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Collusive pricing with capacity constraints in the presence of demand uncertainty

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We explore the response of collusive prices to changing demand conditions when firms operate under capacity constraints in the presence of demand uncertainty. We find support for the conventional view that periods of low demand lead, through the emergence of excess capacity, to a breakdown of collusive pricing. We also find that the nature of price wars depends on the degree of excess capacity in the industry; while small amounts of excess capacity can lead firms to engage in "mild" price wars, characterized by uniform price reductions and market share stability, more "severe" price wars, characterized by price undercutting and market share instability, can emerge if excess capacity is sufficiently great. Finally, our results lend support to the view that market share instability is a symptom of ineffective collusion.

1. Introduction

■ The impact of unexpected shifts in market demand on the ability of firms to maintain tacit collusion is tied fundamentally to the dynamic interaction between business conditions and cost structures. It is on the basis of this observation, and the prevailing evidence on the shape of short-run cost functions, that Scherer and Ross (1990) give theoretical motivation for the conventional empirical wisdom that tacit collusion tends to break down when business conditions turn sour. Focusing on industries characterized by high fixed costs, with marginal cost initially constant but rising steeply as production approaches 100% of capacity, Scherer and Ross argue that the incentive to cut prices is most likely to lead to the breakdown of collusive price agreements and the outbreak of "price wars" when market demand, and thus capacity utilization at the collusive price, is low. As they note, a large body of empirical evidence supports this view.[1]

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[1] See, for example, the evidence on the relationship between prices and business conditions presented in Cowley (1986), Domowitz, Hubbard, and Petersen (1986), and Suslow (1988). Suslow (1986) and Bresnahan and Suslow (1989, 1990) provide support from the primary aluminum and ammonia fertilizer industries that short-run marginal cost is fairly flat below capacity and becomes essentially vertical at capacity.

The notion that unexpectedly low demand leads to price wars has been formalized by Green and Porter (1984) in the setting of an infinitely repeated game. However, capacity utilization in general and capacity constraints in particular play no role in the Green and Porter analysis. Rather, their work is closer in spirit to the earlier work of Stigler (1964), where imperfect observability of demand is the crucial ingredient; firms "agree" to switch to noncooperative Cournot behavior (engage in price wars) when demand is unexpectedly low as a way of mitigating any firm's incentives to secretly defect from collusive quantity choices.

More recently, Rotemberg and Saloner (1986) have explored the relationship between collusive prices and business conditions under the assumption of perfect observability in a a setting in which firms collude tacitly in an infinitely repeated (super) game. They establish that, when firms set prices under conditions of constant marginal cost (and no capacity constraints), a reversal of the logic underlying the conventional wisdom applies; periods of unusually high demand present firms with an unusually large gain from shaving price and "taking the market," and this undermines their ability to maintain tacit collusion. The most collusive equilibrium sustainable during "booms" then involves less than fully collusive prices, behavior that Rotemberg and Saloner interpret as "price wars." These price wars are not periods of reversion to noncooperative behavior, like the price wars of Green and Porter (1984), but they do represent periods of less effective collusion.

While Rotemberg and Saloner's (1986) finding of "price wars during booms" goes against the conventional wisdom, it fits well with anecdotal evidence on the pricing behavior of industries in which orders are lumpy and infrequent.[2] In such industries, "booms" are synonymous with large orders, and by definition are thus of a winner-take-all nature. When this is the case, the notion that booms present firms with an unusually strong temptation to shave price clearly applies. This notion may also apply more generally in industries with chronic excess capacity, and Rotemberg and Saloner do present more general empirical evidence in support of their theory. Nevertheless, whatever the scope of the empirical relevance of the Rotemberg and Saloner result, it is clear that their constant-marginal-cost model does not capture the role of capacity constraints that is central to the conventional view of the relationship between collusive prices and business conditions.

In fact, although the conventional view—that periods of low demand lead, through the emergence of excess capacity, to the breakdown of collusive pricing—is anchored in a large body of empirical work, the associated theoretical problem has never been formalized as a problem of tacit collusion.[3] The aim of this article is to do just that.

We adopt the basic setting of an infinitely repeated game in which firms face stochastic market demand and must choose each period's capacity before the market demand for the period is realized. Once market demand for the period has been observed, and with their capacities for the period fixed, firms then simultaneously choose prices. Within this setting, firms attempt to enforce collusion over capacity and price with credible (subgame perfect) threats to punish defections from the collusive agreement. As punishments, we consider the threat of infinite Nash reversion. Our model can be viewed essentially either as an infinitely repeated version of Kreps and Scheinkman (1983), with firms facing stochastic

[2] See, for example, the discussion of pricing behavior in the market for antibiotic tetracycline in Scherer and Ross (1990).

[3] In this regard, Rotemberg and Saloner (1986) do note the potential importance of capacity constraints or, more generally, increasing marginal costs, in affecting the relationship between collusive prices and business conditions. In the case where demand and marginal costs are linear, they show that the flavor of their result is preserved. However, as they note, in this linear-demand and linear-marginal-cost example, a favorable demand shock is formally equivalent to a favorable marginal-cost shock. Hence it is not surprising that favorable demand shocks (favorable marginal-cost shocks) lead to a greater incentive to cut prices and capture the market, and lead therefore to less effective collusion. But it is equally apparent that the logic of the conventional wisdom, with its focus on the emergence of excess capacity in times of demand slumps, is not captured by the linear-marginal-cost example.

market demand that is realized only after capacity is set for the period, or as a variation on the Rotemberg and Saloner (1986) model, with the introduction of a capacity-setting stage at the start of each period.[4]

Within this basic model we establish three main results. First, in the presence of capacity constraints, we find support for the relationship between collusive prices and business conditions described by Rotemberg and Saloner (1986) and also for the relationship that conforms to conventional wisdom. Intuitively, when demand is sufficiently weak and excess capacity sufficiently large that capacity constraints do not bind even for defecting firms, small improvements in demand can lead to less effective collusion, as in the "price wars during booms" result of Rotemberg and Saloner. However, as demand strengthens further, capacity constraints begin to play a role; over this range, the stronger demand is, the closer colluding firms will operate to capacity and the smaller the current gain from defection will be, strengthening the ability to maintain a higher collusive price in such periods. We show that the conventional wisdom of slack demand implying a greater likelihood of price wars applies best in precisely those industries where capacity costs are high and capacity constraints most important. Thus, the importance of capacity constraints is a crucial determinant of the relationship between collusive prices and business conditions across industries.

Second, our equilibrium price wars can take either of two qualitatively distinct forms, depending on the state of demand for the period. One form of price war is analogous in character to the price wars of Rotemberg and Saloner (1986), in which firms temporarily collude less effectively and adopt a uniform price below the joint monopoly level. We also establish the existence of states of demand in which a second kind of price war arises, characterized by mixed-strategy pricing. As we discuss below, these mixed-strategy price wars may, if firms cannot cooperate in mixed strategies, reflect temporary reversion to noncooperative behavior, as in the price wars of Green and Porter (1984). In any event, these mixed-strategy price wars differ from those of both Green and Porter and Rotemberg and Saloner in that they correspond to periods in which one firm's price actually undercuts the other's in equilibrium, with the low-price firm temporarily stealing market share from the high-price firm. Finally, the changing nature of our price wars as the state of demand changes has a strong intuitive appeal; for moderately bad demand realizations, firms simply initiate a uniform price reduction below the joint monopoly price, but if demand conditions turn sufficiently sour, firms will escalate the price war and undercut one another in equilibrium. Only with a virtual collapse of demand will capacity constraints become irrelevant and uniform pricing reemerge.

Third, we find the possibility that firms will undercut one another's prices in equilibrium arises only if firms are achieving a relatively low level of collusion overall. Since it is only when firms pursue such mixed-strategy pricing that relative market shares across colluding firms differ markedly from one period to the next, our results suggest that, in industries where capacity constraints are important, intertemporal instability in relative market shares across oligopolistic firms is evidence of an inability to collude effectively.[5]

To parameterize the importance of capacity constraints, we also consider an intermediate case, between our rigid capacity constraints and none at all, where firms can make costly additions to capacity after demand is realized. We find that, depending on the unit cost of

[4] Related work on price-setting supergames with capacity constraints but without stochastic demand can be found in Brock and Scheinkman (1985), Benoit and Krishna (1987), and Davidson and Deneckere (1990), among others. See also Rotemberg and Saloner (1989a) for an analysis of the impact of import quotas on collusive behavior, and Rotemberg and Saloner (1989b) on the use of inventories to support collusion.

[5] A referee has noted that this idea would also emerge from a comparison of the results of Osborne and Pitchik (1986), who characterize the noncooperative Nash equilibrium of a capacity-constrained price game in the presence of variable demands, to the pricing behavior of a monopolist in this environment, under the assumption that "effective collusion" would mimic the monopoly price.

additions to capacity relative to the unit cost of initial capacity, equilibria can emerge ranging from the conventional view to those exhibiting price wars during booms.

The remainder of the article is devoted to establishing these points. The next section sets forth the basic model and solves for the static Nash and joint monopoly equilibria. Section 3 considers the dynamic game and establishes our main results. Section 4 extends the model to allow for costly additions to capacity once the state of demand is revealed. Section 5 concludes.

2. The basic model

■ In this section we present the basic model and describe two equilibria: the static Nash equilibrium and the joint monopoly outcome. The latter serves as the collusive ideal toward which the two firms will strive in the repeated game. The former serves both as a benchmark from which to measure collusive gains and as a credible (though not optimal) threat with which to sustain collusion.

□ **Assumptions.** We consider an infinitely repeated game between two firms, labelled 1 and 2, selling in a market where demand is stochastic. At the beginning of any period, firms first must simultaneously set capacity K_1 and K_2, facing per-unit capacity costs $r > 0$. Once capacity choices for the period are made, the state of demand for the period is revealed.[6] We assume for simplicity that demand takes the linear form

$$D = \alpha - P. \tag{1}$$

The parameter α is determined each period by an independent draw from a distribution with full support on the interval $\alpha \in [0, \bar{\alpha}]$ and (commonly known) distribution function $F(\alpha)$.[7] After observing the demand realization for the period, the two firms then simultaneously set prices facing zero marginal costs of production (up to capacity).

□ **The static Nash equilibrium.** We first characterize the unique symmetric static Nash equilibrium of this game. We rely heavily on Kreps and Scheinkman (1983) and therefore on the particular (efficient) rationing rule underlying their results. Specifically, consumers buy first from the cheapest supplier, and income effects from price changes are absent.[8]

In Appendix A we show that there exists a unique symmetric Nash equilibrium for the static game, with symmetric capacity choice characterized by

$$K^n = \frac{E[\alpha] - G(3K^n) - r}{3[1 - F(3K^n)]}, \tag{2}$$

[6] We model capacity choices as being made anew at the beginning of each period, with the period's prices then set after the state of demand is revealed. The assumption that capacity is variable across periods might be appropriate in a setting such as the automobile, computer, or appliance industries, where product life-cycles are relatively short and production capacity is variable with each model year. In fact, since each period looks the same at the time capacity choices are made, equilibrium capacity choices will be constant over time. If we were to adopt the approach of Brock and Scheinkman (1985) and fix capacity exogenously at the outset of the game, or follow the approach of Davidson and Deneckere (1990), also considered by Benoit and Krishna (1987), and have firms make once-for-all capacity choices at the start of the game, the sustainable level of collusion would be altered, since firms could not change capacity levels in the event of a defection from the collusive agreement; but the general properties of our results would remain.

[7] Setting the lower bound on α at zero rather than some $\underline{\alpha} > 0$ reduces the number of cases we must consider but does not alter the flavor of our results in any way. The assumption that demand shocks are independently distributed is potentially important, as Haltiwanger and Harrington (1991) have shown in the context of the model of Rotemberg and Saloner (1986).

[8] See Davidson and Deneckere (1986) for results from a static two-stage game under different rationing rules.

where $E[\alpha]$ is the expected value of α and $G(\alpha) \equiv \int_0^\alpha s dF(s)$. The Nash equilibrium pricing behavior as a function of the realization of α can be characterized as follows. There are three distinct pricing patterns, corresponding to realizations of $\alpha \in [0, K^n]$ (Region I), $\alpha \in (K^n, 3K^n)$ (Region II), and $\alpha \in [3K^n, \bar{\alpha}]$ (Region III). For realizations of α in Region III, capacity constraints bind, and both firms will sell their entire capacity at market-clearing prices $P(\alpha; 2K^n) \equiv \alpha - 2K^n$. For realizations of α in Region I, either firm has sufficient capacity to supply the entire market at a price equal to marginal cost (zero), so that capacity constraints do not bind in the standard Bertrand equilibrium and the equilibrium price is driven to zero. Finally, for realizations of α in Region II, pure-strategy equilibria to the capacity-constrained price game do not exist, and firms play mixed strategies.

Thus the unique symmetric static Nash equilibrium has each firm setting capacity K^n (in every period) and making expected profits of $E\pi^n$ (defined in Appendix A), with both firms selling their entire capacity under high demand realizations in the range $\alpha \in [3K^n, \bar{\alpha}]$, both firms randomizing over price and each having excess capacity with positive probability under medium demand realizations in the range $\alpha \in (K^n, 3K^n)$, and both firms selling at marginal cost (zero) and each having excess capacity with certainty under low demand realizations in the range $\alpha \in [0, K^n]$.

□ **The joint monopoly solution.** Before turning to the dynamic game, we consider the monopoly solution to the static game. This provides the collusive ideal toward which the two firms will strive in the repeated setting.

Consider first the monopolist's profit-maximizing choice of output for a period in which capacity is already in place and does not pose a binding constraint. Here the optimal output choice is simply $\alpha/2$. Thus, for any choice of capacity K, the monopolist sells its unconstrained optimum output level $\alpha/2$ over the range of α given by $\alpha \in [0, 2K]$. For $\alpha \in [2K, \bar{\alpha}]$, the monopolist simply sells its capacity at the market-clearing price $P(\alpha; K)$. Thus, expected monopoly profits are given by

$$E\pi^m(K) = \int_{2K}^{\bar{\alpha}} P(\alpha; K) \cdot K dF(\alpha) + \int_0^{2K} P(\alpha; \alpha/2) \cdot (\alpha/2) dF(\alpha) - rK. \tag{3}$$

The first-order condition for (3) yields the monopoly capacity choice

$$K^m = \frac{E[\alpha] - G(2K^m) - r}{2[1 - F(2K^m)]}, \tag{4}$$

with second-order conditions met globally. Using (2) and (4), it follows that $K^m = 3K^n/2$. Thus, market capacity is smaller under the monopoly solution than in the static Nash equilibrium. Finally, monopoly pricing has

$$P^m(\alpha) = \begin{cases} P(\alpha; K^m) & \text{for} \quad \alpha \in [2K^m, \bar{\alpha}] \\ \alpha/2 & \text{for} \quad \alpha \in [0, 2K^m]. \end{cases} \tag{5}$$

Thus, for $\alpha \in [0, 2K^m]$, profit-maximizing pricing involves excess capacity. We define $E\pi^m \equiv E\pi^m(K^m)$.

3. The dynamic game

■ We are now ready to characterize the dynamic game. We explore an infinitely repeated version of the static game described above. The two firms achieve the most collusive (symmetric) outcome sustainable by the credible (subgame perfect) threat to punish defectors. In our working paper (Staiger and Wolak, 1990) we considered two forms of punishment. The first is a threat to revert forever to the static Nash equilibrium characterized above in the event that either firm defects. Although reverting to noncooperative behavior has the advantage of simplicity and intuitive appeal (as well as conformity with Rotemberg and Saloner (1986) and Green and Porter (1984), with which we compare our results), Abreu (1986, 1988) has shown that credible punishments are generally available that sustain a greater degree of collusion. Hence, in our working paper we also considered symmetric punishments that are optimal in the sense of Abreu (1986). But since our results are not affected by the form of punishment adopted, provided only that perfect collusion (the monopoly outcome) is not sustainable in all states of demand under either punishment, here we present only our results under Nash reversion.

In the collusive equilibrium, neither firm can have any incentive to defect unilaterally from the collusive agreement. Because both firms are completely symmetric, we characterize the "no defection" condition suppressing firm subscripts. A firm may defect from the collusive agreement either in its price choice or in its capacity choice. The latter defection cannot be conditioned on the state of demand (which is unknown at the time), while the former defection can be conditioned on the demand realization for the period. We consider each in turn.

□ **Collusive pricing behavior.** We begin by fixing a (symmetric) collusive capacity level K^c for each firm, defining $P^m(\alpha; K^c)$ as the fully collusive price given K^c (as a function of α) and treating the present discounted value of maintaining the most-collusive arrangement, ω, as a parameter. In analogy with (5),

$$P^m(\alpha; K^c) = \begin{cases} P(\alpha; K^c) & \text{for} \quad \alpha \in [4K^c, \bar{\alpha}] \\ \alpha/2 & \text{for} \quad \alpha \in [0, 4K^c]. \end{cases} \tag{6}$$

The price war region. Now consider a defection from the collusive price $P^m(\alpha; K^c)$. For $\alpha \in [4K^c, \bar{\alpha}]$, both firms sell their entire capacity at the market-clearing (monopoly) price, so neither firm has any incentive to alter its price from $P^m(\alpha; K^c)$. For $\alpha \in [0, 4K^c)$, however, neither firm is selling its entire capacity at the collusive price $P^m(\alpha; K^c)$, and thus either could generate a one-time profit gain by shaving its price below $P^m(\alpha; K^c)$ and selling $\min(K^c, D(\alpha; P^m(\alpha; K^c)))$. Finally, noting that $D(\alpha; P^m(\alpha; K^c)) = \alpha/2$ for $\alpha \in [0, 4K^c]$ allows the current payoff from defection, $\Gamma^m(\alpha; K^c)$, to be written as

$$\Gamma^m(\alpha; K^c) = \begin{cases} 0 & \text{for} \quad \alpha \in [4K^c, \bar{\alpha}] \\ P^m(\alpha; K^c) \cdot [K^c - {}^1\!/_2 D(\alpha; P^m(\alpha; K^c))] & \text{for} \quad \alpha \in [2K^c, 4K^c] \\ P^m(\alpha; K^c) \cdot {}^1\!/_2 D(\alpha; P^m(\alpha; K^c)) & \text{for} \quad \alpha \in [0, 2K^c], \end{cases} \tag{7}$$

which is continuous in $\alpha \in [0, \bar{\alpha}]$, increasing in α for $\alpha \in [0, 2K^c)$, and decreasing in α for $\alpha \in (2K^c, \bar{\alpha}]$. $P^m(\alpha; K^c)$ will be sustainable if and only if the incentive constraint is met:

$$\Gamma^m(\alpha; K^c) \leq \omega \qquad \text{for} \qquad \alpha \in [0, \bar{\alpha}]. \tag{8}$$

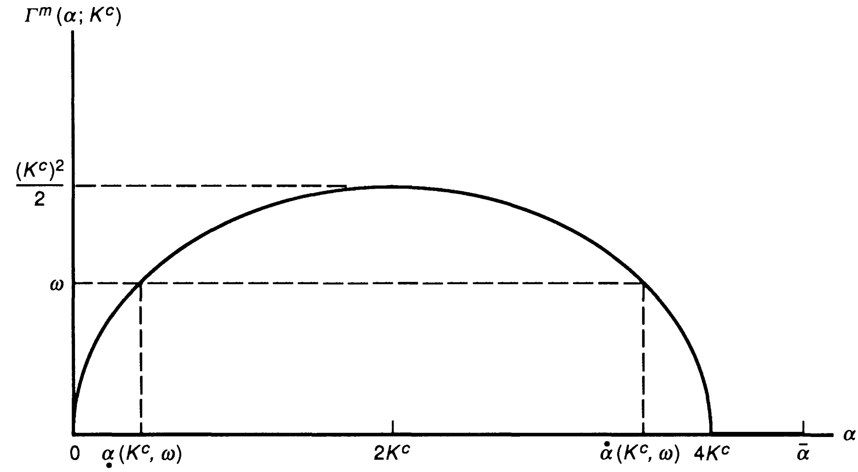
Figure 1 illustrates the range of α over which fully collusive prices are unsustainable, for given K^c and ω. As pictured, $\Gamma^m(\alpha; K^c)$ is a concave function of α over the range $\alpha \in [0, 4K^c]$, which is zero at $\alpha = 0$ and for $\alpha \geq 4K^c$, and which reaches its maximum value of $(K^c)^2/2$ at $\alpha = 2K^c$. For any $\omega \in [0, (K^c)^2/2)$, there exists an $\underset{.}{\alpha}(K^c, \omega) \in [0, 2K^c)$ such that $\Gamma^m(\underset{.}{\alpha}; K^c) = \omega$ and an $\dot{\alpha}(K^c, \omega) \in (2K^c, 4K^c]$ such that $\Gamma^m(\dot{\alpha}; K^c) = \omega$, with $\Gamma^m(\alpha; K^c) > \omega$ in the range $\alpha \in (\underset{.}{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$.

We denote as the "price war region" the range of demand states

$$\alpha \in (\underset{.}{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega)),$$

since the fully collusive price $P^m(\alpha; K^c)$ is unsustainable if and only if the realization of α falls in this range. To see how capacity constraints affect the relationship between the state

FIGURE 1



of demand and the likelihood of price wars, consider first $\alpha \in [0, 2K^c]$. Over this range of demand states, the perfectly collusive price is $\alpha/2$, leading to market sales of $\alpha/2$, which range from zero to K^c as α ranges from zero to $2K^c$. For $\alpha \in [0, 2K^c]$, then, capacity constraints are nonbinding whether a firm colludes and sells $\alpha/4$ or defects, captures the entire market, and sells $\alpha/2$. Thus, as in Rotemberg and Saloner (1986), where capacity constraints are absent, the incentive to defect from the perfectly collusive price is rising in α for $\alpha \in [0, 2K^c]$, preserving the price-wars-during-booms relationship in this range.

However, for $\alpha \in [2K^c, \bar{\alpha}]$, capacity constraints play a role; when they do, there is an inverse relationship between the state of demand and the likelihood of price wars. Over this range, stronger demand leads monotonically to a weaker incentive to defect from the perfectly collusive price (see (7)), so it is in periods of relatively soft demand that price wars break out. The reason is that, starting from $\alpha = 2K^c$, a strengthening of demand conditions (a rising α) will, at a fixed price, leave defection profits unaltered (capacity constraints already bind under a defection) but increase collusive profits through a rise in collusive capacity utilization. Thus, at fixed prices the improvement in demand conditions directly reduces the incentive to defect through the accompanying improvement in collusive capacity utilization. The increase in the collusive price that accompanies the rise in α reduces this direct effect, but it is not sufficient to reverse it.[9] Hence, for demand states characterized by $\alpha \in [2K^c, \bar{\alpha}]$, capacity constraints are relevant, and the relationship between the state of demand and the likelihood of price wars corresponds to conventional wisdom; collusion becomes increasingly difficult as the state of demand worsens.

The nature of price wars. While we have established that perfectly collusive prices are unsustainable in the region $\alpha \in (\underset{\bullet}{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$, we have not yet characterized the nature of the price wars that occur within the region. We now show that firms may choose

[9] The linear demand setup we have adopted here leads to falling demand elasticities at a given price with each rise in α, which is why the perfectly collusive price is rising in α. However, the rate at which collusive prices increase with α is not sufficient to offset the direct impact of a rising α on collusive demand, so that collusive capacity utilization is monotonically increasing in α. More generally, provided that improvements in the state of demand are not associated with too rapid a fall in demand elasticities, the perfectly collusive quantity will still rise with improvements in the state of demand, eventually causing capacity constraints to bind and driving the incentive to defect from the collusive price to zero. Thus our basic finding that, past some level of demand, further improvements in the state of demand lead through increasing collusive capacity utilization to a fall in the incentive to defect from the collusive price, is much more general than the linear demand setup we have used to illustrate it.

to adopt pure-strategy pricing over some ranges of demand and mixed-strategy pricing over others, so that the price wars that occur within this region may involve a uniform reduction in price below the monopoly level, or they may lead one firm to undercut another and steal market share.

For α in the price war region, sustainable price collusion requires that price be lowered below $P^m(\alpha; K^c)$ in order to reduce the current incentive to defect from the collusive price. Once prices below the perfectly collusive price are being considered, however, a new form of defection arises. In addition to shaving price and stealing market share from its competitor, a firm might also consider defecting to a higher price, allowing its competitor to sell at capacity, and setting a monopoly price for the residual market demand. We focus, for the moment, on prices that keep the former incentive to defect in check: then we consider the implications of the latter defection incentive on the nature of collusive pricing behavior.

It is readily shown that for prices below the perfectly collusive price, the incentive to shave price and steal market share is increasing in P. Thus, $P^c(\alpha; K^c, \omega)$, the most collusive price that keeps in check the incentive to shave price, will have the relevant incentive constraint just bind for each $\alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$. Defining $\check{\alpha}(K^c, \omega)$ as the value of α at which a single firm's collusive capacity is just sufficient to supply the entire market at the most collusive price, it follows that, for $\alpha > \check{\alpha}(K^c, \omega)$, a firm that defects by shaving its price below the collusive price will be capacity constrained. With this we can now implicitly define $P^c(\alpha; K^c, \omega)$ by equating the one-time payoff from shaving price with the discounted value of maintaining collusion:

$$P^c(\alpha; K^c, \omega) \cdot {}^1\!/_2 D(\alpha; P^c(\alpha; K^c, \omega)) = \omega \qquad \text{for} \qquad \alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \check{\alpha}(K^c, \omega)]$$

$$P^c(\alpha; K^c, \omega) \cdot [K^c - {}^1\!/_2 D(\alpha; P^c(\alpha; K^c, \omega))) = \omega \qquad \text{for} \qquad \alpha \in [\check{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega)).$$

Explicit calculation yields

$$P^c(\alpha; K^c, \omega) = \begin{cases} [\alpha - \sqrt{\alpha^2 - 8\omega}]/2 & \text{for} \quad \alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \check{\alpha}(K^c, \omega)] \\ [(\alpha - 2K^c) + \sqrt{(\alpha - 2K^c)^2 + 8\omega}]/2 & \text{for} \quad \alpha \in [\check{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega)). \end{cases}$$

However, we have not yet checked to see whether $P^c(\alpha; K^c, \omega)$ keeps in check the incentive to defect to a high price for all α in this range. In fact, for $\alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \check{\alpha}(K^c, \omega))$, defecting to a high price would leave one's rival with sufficient capacity to capture the entire market at $P^c(\alpha; K^c, \omega)$, and such defections would never be worthwhile. However, for $\alpha \in (\check{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$, the one-time gain to raising price, allowing one's competitor to sell at capacity, and setting a monopoly price for the residual market demand is given by $\gamma(\alpha; K^c, P^c) = \left(\frac{\alpha - K^c}{2}\right)^2 - P^c D(\alpha; P^c)/2$, which is the firm's residual market monopoly revenues less its share of total revenues under the collusive price P^c. It is easily checked that $\gamma(\alpha; K^c, P^c)$ is decreasing in P^c. Thus, if $\gamma(\alpha; K^c, P^c(\alpha; K^c, \omega)) > \omega$ for any $\alpha \in (\check{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$, then P^c would have to be raised above $P^c(\alpha; K^c, \omega)$ to keep firms from defecting to a higher price for such demand realizations. But $P^c(\alpha; K^c, \omega)$ is already the highest price that keeps in check the incentive to shave price, so for such demand realizations, there is no pure-strategy collusive price that simultaneously keeps in check the incentives to shave price and to raise price. For such demand realizations, colluding firms must adopt mixed strategies.

In fact, firms will choose mixed-strategy prices whenever $\gamma(\alpha; K^c, P^c(\alpha; K^c, \omega)) > 0$, even if $\gamma(\alpha; K^c, P^c(\alpha; K^c, \omega)) \leq \omega$ and pure-strategy prices can be supported. This is so because whenever $0 < \gamma(\alpha; K^c, P^c(\alpha; K^c, \omega)) \leq \omega$, the incentive to defect to a higher price can be used to raise the expected price under mixing up above the sustainable pure-strategy price.

We now characterize the range of demand realizations for a given ω over which mixing is preferred under the assumption that only noncooperative mixed strategies are feasible, and simply note that any ability to collude over mixed strategies will only widen this range.[10]

Under the assumption that firms are unable to collude in mixed strategies, mixed-strategy pricing will be undertaken in the range $\alpha \in (\underset{\circ}{\alpha}(K^c, \omega), \mathring{\alpha}(K^c, \omega))$, with the critical $\underset{\circ}{\alpha}(K^c, \omega)$ and $\mathring{\alpha}(K^c, \omega)$ defined implicitly by two of the four solutions to

$$^1/_2 P^c(\alpha; K^c, \omega) \cdot D(\alpha; P^c(\alpha; K^c, \omega)) = \tilde{R}(\alpha; K^c, K^c),$$

where $\tilde{R}(\alpha; K^c, K^c)$, the expected firm profit under mixing, is defined in Appendix A and is equal to the Stackelberg follower profits in the symmetric capacity case. Explicit calculation yields

$$\underset{\circ}{\alpha}(K^c, \omega) = \begin{cases} 2K^c - \sqrt{(K^c)^2 - 4(\omega + K^c\sqrt{\omega})} & \text{for} \quad \omega \in \left[0, \dfrac{3 - 2\sqrt{2}}{4}(K^c)^2\right] \\ 2K^c & \text{for} \quad \omega \geq \dfrac{3 - 2\sqrt{2}}{4}(K^c)^2 \end{cases}$$

$$\mathring{\alpha}(K^c, \omega) = \begin{cases} 2K^c + \sqrt{(K^c)^2 - 4(\omega + K^c\sqrt{\omega})} & \text{for} \quad \omega \in \left[0, \dfrac{3 - 2\sqrt{2}}{4}(K^c)^2\right] \\ 2K^c & \text{for} \quad \omega \geq \dfrac{3 - 2\sqrt{2}}{4}(K^c)^2. \end{cases}$$

Hence, for ω below $\dfrac{3 - 2\sqrt{2}}{4}(K^c)^2$, the price war region can be decomposed into two qualitatively different regions. For α in a range that lies at either extreme of the price war region—$\alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \underset{\circ}{\alpha}(K^c, \omega)] \cup [\mathring{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$—the nature of pricing behavior has firms choosing a common collusive price that lies below the joint monopoly price and earning profits that are less than the joint monopoly profit level as a result. Hence, as in Rotemberg and Saloner (1986), these price wars are in fact simply episodes of less effective collusion over a common price.

However, for α in a range that lies symmetrically around $\alpha = 2K^c$—$\alpha \in (\underset{\circ}{\alpha}(K^c, \omega), \mathring{\alpha}(K^c, \omega))$—the pricing behavior is qualitatively quite different. Specifically, since firms pursue mixed strategies in this region, there will be undercutting in equilibrium, with market share temporarily expanding for the low-price firm and contracting for the high-price firm. As such, these price wars differ from those of Rotemberg and Saloner (1986) and Green and Porter (1984). Moreover, unless firms can collude in mixed strategies, these episodic price wars will represent temporary reversions to noncooperative behavior triggered not by imperfect observability, as in Green and Porter, but rather by emerging excess capacity, and they are resolved naturally by higher rates of capacity utilization that come with the recovery of demand, not simply by the passage of a prespecified length of time, as in Green and Porter.

Finally, the nature of price wars that emerges from this analysis has a strong intuitive appeal. As long as demand is not too depressed ($\alpha \in [\mathring{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$), price wars take the relatively mild form of a uniform reduction in price below the joint monopoly level. But when demand conditions turn sufficiently sour ($\alpha \in (\underset{\circ}{\alpha}(K^c, \omega), \mathring{\alpha}(K^c, \omega))$), the nature of price wars changes, with firms undercutting each other in equilibrium. Finally, for a virtual collapse of demand ($\alpha \in [0, \underset{\circ}{\alpha}(K^c, \omega)]$) that leaves either firm with sufficient

[10] In general, collusion in mixed strategies is possible if the outcomes of firms' randomizing devices are jointly observable *ex post*. Collusion in mixed strategies is also possible when firms can only observe each others' past actions, provided that the collusive agreement concerns the support of the mixed strategies (see Fudenberg and Maskin, 1986).

capacity to supply the entire market, capacity constraints become completely irrelevant, and firms return to a uniform pricing policy.

It should be noted that throughout we have considered how the sustainable price compares to the fully collusive price as demand conditions vary. As suggested by a referee, one can also examine how price compares to marginal cost as demand conditions vary. Marginal costs are undefined over $\alpha \in [4K^c, \bar{\alpha}]$, because firms are capacity constrained over this range of demand. For $\alpha \in [0, 4K^c)$, however, marginal costs are well defined, and the behavior of the difference of price and marginal cost can be described. For $\alpha \in [0, \underset{\cdot}{\alpha}(K^c, \omega))$, the incentive to defect is sufficiently weak, given the weak state of demand, that the monopoly price can be sustained, and the difference of price and marginal cost rises as demand conditions improve in this range. Over the range $\alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \check{\alpha}(K^c, \omega))$, however, the behavior of the price–marginal cost difference is consistent with the notion of price wars during booms, and it implies that the amount by which price exceeds marginal cost falls as demand conditions improve. Finally, over the rest of the range of demand conditions for which marginal cost is well defined, the price–marginal cost difference can once again be expected to rise as demand conditions improve.[11]

The relationship between price wars and the ability to collude. We now have defined five critical values for α as a function of K^c and ω that define pricing regions in the collusive play. Using these results, Figures 2a through 2c depict the evolution of the various pricing regions as ω is increased from zero. Figure 2a corresponds to the case where $\omega = 0$. As depicted, $\underset{\cdot}{\alpha}(K^c, 0) = 0$; $\check{\alpha}(K^c, 0) = \underset{\circ}{\alpha}(K^c, 0) = K^c$; $\mathring{\alpha}(K^c, 0) = 3K^c$; and $\dot{\alpha}(K^c, 0) = 4K^c$. This corresponds to the static Nash equilibrium. Thus, firms price at cost for $\alpha \in [0, K^c]$, pursue mixed strategies for $\alpha \in (K^c, 3K^c)$, and sell all capacity at market-clearing prices for $\alpha \in [3K^c, \bar{\alpha}]$. For $\omega \in \left[0, \frac{3 - 2\sqrt{2}}{4}(K^c)^2\right]$, we have

$$0 \leq \underset{\cdot}{\alpha}(K^c, \omega) \leq \check{\alpha}(K^c, \omega) \leq \underset{\circ}{\alpha}(K^c, \omega) \leq 2K^c \leq \mathring{\alpha}(K^c, \omega) \leq \dot{\alpha}(K^c, \omega) \leq 4K^c.$$

Figure 2b depicts the pricing regions when $\omega = \frac{3 - 2\sqrt{2}}{4}(K^c)^2$. Here,

$$\underset{\circ}{\alpha}(K^c, \omega) = 2K^c = \mathring{\alpha}(K^c, \omega),$$

so that firms now choose to play pure-strategy prices over all $\alpha \in [0, \bar{\alpha}]$. Thus, firms sustain monopoly pricing for $\alpha \in [0, \underset{\cdot}{\alpha}(K^c, \omega)]$, collude as best they can for $\alpha \in (\underset{\cdot}{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega))$, sustain monopoly pricing for $\alpha \in [\dot{\alpha}(K^c, \omega), 4K^c]$, and set market-clearing prices for $\alpha \in [4K^c, \bar{\alpha}]$. Finally, as ω rises from $\frac{3 - 2\sqrt{2}}{4}(K^c)^2$ to $(K^c)^2/2$, $\dot{\alpha}(K^c, \omega)$ is falling continuously while $\underset{\cdot}{\alpha}(K^c, \omega)$ and $\check{\alpha}(K^c, \omega)$ continue to rise. Figure 2c depicts the pricing regions when $\omega = (K^c)^2/2$. Here,

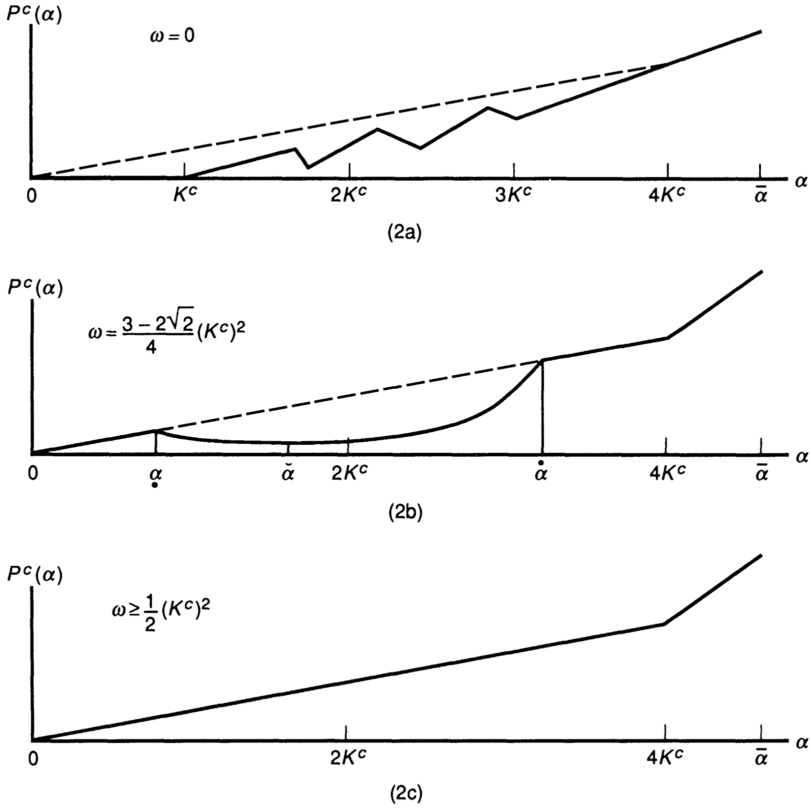
$$\underset{\cdot}{\alpha}(K^c, \omega) = \check{\alpha}(K^c, \omega) = 2K^c = \dot{\alpha}(K^c, \omega),$$

so that firms now sustain the monopoly-pricing outcome. Thus, monopoly prices are sustained over the range $\alpha \in [0, 4K^c]$, with market-clearing prices charged for $\alpha \in [4K^c, \bar{\alpha}]$.

Observe that when collusion is completely ineffective ($\omega = 0$), price wars that lead to equilibrium undercutting and market-share volatility will occur over a broad range of demand states ($\alpha \in (K^c, 3K^c)$). In general, the more effectively firms are able to collude overall (the higher is ω), the narrower will be the range of demand realizations giving rise to price wars of this nature; in particular, for $\omega \geq \frac{3 - 2\sqrt{2}}{4}(K^c)^2$, the occurrence of such price wars

[11] The relationship between price–marginal cost differences and demand conditions holds only in an expected sense over this range, due to the possibility of mixed-strategy pricing discussed above.

FIGURE 2



disappears. These results provide theoretical justification for the belief that frequent episodes of price cutting and market-share volatility are characteristic of an industry that is relatively ineffective in maintaining collusion.[12]

While conforming to common sense, this result does not in fact emerge from other attempts to model price wars, where undercutting is never an equilibrium phenomenon. It arises in the present model only because changing demand conditions interact with capacity constraints in a way that can move collusive firms naturally into mixed-strategy regions, with undercutting and "business stealing" occurring in equilibrium as a result.

We summarize the discussion thus far by defining the most-collusive revenue function as

$$R^c(\alpha; K^c, \omega) = \begin{cases} P(\alpha; 2K^c)K^c & \text{for } \alpha \in [4K^c, \bar{\alpha}] \\ {}^1\!/_2 P^m(\alpha) \cdot D(\alpha; P^m(\alpha)) & \text{for } \alpha \in [\dot{\alpha}(K^c, \omega), 4K^c] \\ {}^1\!/_2 P^c(\alpha; K^c, \omega) \cdot D(\alpha; P^c(\alpha; K^c, \omega)) & \text{for } \alpha \in [\mathring{\alpha}(K^c, \omega), \dot{\alpha}(K^c, \omega)] \\ \tilde{R}(\alpha; K^c) & \text{for } \alpha \in [\underset{\circ}{\alpha}(K^c, \omega), \mathring{\alpha}(K^c, \omega)] \\ {}^1\!/_2 P^c(\alpha; K^c, \omega) \cdot D(\alpha; P^c(\alpha; K^c, \omega)) & \text{for } \alpha \in [\check{\alpha}(K^c, \omega), \underset{\circ}{\alpha}(K^c, \omega)] \\ {}^1\!/_2 P^c(\alpha; K^c, \omega) \cdot D(\alpha; P^c(\alpha; K^c, \omega)) & \text{for } \alpha \in [\underset{\cdot}{\alpha}(K^c, \omega), \check{\alpha}(K^c, \omega)] \\ {}^1\!/_2 P^m(\alpha) \cdot D(\alpha; P^m(\alpha)) & \text{for } \alpha \in [0, \underset{\cdot}{\alpha}(K^c, \omega)], \end{cases}$$

[12] Note that we are not claiming that undercutting and market-share instability necessarily correspond to episodes of noncooperative behavior, but rather that the frequency of such episodes is directly related to the overall success with which firms collude (as measured by the magnitude of ω).

and we note that $R^c(\alpha; K^c, \omega)$ is continuous in α, K^c, and ω. Most-collusive expected profits are then given by

$$E\pi^c(K^c, \omega) = \int_0^{\bar{\alpha}} R^c(\alpha; K^c, \omega)dF(\alpha) - rK^c. \tag{9}$$

□ **The collusive capacity choice.** Having defined most-collusive profits for any K^c as a function of ω in (9), we next describe the most-collusive capacity choice $\hat{K}^c$ as a function of ω, which for now we still take as a parameter. (See Staiger and Wolak (1990) for a formal derivation of $\hat{K}^c$.) We first describe the optimal collusive capacity choice that maximizes $E\pi^c(K^c, \omega)$, ignoring for the moment whether it can in fact be sustained. The first-order condition of (9) is

$$E\pi^c_{K^c}(K^c, \omega) = \int_0^{\bar{\alpha}} R^c_K(\alpha; K^c, \omega)dF(\alpha) - r = 0. \tag{10}$$

Provided that capacity costs (r) are sufficiently high, second-order conditions will hold at $K^c(\omega)$, the solution to (10), for $\omega \in [0, \infty]$.[13]

However, collusive capacity $K^c(\omega)$ will be sustainable if and only if $\Omega(K^c(\omega), \omega)$, the current incentive to defect from K^c, satisfies

$$\Omega(K^c(\omega), \omega) \equiv E\pi^d(K^d(K^c(\omega)), K^c(\omega)) - E\pi^c(K^c(\omega), \omega) \leq \omega, \tag{11}$$

where $K^d(K^c(\omega))$ is the optimal defection capacity, $E\pi^d(K^d(K^c(\omega)), K^c(\omega))$ is current defection profits of a defecting firm, and it is assumed that firms play noncooperative Nash prices immediately following a capacity defection. When (11) holds, the (sustainable) collusive capacity choice is defined implicitly by (10). When (11) is violated under the optimal choice, then capacity must be increased beyond the optimal capacity choice. The most-collusive choice $\bar{K}^c(\omega)$ will then have (11) bind, and it is defined implicitly by

$$\Omega(\bar{K}^c, \omega) \equiv E\pi^d(K^d(\bar{K}^c), \bar{K}^c) - E\pi^c(\bar{K}^c, \omega) = \omega. \tag{12}$$

It can be shown that there exists a unique $\bar{\omega} \in (0, (K^m)^2/8)$ such that the optimal capacity $K^c(\omega)$ is sustainable for $\omega \in [\bar{\omega}, \infty]$ and unsustainable for $\omega \in [0, \bar{\omega})$. We can now write the most-collusive capacity choice as a function of ω as

$$\hat{K}^c(\omega) = \begin{cases} \bar{K}^c(\omega) & \text{for} \quad \omega \in [0, \bar{\omega}) \\ K^c(\omega) & \text{for} \quad \omega \in [\bar{\omega}, \infty]. \end{cases} \tag{13}$$

By substituting (13) into (9), we now have most-collusive expected profits as a function of ω:

$$E\pi^c(\hat{K}^c(\omega), \omega) = \int_0^{\bar{\alpha}} R^c(\alpha; \hat{K}^c(\omega), \omega)dF(\alpha) - r\hat{K}^c(\omega). \tag{14}$$

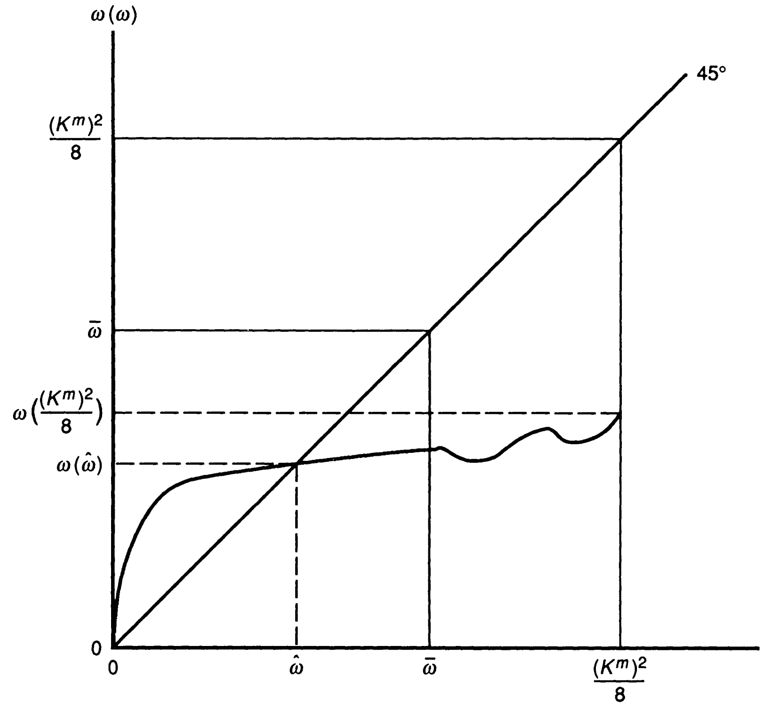
□ **Completing the characterization of equilibrium.** While (14) gives most-collusive profits as a function of ω, the present discounted gain from maintaining the collusive arrangement into the infinite future, ω is in fact itself a function of the most-collusive profits, and it is defined in the case of infinite Nash reversion by

$$\omega(\omega) \equiv \frac{\delta}{1-\delta}[E\pi^c(\hat{K}^c(\omega), \omega) - E\pi^n], \tag{15}$$

where $\delta \in (0, 1)$ is the discount factor. Thus, the last step is to solve for a fixed point ω to (15) such that $\omega(\hat{\omega}) = \hat{\omega}$. This is contained in Appendix B. Since our primary focus is on

[13] Intuitively, this restriction ensures that the optimum capacity choice will bind with sufficient frequency to make $E\pi^c(K^c, \omega)$ concave in K^c.

FIGURE 3



characterizing the behavior of imperfectly colluding firms, we simply establish in Appendix B that for δ sufficiently small, a unique positive fixed point $\hat{\omega}$ exists with $\hat{\omega} \in (0, \bar{\omega})$. For $\hat{\omega}$ in this range, firms imperfectly collude in both capacity and price.

Figure 3 illustrates the determination of $\hat{\omega} \in (0, \bar{\omega})$. For $\omega \in [0, \bar{\omega}]$, Appendix B establishes that $\omega(\omega)$ rises with infinite slope out of the origin and is strictly concave. The value of $\omega(\omega)$ is highest at $(K^m)^2/8$, where perfectly collusive price and capacity choices are supported; provided δ is sufficiently small, $\omega((K^m)^2/8) < \bar{\omega}$. As depicted, the unique positive fixed point under Nash reversion is $\hat{\omega} \in (0, \bar{\omega})$, with the relationship between demand conditions (α) and collusive pricing as described above.[14]

4. Adding to capacity

■ Thus far we have ruled out the possibility of adding to capacity once the state of demand for the period has been observed. While modelling capacity constraints as an absolute has served as a useful benchmark, few industries are likely to face such inflexible constraints.[15] Rather, a more plausible scenario would allow firms to make additions to capacity after observing demand, but at a higher per-unit cost than capacity that is installed well in advance of sales.[16] In this section, we extend the model to capture this scenario and show that the

[14] Additional conditions must be imposed to ensure the concavity of $\omega(\omega)$ over the range $\omega \in (\bar{\omega}, (K^m)^2/8]$, since over this range capacity is being chosen optimally without a binding incentive constraint. For our focus, however, concavity of $\omega(\omega)$ over this range is not required.

[15] However, as discussed in footnote 1, Suslow (1986) and Bresnahan and Suslow (1989, 1990) provide evidence that the primary aluminum and ammonia fertilizer industries might face such capacity constraints. In addition, there are instances in which firms have sold out of a product line for an entire model year; e.g., the RISC System/6000, IBM's initial entry into the workstation market, sold out during its first year of production.

[16] Rotemberg and Saloner (1989b) allow costly additions to capacity after demand is revealed in a model of strategic inventories, and they note that this additional freedom makes collusion more difficult to sustain in periods of strong demand.

character of collusive pricing behavior approaches that depicted by Rotemberg and Saloner (1986) as the importance of capacity constraints declines.

To this end, we suppose that capacity can be added after observing the state of demand α (but before setting price) at a per-unit cost of $R \geq r$. Note that prior commitments to capacity made in advance of the observation of demand cannot be undone once the state of demand is observed. Thus, it is only the expansion of capacity that is feasible (at a per-unit cost R) at this later stage, not a capacity-cost-saving reduction.

Consider first the monopolist's behavior given this additional flexibility. Calculations similar to those above establish that the initial monopoly capacity choice $K^m(R)$ is given by the maximum of zero and the solution to

$$K^m = \frac{\int_{2K^m}^{2K^m+R} \alpha dF(\alpha) - [r - [1 - F(2K^m + R)]R]}{2[F(2K^m + R) - F(2K^m)]}, \tag{16}$$

while additions to monopoly capacity are given by

$$Q^m(\alpha; R) = \frac{\alpha - R}{2} - K^m. \tag{17}$$

In the collusive ideal, waiting to build capacity until demand is revealed in an effort to avoid excessive capacity installation carries with it the risk of eventually having to add capacity when the costs of doing so are higher. Clearly, for R above a critical level $\bar{R}$, the ability to add capacity after observing demand will not be exercised even for the strongest of demand realizations $\bar{\alpha}$, and $K^m(R)$ in (16) will coincide with K^m defined by (4). Calculations yield $\bar{R} = \bar{\alpha} - 2K^m(\bar{R})$. On the other hand, for R below a critical level $\underline{R}$, all capacity will be built after observing demand. Calculations yield

$$\underline{R} = \left[r - \int_0^{\underline{R}} \alpha dF(\alpha)\right] \Big/ [1 - F(\underline{R})],$$

with $\underline{R} > r$. Finally, we define $\Delta \equiv R - r$, $\bar{\Delta} \equiv \bar{R} - r$, and $\underline{\Delta} \equiv \underline{R} - r$. The parameter Δ can be used to capture the importance of capacity constraints, with higher Δ corresponding to the increased importance of capacity constraints. That is, a "flexible" supply response to changing demand conditions becomes increasingly costly as Δ rises.

It is now straightforward to establish that with capacity constraints sufficiently unimportant—for $\Delta \leq \underline{\Delta}$—all capacity in the most-collusive equilibrium is installed after observing demand. In this case, the repeated capacity-constrained price game collapses to a repeated Cournot game, and the results of Rotemberg and Saloner (1986) apply directly; with linear demand and constant marginal costs, firms engage in price wars during booms, a characterization that applies over all α. Alternatively, when capacity constraints are sufficiently important—for $\Delta \geq \bar{\Delta}$—all capacity in the most-collusive equilibrium is installed prior to the realization of demand. In this case, the repeated capacity-constrained price game is exactly as modelled in the previous sections. Finally, for $\underline{\Delta} < \Delta < \bar{\Delta}$, it can be shown that the pricing behavior described in the previous sections is preserved over the range $\alpha \in [0, 3K^c + R]$, while pricing for $\alpha \in [3K^c + R, \bar{\alpha}]$ corresponds to the repeated Cournot game studied by Rotemberg and Saloner and thus exhibits price wars during booms over this range.

So as Δ increases from $\underline{\Delta}$, firms become increasingly constrained by their initial capacity decisions, and most-collusive pricing behavior looks increasingly like that characterized in the previous sections. In this sense, we have now established that the conventional view—that periods of low demand lead to the breakdown of collusive pricing—applies best in precisely those industries where incremental additions to capacity are most costly and capacity constraints are most important.

5. Conclusion

■ We have presented a model that attempts to capture the role of capacity constraints in determining the relationship between demand conditions and the ability of firms to maintain tacit collusion. Our findings lend support to the conventional view of the relationship between the ability to maintain collusion and demand conditions. In particular, the presence of capacity constraints sets up a range of demand conditions over which the ability to maintain collusive prices weakens as business conditions turn sour. We have shown that the conventional wisdom is most relevant in precisely those industries where capacity costs are high and capacity constraints most important. Thus, the importance of capacity constraints is a crucial determinant of the relationship between collusive prices and business conditions across industries.

Moreover, we find that when capacity constraints are present, the nature of equilibrium price wars can be affected in interesting ways. Specifically, emerging excess capacity associated with unexpectedly low demand can turn mild price wars, consisting of a uniform reduction in price below the joint monopoly level and the maintenance of stable market shares, into episodes of price undercutting and market-share instability.

Finally, we find that this latter form of pricing behavior arises only if firms are relatively ineffective in their attempts to collude; that is, if in equilibrium the discounted gains from maintaining collusion are low. Hence, in this sense, our results suggest that intertemporal instability in market shares is a signal of relatively unsuccessful collusion.

Appendix A

■ In this appendix, we characterize the unique symmetric Nash equilibrium to the static game of Section 2. Following Kreps and Scheinkman (1983), we begin by defining $q_1(\alpha; q_2)$ and $q_2(\alpha; q_1)$ as the Cournot duopoly best (output) response functions for firms 1 and 2, respectively, facing the linear demand function given in (1) and zero marginal costs. It is straightforward to show that these best response functions are given by $q_1(\alpha; q_2) = (\alpha - q_2)/2$ and $q_2(\alpha; q_1) = (\alpha - q_1)/2$. For any q_1 and q_2, we next define $\underset{.}{\alpha}_1$ and $\underset{.}{\alpha}_2$ implicitly by $q_1 = q_1(\underset{.}{\alpha}_1; q_2)$ and $q_2 = q_2(\underset{.}{\alpha}_2; q_1)$, which yield explicit expressions for $\underset{.}{\alpha}_1(q_1, q_2)$ and $\underset{.}{\alpha}_2(q_1, q_2)$ of $\underset{.}{\alpha}_1(q_1, q_2) = 2q_1 + q_2$ and $\underset{.}{\alpha}_2(q_1, q_2) = 2q_2 + q_1$. For any q_1 and q_2, $\underset{.}{\alpha}_1(\cdot)$ gives the value of α above which $q_1 < q_1(\alpha; q_2)$ and below which $q_1 > q_1(\alpha; q_2)$, and similarly for $\underset{.}{\alpha}_2(\cdot)$. With this, we now define $\hat{\alpha}(q_1, q_2)$ by $\hat{\alpha}(q_1, q_2) \equiv \max(\underset{.}{\alpha}_1(q_1, q_2), \underset{.}{\alpha}_2(q_1, q_2))$, or

$$\hat{\alpha}(q_1, q_2) = \begin{cases} 2q_1 + q_2 & \text{if} \quad q_1 \geq q_2 \\ 2q_2 + q_1 & \text{if} \quad q_1 < q_2. \end{cases} \tag{A1}$$

Finally, we define

$$\tilde{\alpha}(q_1, q_2) \equiv \min(q_1, q_2). \tag{A2}$$

The critical values of $\alpha(q_1, q_2)$ defined in (A1) and (A2) allow us to employ the results of Kreps and Scheinkman (1983) and characterize Nash equilibrium profits for each firm in the final (price-setting) stage of the game for given capacity choices. In particular, fix K_1 and K_2 at any value such that

$$\hat{\alpha}(K_1, K_2) \in (0, \bar{\alpha}), \tag{A3}$$

and consider the price-setting subgame that ensues. If the realization of α falls in the range $\alpha \in [\hat{\alpha}(K_1, K_2), \bar{\alpha}]$, then we have $K_1 \leq q_1(\alpha; K_2)$ and $K_2 \leq q_2(\alpha; K_1)$. Appealing to Proposition I(a) of Kreps and Scheinkman (1983), we then have that realizations of $\alpha \in [\hat{\alpha}(K_1, K_2), \bar{\alpha}]$ result in revenues for the two firms of $R_1(\alpha; K_1, K_2) = K_1 \cdot P(\alpha; K_1 + K_2)$ and $R_2(\alpha; K_1, K_2) = K_2 \cdot P(\alpha; K_1 + K_2)$. That is, each firm sells its entire capacity at the market-clearing price. Alternatively, if the realization of α falls in the range $\alpha \in (\tilde{\alpha}(K_1, K_2), \hat{\alpha}(K_1, K_2)]$, then we have $K_1 > q_1(\alpha; K_2)$ and/or $K_2 > q_2(\alpha; K_1)$. Appealing to Proposition I(b) and I(c) of Kreps and Scheinkman (1983), we then have that firms play mixed strategies in the final (price-setting) stage of the game, but that revenues for each firm are given by the functions $\tilde{R}_1(\alpha; K_1, K_2)$ and $\tilde{R}_2(\alpha; K_1, K_2)$, where these functions are continuous in K_1 and K_2 and satisfy

$$\begin{aligned} \tilde{R}_1(\hat{\alpha}(K_1, K_2); K_1, K_2) &= R_1(\hat{\alpha}(K_1, K_2); K_1, K_2) \\ \tilde{R}_2(\hat{\alpha}(K_1, K_2); K_1, K_2) &= R_2(\hat{\alpha}(K_1, K_2); K_1, K_2) \end{aligned} \tag{A4}$$

$$\tilde{R}_1(\tilde{\alpha}(K_1, K_2); K_1, K_2) = 0; \qquad \tilde{R}_2(\tilde{\alpha}(K_1, K_2); K_1, K_2) = 0 \tag{A5}$$

$$\tilde{R}_1(\alpha; K_1 > K_2, K_2) = \tilde{R}_1(\alpha; K_1 = K_2, K_2) = P(\alpha; q_1(\alpha; K_2), K_2) \cdot q_1(\alpha; K_2)$$

$$\tilde{R}_2(\alpha; K_1, K_2 > K_1) = \tilde{R}_2(\alpha; K_1, K_2 = K_1) = P(\alpha; K_1, q_2(\alpha; K_1)) \cdot q_2(\alpha; K_1). \tag{A6}$$

Finally, for realizations of α in the range $\alpha \in [0, \tilde{\alpha}(K_1, K_2)]$, we have that $K_1 \geq D(\alpha; P \geq 0)$ and $K_2 \geq D(\alpha; P \geq 0)$. Thus, either firm can satisfy the entire market for any nonnegative price. In this range, capacity constraints do not upset the standard Bertrand equilibrium in which price equals marginal cost (which is zero), so that revenues for each firm will be zero.

We are now in a position to write down expected profits for each firm as a function of K_1 and K_2 in the range of K_1 and K_2 satisfying (A3):

$$E\pi_1(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_1(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_1(\alpha; K_1, K_2)dF(\alpha) - rK_1 \tag{A7}$$

$$E\pi_2(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_2(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_2(\alpha; K_1, K_2)dF(\alpha) - rK_2. \tag{A8}$$

Assuming for the moment that all relevant capacity choices satisfy (A3), and using (A4) and (A5), the first- and second-order conditions of (A7) and (A8) are

$$E\pi_{1K1}(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_{1K1}(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_{1K1}(\alpha; K_1, K_2)dF(\alpha) - r = 0 \tag{A9}$$

$$E\pi_{1K1K1}(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_{1K1K1}(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_{1K1K1}(\alpha; K_1, K_2)dF(\alpha) < 0$$

$$E\pi_{2K2}(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_{2K2}(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_{2K2}(\alpha; K_1, K_2)dF(\alpha) - r = 0 \tag{A10}$$

$$E\pi_{2K2K2}(K_1, K_2) = \int_{\hat{\alpha}(K_1,K_2)}^{\bar{\alpha}} R_{2K2K2}(\alpha; K_1, K_2)dF(\alpha) + \int_{\tilde{\alpha}(K_1,K_2)}^{\hat{\alpha}(K_1,K_2)} \tilde{R}_{2K2K2}(\alpha; K_1, K_2)dF(\alpha) < 0,$$

where subscripts denote partial derivatives.

Provided that second-order conditions hold at the optimum, expressions (A9) and (A10) implicitly define the best capacity response functions $K_1(K_2)$ and $K_2(K_1)$. The symmetric static Nash equilibrium capacity choice K^n will satisfy $K_1(K^n) = K^n = K_2(K^n)$. However, from (A6), $\tilde{R}_{1K1}(\alpha; K_1, K_2) = \tilde{R}_{1K1K1}(\alpha; K_1, K_2) = 0$ for any $K_1 = K_2$. With this, and using (A9) and (A10), we have

$$K^n = \frac{E[\alpha] - G(3K^n) - r}{3[1 - F(3K^n)]}, \tag{A11}$$

where $E[\alpha]$ is the expected value of α and $G(\alpha) \equiv \int_0^\alpha s dF(s)$. Note also that second-order conditions are met: $E\pi_{1K1K1}(K_1 = K^n, K_2 = K^n) = E\pi_{2K2K2}(K_1 = K^n, K_2 = K^n) = -2[1 - F(3K^n)] < 0$.

Finally, recall that to derive (A11) we have restricted the set of capacity choices to those satisfying (A3). We must now establish conditions under which this restricted set of capacity choices is indeed the relevant set from which firms will make their Nash equilibrium choices. It is straightforward to establish by contradiction that combinations of K_1 and K_2 that lead to $\hat{\alpha}(K_1, K_2) = \bar{\alpha}$ imply that at least one firm must be off its best capacity response function, and can thus be ruled out. For example, suppose that $\hat{\alpha}(\tilde{K}_1, \tilde{K}_2) = \bar{\alpha}$ for some $\tilde{K}_1$ and $\tilde{K}_2$. Then we must have $\tilde{K}_1 > q(\alpha; \tilde{K}_2)\ \forall \alpha \in [0, \bar{\alpha}]$ and/or $\tilde{K}_2 > q_2(\alpha; \tilde{K}_1)\ \forall \alpha \in [0, \bar{\alpha}]$, which violates optimality provided that $r > 0$. To rule out $\hat{\alpha}(K_1, K_2) = 0$, we simply note that this would imply $K_1 = K_2 = 0$. However, under these capacity choices, both firms must be off their best capacity response functions provided only that

$$E[\alpha] - r > 0. \tag{A12}$$

The left-hand side of condition (A12) is expected market demand at a price (r) that reflects long-run production costs. Thus, according to (A12), combinations of K_1 and K_2 that imply $\hat{\alpha}(K_1, K_2) = 0$, i.e., $K_1 = K_2 = 0$, can be ruled out provided that expected market demand is strictly positive at a price reflecting long-run costs. We assume that (A12) is met. Using (A7), (A8), and (A11), we define $E\pi^n \equiv E\pi(K^n, K^n)$.

Appendix B

■ In this appendix, we solve for a fixed point ω to (15) such that $\omega(\hat{\omega}) = \hat{\omega}$. We note first that $\omega(0) = 0$, so that one fixed point involves continual play of the static Nash capacity and prices. This can be verified by noting that for $\omega \in [0, \bar{\omega})$, (12) must hold, so substitution into (15) and simplifying yields

$$\omega(\omega) = \delta[E\pi^d(K^d(\bar{K}^c(\omega)), \bar{K}^c(\omega)) - E\pi^n] \qquad \text{for} \qquad \omega \in [0, \bar{\omega}). \tag{B1}$$

But at $\omega = 0$, collusive pricing is identical to noncooperative pricing for a given capacity choice, so that (12) implies $\bar{K}^c(\omega = 0) = K^n$. With this, (B1) implies $\omega(\omega = 0) = 0$.

We turn next to the existence of a positive fixed point $\hat{\omega} > 0$. Since our primary focus is on characterizing the behavior of imperfectly colluding firms, we simply establish that for δ sufficiently small, a unique positive fixed point $\hat{\omega}$ exists with $\hat{\omega} \in (0, \bar{\omega})$, that is, with firms imperfectly colluding in capacity and price.

To establish the existence of a collusive fixed point $\hat{\omega} \in (0, \bar{\omega})$, we note, using (B1) and the envelope theorem, that

$$\omega_\omega(\omega) = \delta E\pi^d_{\bar{K}^c}(K^d, \bar{K}^c(\omega))d\bar{K}^c/d\omega \qquad \text{for} \qquad \omega \in [0, \bar{\omega}). \tag{B2}$$

It can be shown that $E\pi^d_{\bar{K}^c}(\cdot) < 0$. Moreover, totally differentiating (12), using the envelope theorem, and solving for $d\bar{K}^c/d\omega$ yields

$$d\bar{K}^c/d\omega = \frac{1 + E\pi^c_\omega(\bar{K}^c, \omega)}{[E\pi^d_{\bar{K}^c}(K^d, \bar{K}^c) - E\pi^c_{\bar{K}^c}(\bar{K}^c, \omega)]}. \tag{B3}$$

It can be verified that the denominator of (B3) is negative for $\omega \in (0, \bar{\omega})$ and zero at $\omega = 0$, while the numerator is strictly positive for $\omega \in [0, \bar{\omega})$. Thus, $d\bar{K}^c/d\omega = -\infty$ at $\omega = 0$, implying through (B2) that $\omega_\omega(\omega = 0) = \infty$. Finally, differentiating (B2) with respect to ω establishes that $\omega_{\omega\omega}(\omega) < 0$ for $\omega \in [0, \bar{\omega})$ provided that, as noted in Section 3, capacity costs are sufficiently high.

Note that $\omega(\omega)$ reaches its maximum value when evaluated at $(K^m)^2/8$, the value of ω that supports full collusion in price and capacity. Since $\omega = (K^m)^2/8$ supports the joint monopoly outcome, this maximum value is

$$\omega((K^m)^2/8) = \frac{\delta}{1-\delta}[E\pi^m/2 - E\pi^n]. \tag{B4}$$

The properties of $\omega(\omega)$ for $\omega \in [0, \bar{\omega}]$ established above assure that a unique positive fixed point $\hat{\omega}$ exists, with $\hat{\omega} \in (0, \bar{\omega})$, provided that $\omega((K^m)^2/8) < \bar{\omega}$. Using (B4), this condition reduces to

$$\frac{\delta}{1-\delta} < \bar{\omega}/[E\pi^m/2 - E\pi^n]. \tag{B5}$$

Since the right-hand side of (B5) is independent of δ, (B5) amounts to a condition that δ is sufficiently small.

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