RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 50, NUMBER 6 DECEMBER 1994

Dark and bright photovoltaic spatial solitons

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(Received 14 April 1994)

Dark (bright) planar spatial solitons are predicted for photovoltaic photorefractive materials when the diffraction of an optical beam is exactly compensated by nonlinear self-defocusing (focusing) due to the photovoltaic field and electro-optic effect. These solitons may have steady-state irradiances of microwatts to milliwatts per square centimeter and widths as small as 10 μm in lithium niobate. Optical control is provided by incoherent illumination, and the nonlinear index of a dark soliton may be used to trap a bright soliton by rotating the plane of polarization of the soliton field.

PACS number(s): 42.50.Rh

Self-trapping of light beams in nonlinear Kerr media has been the subject of intense theoretical and experimental investigation for three decades [1–5]. The self-trapped beams, which may behave as spatial solitons, evolve from nonlinear changes in the refractive index of the material induced by the intensity distribution of the light when the confining effect of the refractive index exactly compensates diffraction. The typical powers required for observation are hundreds of kilowatts (pulsed) in optical Kerr media [4] or watts (cw) in thermal-nonlinear media [5].

More recently, theoretical and experimental work has demonstrated a different spatial soliton using the photorefractive nonlinearity in an electro-optic crystal to compensate for diffraction [6–10]. The intensity profile of the beam modulates the refractive index via the photorefractive effect such that exact compensation for diffraction allows the light beam to propagate with a constant profile in two transverse dimensions. The photorefractive solitons have exceptionally low optical powers in the microwatt range, disappear at steady state, and require an externally applied electric field.

Here we predict the existence of a different spatial soliton that results from the photovoltaic effect [11] in a photorefractive material. Solitons in photovoltaic-photorefractive materials differ from both the Kerr and photorefractive solitons in physical origin, dependence on the light intensity and material properties, and temporal behavior. Our work is motivated by numerous observations [12–14] of dark features that appear in the transverse profile of a single beam propagating in a bulk photovoltaic medium, $LiNbO_3$. We derive and solve equations for beam propagation in photovoltaic media and obtain the properties of the dark (or bright) spatial soliton in one transverse dimension. Then we derive the conditions necessary for experimental observation and compare photovoltaic solitons to Kerr and photorefractive solitons.

We start with the standard set of rate, current, and Poisson's equations that describes the photorefractive effect in a medium in which the photovoltaic current is nonzero and electrons are the sole charge carriers. In the steady state and one dimension these equations are [15]

$$(sI+\beta)(N_d-N_d^i)-\gamma_e nN_d^i=0, \tag{1}$$

$$q\mu nE+k_BT\mu\,\frac{dn}{dx}+\kappa s(N_d-N_d^i)I=0, \tag{2}$$

$$\frac{dE}{dx}+(q/\varepsilon)(n+N_A-N_d^i)=0, \tag{3}$$

where n is the electron number density, N_d is the total donor number density, N_d^i is the number density of ionized donors, and N_A is the number density of negatively charged acceptors that compensate for the ionized donors. Further, I is the irradiance of the optical beam and E is the space-charge field inside the crystal, and all variables depend on the distance x perpendicular to the direction of propagation z of the optical beam. The parameters of the crystal are the photoionization cross section s, the dark generation rate β, the recombination rate coefficient γ_e, the electron mobility μ, the photovoltaic constant κ, and the dielectric constant ε; q is the charge on the electron, k_B is Boltzmann's constant, and T is the absolute temperature.

These material equations must be supplemented by the equation for the optical field $E_{\mathrm{opt}}(x,z,t)=A(x,z)\exp(ikz-i\omega t)+\text{c.c.}$ In the paraxial approximation the slowly varying amplitude, $A(x,z)$ is determined by

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$$\left(\frac{\partial}{\partial z}-\frac{i}{2k}\frac{\partial^2}{\partial x^2}\right)A(x,z)=\frac{ik\Delta n}{n_b}A(x,z), \tag{4}$$

where Δn is the nonlinear change in the refractive index, n_b is the background refractive index, and k is the optical wave number in the material, $k=2\pi n_b/\lambda$, where λ is the wavelength. In electro-optic materials Δn is related to the electric field through the effective electro-optic coefficient r by [15]

$$\Delta n=-n_b^3 rE/2, \tag{5}$$

and this relation plus appropriate boundary conditions specifies the problem. For simplicity, we assume that we deal with a crystalline geometry such that Δn is scalar. This implies $\kappa=\kappa_{ii}$ and $r=r_{ii}$, where i is a crystalline principal axis parallel to x.

Solution of Eqs. (1)–(5) generally requires approximation or numerical work. Here we approximate the space-charge field in terms of the optical irradiance:

$$E\approx -E_p\frac{I/I_{\text{dark}}}{1+I/I_{\text{dark}}}, \tag{6}$$

where $E_p=\kappa\gamma_e N_A/(q\mu)$ is the photovoltaic field constant, $I(x,z)=|A(x,z)|^2$, and the dark irradiance is defined by $I_{\text{dark}}=\beta/s$. We substitute Eq. (6) in Eqs. (5) and (4) and obtain a solution for the soliton amplitude.

Before solving the wave equation, we derive Eq. (6), which involves neglect of the diffusion current relative to drift and photovoltaic currents in Eq. (2) and approximation of the ionized donor number density by its dark value ($N_d^i\approx N_A$). First, we solve Eq. (2) for E,

$$E=E_D(x)+E_{\text{PV}}(x). \tag{7}$$

where the diffusion field is given by $E_D(x)=-k_BT/qd\ \ln n/dx$ and the photovoltaic field by $E_{\text{PV}}(x)=-\kappa s(N_d-N_d^i)I/(q\mu n)$. Second, we solve Eq. (1) for n in terms of N_d^i:

$$n=(sI+\beta)(N_d-N_d^i)/\gamma_e N_d^i. \tag{8}$$

Finally, we substitute $N_d^i=N_A+\Delta N_d^i$ and expand about N_A. This yields

$$\begin{aligned}n=&(sI+\beta)(N_d-N_A)/(\gamma_e N_A)[1-\Delta N_d^i/(N_d-N_A)]\\&\times[1-\Delta N_d^i/N_A+(\Delta N_d^i/N_A)^2+\cdots]\end{aligned} \tag{9}$$

and

$$E_{\text{PV}}=-E_p\frac{I/I_{\text{dark}}}{1+I/I_{\text{dark}}}(1+\Delta N_d^i/N_A). \tag{10}$$

We note that if $\Delta N_d^i/N_A\ll 1$, the magnitude of the diffusion field is simply $k_BT/(qL_s)$, where L_s is the spatial scale of I. Then $E_D(x)$ may be neglected relative to $E_{\text{PV}}(x)$ if L_s is much greater than $k_BT/(qE_p)$. Photovoltaic fields are on the order of 10^4 to 10^7 V/m and $k_BT/q=0.025$ V at room temperature. Even for fields as low as 10^4 V/m, neglect of the diffusion field is a good approximation for $L_s\gg 2.5\ \mu$m.

It remains to show $\Delta N_d^i/N_A\ll 1$ to obtain Eq. (6) from (10). Substituting $N_d^i=N_A+\Delta N_d^i$ into Eq. (3) yields

$$\Delta N_d^i=n+(\varepsilon/q)\frac{dE}{dx}. \tag{11}$$

Thus we require $n\ll N_A$ and $\varepsilon/q(dE/dx)\ll N_A$. Substitution of Eq. (11) into Eq. (10) yields

$$E_{\text{PV}}=-E_p\frac{I/I_{\text{dark}}}{1+I/I_{\text{dark}}}\left(1+\frac{n}{N_A}+\frac{\varepsilon}{qN_A}\frac{dE}{dx}\right). \tag{12}$$

For irradiances typical of cw lasers, less than 10 W/cm^2, the second term in parentheses, n/N_A, is always much less than 1 [15]. The third term, $(\varepsilon dE/dx)/(qN_A)$, can be evaluated using Eq. (6) for $E(x)$, the zeroth-order term in the expansion, to see if the expansion converges. This yields the expression

$$\frac{\varepsilon}{qN_A}\frac{d}{dx}\left|\left(\frac{-E_pI/I_{\text{dark}}}{1+I/I_{\text{dark}}}\right)\right|\ll 1. \tag{13}$$

In the absence of a solution for the soliton irradiance $I(x)$, Eq. (13) can be evaluated dimensionally, and one obtains that the soliton dimension L_s must be much greater than $E_P\varepsilon/(qN_A)$. One can restate this condition as requiring that the limiting space charge field in the material be large compared to the photovoltaic field. For typical materials, $N_A=10^{16}$ cm^{-3}, $\varepsilon=30\varepsilon_0$ (ε_0 is the permittivity of free space), and $E_P=10^7$ V/m; this yields $L_s\gg 1.66\ \mu$m.

Substitution of the usual soliton ansatz,

$$A(x,z)=u(x)\exp(i\gamma z)I_{\text{dark}}^{1/2}, \tag{14}$$

and Eqs. (5) and (6) in Eq. (4) yields

$$u-u''/(2k\gamma)=(a/\gamma)u^3/(1+u^2), \tag{15}$$

where $a=kn_b^2 rE_p/2$. Soliton solutions to equations like (15) have been investigated theoretically in the context of saturable absorbers [16,17]. There are two cases of interest in Eq. (15). If a is negative (defocusing nonlinearity), then γ must be negative to obtain a physical solution. In this case, one can set $\xi=(-2k\gamma)^{1/2}x$ and $\delta=a/\gamma$ to obtain

$$u''+u-\delta u^3/(1+u^2)=0, \tag{16}$$

where the double prime refers to the variable ξ. This equation yields dark solitons. On the other hand, if a is positive (focusing nonlinearity), then γ must be positive, and one can set $\xi=(2k\gamma)^{1/2}x$ and $\delta=a/\gamma$ to obtain

$$u''-u+\delta u^3/(1+u^2)=0, \tag{17}$$

and this equation yields bright solitons. If the denominator in the third term were replaced by 1, Eq. (16) would have the Kerr solution $u(\xi)=(1/\delta)^{1/2}\tanh(\xi/2^{1/2})$ while, Eq. (17) would have the solution $u(\xi)=(2/\delta)^{1/2}\text{sech}(\xi)$.

A first integral of Eq. (16) or (17) is obtained by quadrature methods:

$$p^2-p_0^2=\pm\{(\delta-1)(u^2-u_0^2)-\delta\ln[(1+u^2)/(1+u_0^2)]\}, \tag{18}$$

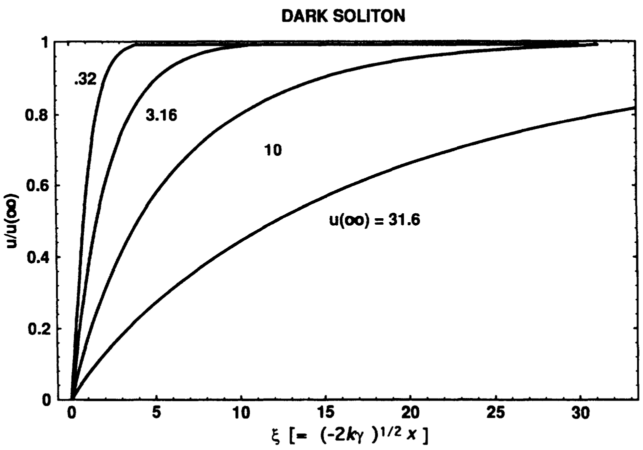

FIG. 1. Dark photovoltaic soliton amplitude $u/u(\infty)$ as a function of ξ for several values of $u(\infty)$. As for Kerr media, the dark photovoltaic soliton is an odd function of ξ. Note that the horizontal scales in dimensional units are different for each curve and correspond to $x=\xi/\sqrt{-2kau_\infty^2/(u_\infty^2+1)}$.

where $p=du/d\xi$ and the upper (lower) sign is for dark (bright) solitons. The boundary conditions for the dark soliton are $p(0)=p_0$, $u(0)=0$, $p(\infty)=0$, and $u(\infty)=u_\infty$, while for the bright soliton $p(0)=0$, $u(0)=u_0$, $p(\infty)=0$, and $u(\infty)=0$. For the dark soliton $u''(\infty)=0$, Eq. (16) implies $u(\infty)=1/(\delta-1)^{1/2}$, and Eq. (18) gives $p_0^2=\delta\ \ln[\delta/(\delta-1)] - 1$. For bright solitons u_0 is obtained from Eq. (18) evaluated from 0 to ∞:

$$\delta[u_0^2-\ln(1+u_0^2)]-u_0^2=0. \tag{19}$$

Equation (18) can be integrated numerically to obtain the photovoltaic dark soliton profile as a function of ξ shown in Fig. 1. Note that, unlike the Kerr soliton, which has a constant width as a function of δ in units of ξ, the photovoltaic soliton broadens substantially for δ near 1. For the bright solitons, we use the boundary condition $u(0)$ from Eq. (19),

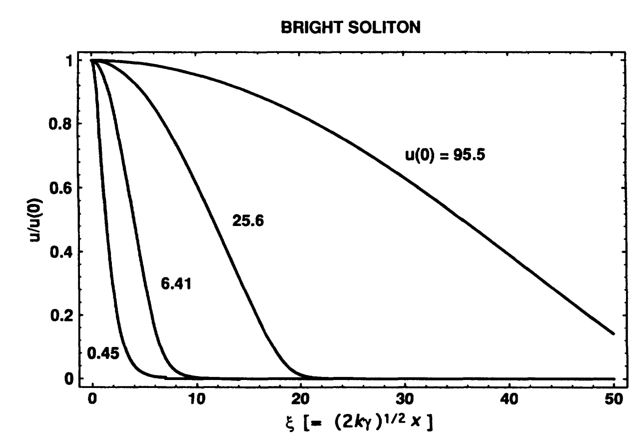

FIG. 2. Bright soliton amplitude u/u_0 as a function of ξ for several values of u_0. As for Kerr media, the bright photovoltaic soliton is an even function of ξ. Note that the horizontal scales in dimensional units are different for each curve and correspond to $x=\xi/\sqrt{2ka(u_0^2-\ln(1+u_0^2))/u_0^2}$.

and Fig. 2 shows the amplitude profiles normalized to u_0 as a function of ξ for several values of u_0.

Now we return to the condition $(\varepsilon/q)(dE/dx)\ll N_A$. Self-consistency requires

$$\frac{\varepsilon E_p}{q}\frac{d}{dx}\left(\frac{u^2}{1+u^2}\right)\ll N_A. \tag{20}$$

Performing the derivatives yields

$$\frac{2up(2k|\gamma|)^{1/2}}{(1+u^2)^2}\frac{\varepsilon E_p}{qN_a}\ll 1, \tag{21}$$

where p is related to u by Eq. (18). Recalling $\gamma=a/\delta$, $a=k\Delta n_{\max}/n_b$, and the relation between δ and u_0 for bright solitons [Eq. (19)] or $u(\infty)^2=1/(\delta-1)$ for dark solitons, one can evaluate the left-hand side of Eq. (21). The factor $up/(1+u^2)^2$ is large for intensities much greater than the equivalent dark irradiance and has a maximum value of 0.18 for the bright solitons and about 2 for the dark solitons. Substitution into Eq. (21) then yields

$$C\frac{E_p}{E_q(k)}\left(\frac{\Delta n_{\max}}{n_b}\right)^{1/2}\ll 1, \tag{22}$$

where $C\sim 0.5$ for bright and ~ 5.0 for dark solitons, and $E_q(k)$ is the limiting space charge field evaluated at the optical wave number, $qN_A/\varepsilon k$. For $N_A=10^{16}\ \mathrm{cm}^{-3}$, $n_b=2.2$, $\Delta n_{\max}=10^{-4}$, and $\lambda=0.5\ \mu$m, the criterion becomes $E_p\ll 1.4\times 10^8$ V/m (bright) and 1.4×10^7 V/m (dark). Typically, photovoltaic fields are less than 10^7 V/m and the approximation is very good for bright solitons and satisfied for dark solitons of moderate irradiance.

It is instructive to plot the dark soliton half-width as a function of irradiance in more physical units for an illustrative photovoltaic material. Consider a $LiNbO_3$ crystal characterized by a large photovoltaic (Glass) constant [11] $G_{33}\sim 3\times 10^{-9}$ cm/V and an electro-optic coefficient $r_{33}\sim 30$ pm/V. Since we have assumed a scalar Δn, this implies that our x and z directions must be parallel to the principal crystalline axes. For large photovoltaic effects in $LiNbO_3$ it is useful to propagate along the crystalline a axis with a beam that is narrow in the direction parallel to the c axis and linearly polarized parallel to c. For typical photovoltaic fields [16], one obtains refractive-index perturbations of $\Delta n_{\max}=10^{-4}$ to 10^{-3}. For $\lambda=0.5\ \mu$m and $n_b=2.2$, this yields $(2ka)^{-1/2}=3.79$ ($\Delta n_{\max}=10^{-4}$) and 1.20 ($\Delta n_{\max}=10^{-3}$) μm. Typically the dark irradiance in $LiNbO_3$ is about 1 μW/cm^2. Since all observations [12–14] indicate a negative perturbation in the index ($a<0$), we give illustrative results for the dark soliton. Figure 3 shows the half-width in units of $(2ka)^{-1/2}$ as a function of irradiance divided by dark irradiance. Note that photovoltaic solitons with a dimensionless half-width of less than about 1.2 do not exist, and for widths greater than about 1.2 there exist solitons at two values of the optical irradiance.

Next, we discuss the properties of the photovoltaic spatial solitons. The most distinctive property is sensitivity to the ratio of the optical irradiance to the equivalent dark irradiance—unlike Kerr solitons that depend on the absolute irradiance [1,2] or the photorefractive solitons that are inde-

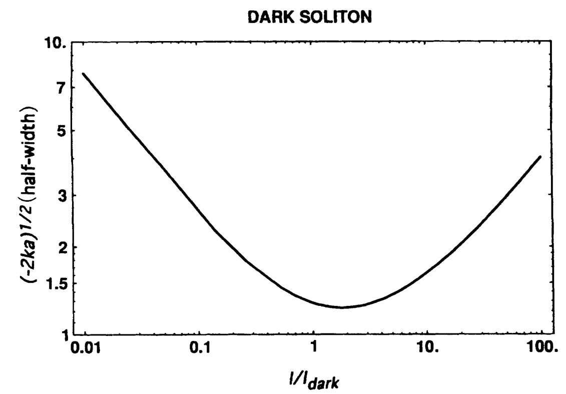

FIG. 3. The dimensionless half-width of the dark soliton as a function of irradiance divided by the dark irradiance.

pendent of the irradiance [6–10]. This property permits observation of photovoltaic solitons at moderate powers and facilitates control of their width through the intensity. Material absorption modifies the photovoltaic soliton since it reduces $I/I_{\rm dark}$; however, absorption can be balanced by modulating the dark irradiance artificially with incoherent uniform illumination (note that the time response to reach the steady state dramatically depends on the intensity [18]). Another special property of the photovoltaic soliton is that, in principle, one may switch from a bright to a dark soliton simply by rotating the polarization of the light. Some photovoltaic materials (for example, $BaTiO_3$ [18]) possess photovoltaic constants that change sign under polarization rotation. This results in an equivalent change of sign in the refractive-index perturbation and means that a given laser intensity can be used either for bright or dark solitons depending on polarization of the light. On the other hand, in Kerr media the regimes of dark and bright solitons appear at different wavelength ranges, while in photorefractive media, the solitons may be switched from bright to dark by changing the polarity of the external electric field.

Finally, it is interesting to find the lowest optical irradiance (intensity) that can lead to a photovoltaic soliton. Consequently, we consider a dark photovoltaic soliton in the regime where $|u^2| \ll 1$ for all ξ. This leads to the limit where photovoltaic solitons behave as Kerr solitons, with one difference: the nonlinear change in the refractive index is due to the photovoltaic field and electro-optic effect rather than the optical Kerr effect. The solutions are therefore of the form $u = u(\infty)\tanh(\xi/2^{1/2})$. Recalling that the soliton size $L_s = 1/(k\gamma)^{1/2}$ and substituting for γ yields $u(\infty)^2 = (\lambda/L_s)^2\,(2\pi^2 E_p n^4 r)^{-1}$. For $u(\infty)^2 = 0.1$ (maximum irradiance equal to 0.1 times the dark irradiance) and $L_s \sim 20\ \mu$m, we find that a minimum value of $E_p = 40$ kV/cm is required to trap a minimum irradiance dark soliton, which is physically accessible [18].

In conclusion, we have shown theoretically that photorefractive crystals with a photovoltaic current can support a unique type of spatial soliton.

ACKNOWLEDGMENTS

The work of G.C.V. was supported by Hughes Aircraft Company Internal Research and Development; M.S. and A.Y. were supported by the Advanced Research Projects Agency (ARPA), the U.S. Army Research Office, and the U.S. Air Force Office of Scientific Research; M.M.F. acknowledges the support of ARPA through the Center for Nonlinear Optical Materials at Stanford University.

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