tot}} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} . \tag{16}$$

Once the total transfer matrix has been calculated, it is straightforward to apply appropriate boundary conditions and solve for the output electric fields in terms of the input field amplitude a_1. For a system with two separate leads, the boundary condition is $a_2 = 0$ (see Fig. 11a). For a system with only a single input/output lead, the appropriate boundary condition is $a_2 = b_2 \exp(i\phi_{\text{ring}})$. In this case, the final ring of the CROW is made by connecting the second lead to itself, and ϕ_{ring} includes the loss, rotation-induced phase, and propagation induced phase in a single ring (see Fig. 11b).

For this study, we looked at unidirectional CROW gyroscopes with a single input/output lead, as shown in Fig. 1d. Once again, we assumed that all rings had identical radius and loss, and that all the couplers were identical. For any N-ring unidirectional gyro, the only phase that needs to be kept track of is ϕ_{ring}, the total phase (including the rotation-induced phase) that a signal accumulates in a single roundtrip around one of the ring resonators. The transmission spectrum the N-resonator CROW structure is similar to that of the two-ring structure (Fig. 8), with the two resonant dips in transmission replaced by N resonances. The position, depth and sharpness of each of the N resonances depends on the coupling κ. Fig. 12 shows two plots of the transmission spectrum for the same $N = 10$ CROW gyro with $R = 5$ cm, $\lambda = 1550$ nm, and $\alpha = 2.3 \cdot 10^{-4}\,\text{m}^{-1}$ with two different values for κ.

We numerically investigated several unidirectional CROW gyroscopes (for various values of N up to $N = 27$) by varying the coupling and the source frequency in order to maximize the sensitivity to small rotations. The results of this extensive numerical search agreed with our earlier predictions; no CROW gyro, for any $N > 1$, outperformed an RFOG with the same radius and material loss. Unlike in the two-ring case, where only one value of κ maximizes the rotation sensitivity, in the N-ring case there are gener-

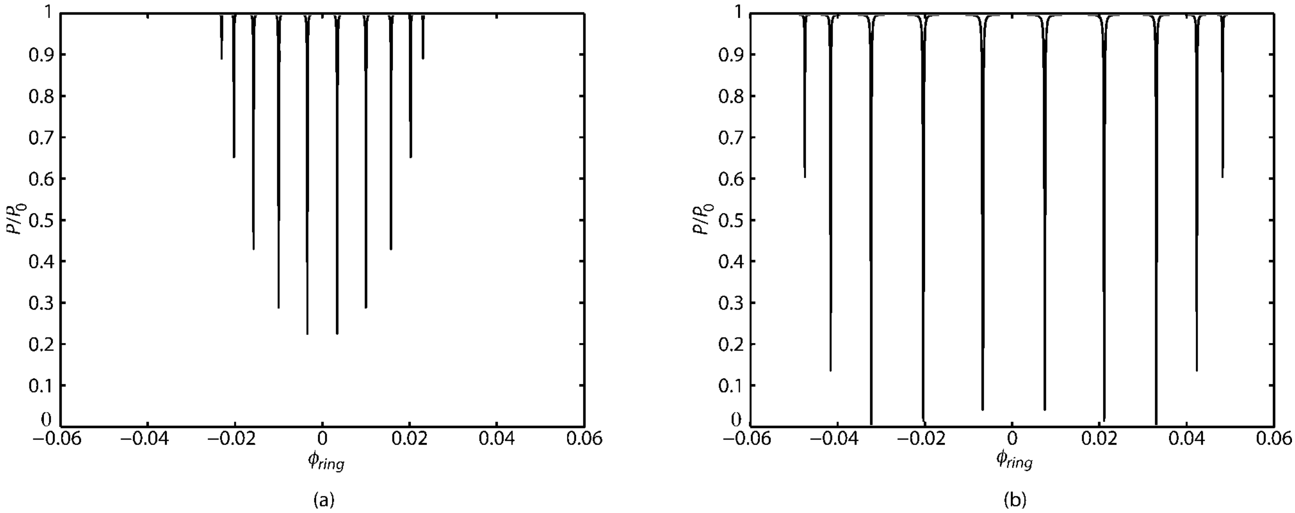

Figure 12 Transmission spectra for the $N = 10$ CROW gyro with $R = 5$ cm, $\lambda = 1550$ nm, and $\alpha = 2.3 \cdot 10^{-4}\,\mathrm{m}^{-1}$. a) $\kappa = 1.45 \cdot 10^{-5}$. b) $\kappa = 6.22 \cdot 10^{-5}$.

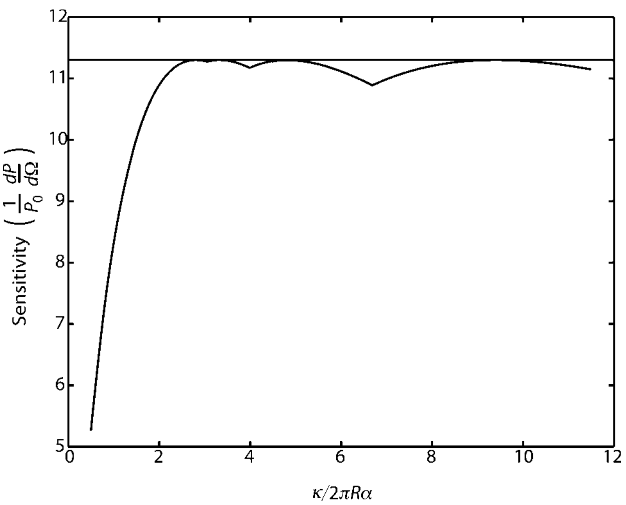

Figure 13 The sensitivity of a phase-biased 10-ring unidirectional CROW gyro as a function of coupling κ. The horizontal line is the sensitivity of a single-ring RFOG with the same material loss and ring radius.

ally several values of κ that give good rotation sensitivity. This is because, as mentioned earlier, the relative strengths of the N resonances depend on κ (see Fig. 12). Different values of κ optimize the rotation sensitivity of different resonances (see Fig. 13). However, we emphasize that no CROW gyro, with any value of κ, ever surpassed the sensitivity of an RFOG.

10. Sagnac configuration coupled-resonator gyroscopes

There is one additional class of coupled-resonator gyroscope that has been studied in the literature: Sagnac-configuration CROW gyroscopes. These sensors operate similarly to the FOG, in that two counterpropagating signals are sent through the CROW, and are then interfered to measure the rotation rate. Fig. 1b shows one such Sagnac configuration CROW gyro, first proposed by Scheuer et al. [14]. It is similar to a conventional FOG, except that the FOG coil is replaced by a CROW. The primary advantage of such a Sagnac configuration CROW gyroscope over other CROW gyros is that it is reciprocal. Reciprocity is useful because it greatly reduces a number of deleterious non-reciprocal effects, such as slow drifts in the sensor output due to slowly varying asymmetric temperature gradients [26]. One disadvantage of the particular configuration of Fig. 1b is that while the CROW takes up a large area, the space inside the CROW loop that is not covered by the individual ring resonators does not contribute significantly to the rotation sensitivity. This disadvantage can be overcome by packing the rings together or by stacking the rings on top of one another.

The original analysis [14] of the Sagnac configuration CROW gyro of Fig. 1b assumed no phase bias. In the unbiased gyroscope considered in [14], the rotation sensitivity is properly predicted to be proportional to N^2, which led the authors to conclude that a significant sensitivity enhancement was possible by increasing N. However, we showed [15] that the response of this unbiased gyro is also proportional to Ω^2, so that the sensitivity to small rotation rates is vanishingly small. A thorough analysis of this gyro with an appropriate bias revealed that it offers no sensitivity enhancement over a conventional FOG of same loss and footprint [15].

It is also possible to make a reciprocal Sagnac configuration gyroscope by using a CROW with a single input/output lead (see Fig. 14). Although the response of a Sagnac configuration CROW gyro is somewhat different than that of the CROW gyros considered so far, this Sagnac

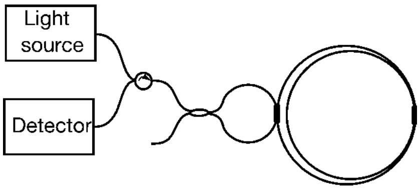

Figure 14 A reciprocal Sagnac configuration two-ring unidirectional CROW gyro.

configuration does not offer any fundamental sensitivity enhancement. Just as for any other optical gyro, the rotation sensitivity is ultimately limited by the waveguide loss. We also note that without proper phase bias, the response of the gyro shown in Fig. 14 is proportional to Ω^2, and thus the sensitivity is vanishingly small for small rotation rates.

11. Slow light and effective length

In all of the above analysis of CROW gyroscopes, it was never necessary to explicitly consider the group velocity of a signal propagating through the CROW. This is because, as has been pointed out for three specific gyro configurations [10, 11, 13, 15], there is no fundamental connection between group index and rotation sensitivity. Rather, the connection is coincidental; both high rotation sensitivity and high apparent group index occur when a signal circulates many times around a high-finesse resonator. The apparent group index is large in this case because the signal spends a long time circulating in one ring of the CROW before moving on to the next one. The rotation sensitivity is high because the total rotation-induced Sagnac phase shift is proportional to the number of times that the signal circulates around each resonator. To avoid any confusion arising from this coincidental connection, it is useful to characterize a CROW by its effective length [3, 4, 14] rather than by its apparent group index. The effective length is defined as its actual physical length ($2NR$) multiplied by π times the number of times a signal circulates around each resonator. Hence, the loss and group delay that a signal experiences in propagating through a CROW depend on the effective length, not the actual length. In particular, the power attenuation $P_{\text{out}}/P_{\text{in}}$ and group delay τ_g in a CROW with effective length L_{eff} are given by $P_{\text{out}}/P_{\text{in}} = \exp(-\alpha L_{\text{eff}})$ and $\tau_g = nL_{\text{eff}}/c$, respectively [3]. Characterizing a CROW by its effective length rather than the apparent group index makes it clear that while a CROW allows for a large effective length to be packed inside a small physical length [3], the relationship between loss, Sagnac phase, and rotation sensitivity is unchanged.

12. Ultra-low-loss materials

We have demonstrated that coupling multiple ring resonators together offers no potential for slow-light enhancement of rotation sensitivity. This is because coupling multiple resonators together does not change the fact that rotation sensitivity is always inversely proportional to loss. Thus, rather than coupling multiple resonators together, the best way to achieve high rotation sensitivity in a miniature passive resonant gyroscope is to use a single resonator (with the proper phase bias) with as little loss as possible. While low-loss optical fibers have a low loss record around $4.6{\cdot}10^{-7}\,\text{cm}^{-1}$ (0.2 dB/km), other materials offer the potential of much lower loss. For example, it has been predicted [21] that the loss of ideal CaF_2 can be as low as $10^{-9}\,\text{cm}^{-1}$ at room temperature. Since the maximum sensitivity of an RFOG is proportional to R/α, it may be possible to create miniature ($R \approx 100\,\mu$m) RFOGs from such an ultra-low-loss material with the same sensitivity as much larger ($R \approx 5$ cm) fiber gyros. However, the fabrication of small high-Q resonators poses a number of difficult challenges, and the current record for the lowest measured loss in a CaF_2 ring resonator is only $1.4{\cdot}10^{-6}\,\text{cm}^{-1}$ [27], which is significantly greater than the theoretical minimum. Until this loss can be made much smaller than that of low-loss optical fibers, gyros made from small high-Q resonators cannot have a higher sensitivity than conventional RFOGs. While resonant gyros made of ultra-low-loss material may present their own set of practical challenges, such as fabrication difficulties, stabilization issues and signal drift due to optical nonlinearities, they possess the potential for high rotation sensitivity in a small footprint, and research on ultra-low-loss materials is ongoing.

13. Conclusions

In summary, we have analyzed a number of optical gyroscope designs and found that while the configuration and mode of operation may differ significantly between designs, the maximum sensitivity of every design is limited by material loss. We showed that the maximum sensitivity of the FOG and RFOG are comparable when both are optimized. We predicted that since the relationship between Sagnac phase and loss is the same for all optical gyros, coupling multiple resonators together cannot enhance rotation sensitivity. Using numerical and analytical methods, we demonstrated that this prediction is true. No CROW gyro is more sensitive than an RFOG with the same footprint and loss. While the group velocity of a signal propagating through a CROW can be quite low, group velocity is ultimately unrelated to rotation sensitivity. Rather than characterizing a CROW by its apparent group index, it is useful to describe it instead by its effective length, which is determined by the loss. Since loss ultimately limits the maximum sensitivity of all passive optical gyros, ultra-low loss materials offer the potential of high rotation sensitivity in a compact gyro with a single ring resonator.

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