Enhancing or suppressing self-focusing in nonlinear photonic crystals

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The authors show that the effect of self-focusing can be controlled by exploiting spatial dispersion effects in a photonic crystal. In the positive refraction region, the critical field value for self-guiding can be significantly reduced. In the negative refraction region, the self-focusing effect can be completely suppressed in spite of a positive Kerr coefficient. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2724905]

The propagation of an intense optical beam in general is strongly influenced by diffraction and nonlinear effects. In a Kerr medium, the interplay between these two effects can result in self-focusing or propagation of a self-guided finite-size wave front.[1,2] Self-guiding of an optical beam is useful for channeling optical information on-chip without using waveguides. For this purpose it is of interest to reduce its power threshold. On the other hand, in high power laser applications, developing approaches to suppress self-focusing (i.e., to increase its power threshold) is also of great interest,[3,4] since self-focusing leads to filament formation, which is the precursor of optical damage. In this letter, we show that self-focusing effects can be controlled with appropriate design of the spatial dispersion effects in a photonic crystal system.

In order to describe the onset of self-guiding in photonic crystals, we first develop an analytic model. Our approach is a generalization of a standard semiquantitative description which considers the propagation of a wide monochromatic beam with a flattop intensity distribution.[5] For simplicity, consider a beam propagating along a symmetry direction of a two-dimensional photonic crystal. We define the wave vector components parallel or perpendicular to the beam propagation directions as $k_\parallel$ or $k_\perp$, respectively. In the absence of nonlinearity, the dispersion relation of the linear crystal is $\omega(k_\parallel, k_\perp)$. With Kerr nonlinearity, where the material is characterized by an intensity dependent dielectric constant $\varepsilon=\varepsilon_0+\varepsilon_2|E|^2$, the dielectric constant shifts by $\delta\varepsilon=\varepsilon_2|E|^2$ inside the beam region. The dispersion relation for the crystal inside the beam region thus becomes

$$\omega'(k_\parallel,k_\perp)=\omega(k_\parallel,k_\perp)-\delta\omega, \tag{1}$$

where $\delta\omega=\gamma\omega\delta\varepsilon=\gamma\omega\varepsilon_2|E|^2>0$, and γ is an overlap factor that is related to the fraction of the beam power in the nonlinear material.

At a given operating frequency ω_0, we suppose that for the linear crystal $\omega_0=\omega(k_\parallel^0,0)$, i.e., $(k_\parallel^0,0)\equiv\mathbf{k}_0$ is on the constant frequency contour of the linear crystal. In the vicinity of $\mathbf{k}_0$, the constant frequency contour for the nonlinear crystal at the beam center can then be described as

$$\omega_0=\omega'(k_\parallel^0+\delta k_\parallel,\delta k_\perp)=\omega(k_\parallel^0+\delta k_\parallel,\delta k_\perp)-\delta\omega. \tag{2}$$

Since typically $\delta\varepsilon \ll \varepsilon_0$, by Taylor expanding $\omega(k_\parallel^0+\delta k_\parallel,\delta k_\perp)$ around the k point $(k_\parallel^0,0)$ and by taking into account $\omega_0=\omega(k_\parallel^0,0)$ and $\partial\omega/\partial k_\perp|_{k_\parallel^0,0}=0$ due to mirror symmetry, Eq. (2) can be further simplified as

$$\delta\omega=\left.\frac{\partial\omega}{\partial k_\parallel}\right|_{k_\parallel^0,0}\delta k_\parallel+\left.\frac{1}{2}\frac{\partial^2\omega}{\partial k_\perp}\right|_{k_\parallel^0,0}\delta k_\perp^2. \tag{3}$$

In order to achieve self-guiding, all wave vector components at the beam center should undergo total internal reflection when incident upon the linear crystal, i.e., $\delta k_\parallel>0$ for all $\delta k_\perp$. The maximum $\delta k_\perp$ is related to the beam width d as $k_{\perp,\max}=2\pi/d$. Thus, in Eq. (3), setting $k_\parallel=0$ at $k_{\perp,\max}$, we have

$$\delta k_\perp|_{\delta k_\parallel=0}=\sqrt{\frac{2\delta\omega}{\partial^2\omega/\partial k_\perp^2|_{0,k_\parallel^0}}}=k_{\perp,\max}=2\pi/d. \tag{4}$$

Hence the critical field value of the self-guiding is

$$E_{\text{critical}}=\frac{1}{d}\sqrt{\frac{\partial^2\omega/\partial k_\perp^2|_{0,k_\parallel^0}}{2\gamma\varepsilon_2\omega}}. \tag{5}$$

For a uniform nonlinear medium, Eq. (5) can be simplified as

$$E_{\text{critical}}=\frac{\lambda_0}{d}\frac{1}{\sqrt{\varepsilon_2}}, \tag{6}$$

where λ_0 is the wavelength in vacuum. Equation (6) is in agreement with the result for a uniform two-dimensional system in Ref. 5.

In a photonic crystal, on the other hand, $\partial^2\omega/\partial k_\perp^2$ is no longer directly proportional to ε_0 but instead becomes strongly structure dependent. As an example, we consider a photonic crystal with a square lattice of air holes introduced into a medium with a dielectric constant $\varepsilon_0=12$ and plot in Fig. 1 the constant frequency contours of the first band (TE mode). The radius of the air hole is $0.35a$. For a beam propagating along the (11) direction, self-collimation[6,7] occurs at a

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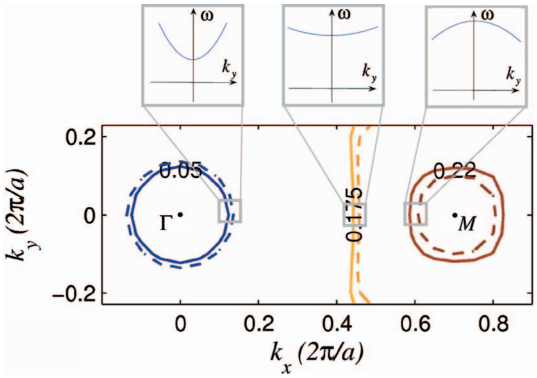

FIG. 1. (Color) Constant frequency contours of the first band (TE mode) of a photonic crystal at frequencies $\omega=0.05, 0.175, 0.22(2\pi c/a)$ (solid lines) and frequencies $\omega=0.055, 0.18, 0.225(2\pi c/a)$ (dashed lines). The crystal consists of square lattice of air holes introduce into a dielectric ($\varepsilon=12$). The radius of the hole is $0.35a$, where a is the lattice constant. The x direction is along the (11) direction of the crystal. The insets are $\omega(k_y)$ in the vicinity of $k_y=0$ at the three frequencies $\omega=0.05, 0.175, 0.22(2\pi c/a)$.

frequency $\omega_s=0.185(2\pi c/a)$. Hence at this frequency, $\partial^2\omega/\partial k_\perp^2=0$. In the frequency region $\omega<\omega_s$ the crystal exhibits positive refraction and $\partial^2\omega/\partial k_\perp^2>0$. Above ω_s the crystal becomes a negatively refracting medium[8,9] and $\partial^2\omega/\partial k_\perp^2<0$. Therefore Eq. (5) predicts different nonlinear behaviors in frequency regions below and above ω_s. In the frequency region $\omega<\omega_s$, self-guiding should occur. However, the threshold for self-guiding is now determined by the lattice. In particular, as $\omega\to\omega_s^-$, $\partial^2\omega/\partial k_\perp^2|_{k_\parallel^0,0}\approx\alpha(\omega_s-\omega)$. From Eq. (5) the critical peak field value for an optical beam becomes

$$E_{\text{critical}} \propto \frac{1}{d}\sqrt{\frac{(\omega_s-\omega)}{\omega_s\varepsilon_2}} \tag{7}$$

and asymptotically approaches zero as the frequency approaches the self-collimation frequency from below. On the other hand, in the frequency region of the first band with $\omega>\omega_s$, self-focusing can be completely suppressed.

Below, we seek to numerically demonstrate these nonlinear effects in a photonic crystal by nonlinear finite defference time domain simulations.[10] We first test our method in a uniform nonlinear medium. We simulate a monochromatic beam with a transverse Gaussian profile propagating through a two-dimensional uniform medium with Kerr nonlinearity (i.e., $\varepsilon=\varepsilon_0+\varepsilon_2|E|^2$). The width of the Gaussian beam is $14a$. At each operating wavelength, we gradually change the peak field amplitude of the input and obtain the critical peak value by examining at the field pattern until no significant diffraction occurs within the computation cell. Figure 2 shows the results of the critical peak field value versus the wavelength. (The electric fields in all plots are normalized to $1/\sqrt{\varepsilon_2}$). The slope of the fitting line differs with the slope predicted from Eq. (6) by about 25%, due largely to the fact that Eq. (6) assumes a flattop beam while in our simulation we use a Gaussian beam. Nevertheless, the linear dependence of the critical field on wavelength is clearly seen, in agreement with the analytic results of Eq. (6).

Using the same numerical methods, we obtain the critical field value for self-guiding in the photonic crystal shown in Fig. 1. Figure 3 shows the wavelength dependency of the critical field value, for the same Gaussian input pulse with a width of $14a$. As predicted from Eq. (5), self-guiding occurs in the positive refraction region with $\omega<\omega_s$. When $\omega\ll\omega_s$, the critical field value for self-guiding is much larger in the crystal compared with the uniform medium, since the air holes in the crystal reduce the amount of nonlinear materials present [i.e., in Eq. (5), the overlap factor γ is reduced]. The critical field value reduces as frequency increases and indeed vanishes when $\omega\to\omega_s^-$. In the vicinity of ω_s^-, which is the regime where Eq. (7) is valid, indeed the numerical results agree very well with the fit using Eq. (7).

The numerical simulations also demonstrate that self-guiding is completely suppressed in the negative region at $\omega>\omega_s$. Figure 4(a) shows the strong defocusing of an optical beam at $\omega=0.21(2\pi c/a)$, which is above the self-collimation frequency in the crystal, with an initial field amplitude ($\varepsilon_2|E|^2\approx0.98$). In comparison, a clear self-guiding effect is

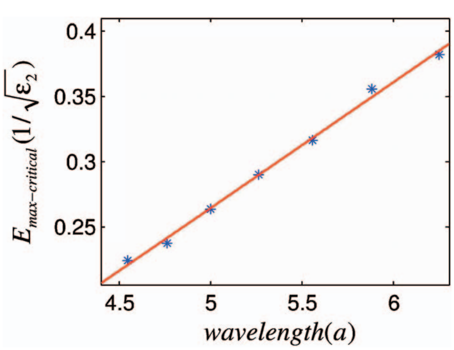

FIG. 2. (Color) Critical field value vs wavelength in a nonlinear uniform medium for an incident optical beam with a width of $14a$. The medium has a nonlinear dielectric constant $\varepsilon=\varepsilon_0+\varepsilon_2|E|^2$, where $\varepsilon_0=12$. The blue dots are the simulation results and the red line is a linear fit.

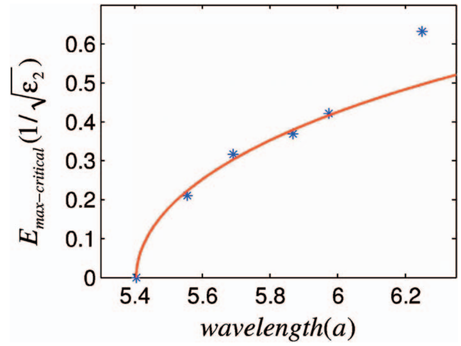

FIG. 3. (Color) Critical field value vs wavelength in a nonlinear photonic crystal. The crystal has the same parameters as in Fig. 1. The incident optical beam has a width of $14a$. The blue dots are the simulations result and the red line is generated by fitting using Eq. (7).

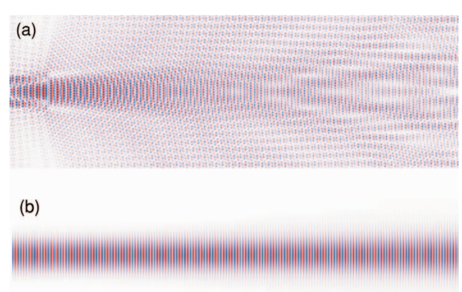

FIG. 4. (Color) Magnetic field distribution in a (a) nonlinear photonic crystal and (b) nonlinear uniform medium at a frequency of $0.21(2\pi c/a)$, where a is the lattice constant in the crystal. The width of the incident optical beam is $14a$. The maximum amplitudes of the input electric field are (a) $0.98(1/\sqrt{\varepsilon_2})$ and (b) $0.06(1/\sqrt{\varepsilon_2})$.

observed in a corresponding uniform medium with a much lower input field amplitude ($\varepsilon_2|E|^2 \approx 0.06$) [Fig. 4(b)].

Finally, we estimate the power level required to observe the effects predicted here in the AlGaAs system, in which spatial soliton propagation has been widely studied. Assuming a Kerr coefficient $n_2 = 1.4 \times 10^{-13} \text{cm}^2/\text{W}$, which is appropriate for AlGaAs at 1.55 μm,[11] our estimation indicates that when the power exceeds a threshold of 5 kW, a spatial soliton with a beam width of 4.5 μm forms in a uniform slab with a thickness of 0.5 μm. (Consist with the experiments in Ref. 12 at a different thickness and beam width.) In such a slab system, our theory predicts significant modification of the power threshold for spatial soliton formation once the photonic crystal is introduced.

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