SPECIES DEPENDENCY OF THE COMPOSITIONAL INDIRECT NOISE MECHANISM

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ABSTRACT
Indirect combustion noise is a growing concern for aviation engine designers. It is typically associated with the distortion of “hot spots” (entropy structures) and vortical structures that generate excess noise as they are convected by an accelerating mean flow. Recently, it has been shown theoretically that fluctuations in a gas’s chemical composition can act as an additional source of indirect noise in aviation engines. This work examines this composition noise mechanism, focusing on the underlying chemical effects that drive this source of indirect noise. Since the mechanism has yet to be confirmed experimentally, this paper begins by applying the theory to inert mixtures of noble gases and air, in an attempt to guide experimentalists by identifying the set of operating conditions that will produce the strongest acoustic response. Turning from non-reacting to reacting flows, the paper next examines the sensitivity of the compositional noise mechanism to fuel type, testing several common fuels. It is found that, while there is a substantial difference between hydrogen and hydrocarbon fuels, overall noise levels vary only slightly between different hydrocarbon fuels. Additionally, there appears to be a common underlying structure to the response of a product-gas mixture generated by burning a fuel, which is explained through linearized theory and confirmed with numerical results. Lastly, the physics of composition noise is examined at the species-specific level, attempting to provide a link between individual combustion products and changes in a mixture’s propensity to generate indirect noise. The sensitivity of individual species can be explained by a combination of differences between the species and mixture’s Gibbs free energy and strong gradients in product gas concentration with mixture fraction. However, by analyzing the species dependency of combustion products at several different mean mixture fractions, it is found that no single species dominates the noise generation over the combustor’s entire range, but rather the most acoustically active species varies strongly with local stoichiometry.

NOMENCLATURE

Roman

Da Dahmköhler number
He Helmholtz number
L Nozzle length
M Mach number
Ns Number of species
R Universal gas constant
T Temperature
W Molecular weight
Yi Mass fraction of species i
Z Mixture fraction
Z* Rescaled mixture fraction
a Speed of sound
cp Isobaric specific heat
cv Isochoric specific heat
g Specific Gibbs free energy
ṁ Mass flow rate
p Pressure
s Entropy
u Velocity

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Greek

\( \Psi \) Normalized chemical potential
\( \gamma \) Ratio of specific heats, \( c_p/c_v \)
\( \mu \) Chemical potential
\( \xi \) Composition wave
\( \pi \) Pressure wave
\( \sigma \) Entropy wave
\( \tau \) Timescale
\( \chi \) Scalar dissipation rate

Subscripts

+ Downstream propagating wave
- Upstream propagating wave
\( a \) Gas state at nozzle inlet
\( b \) Gas state at nozzle exit
\( st \) Stoichiometric

INTRODUCTION

As engineering efforts have successfully reduced the intensity of fan noise and jet noise emitted by aviation engines, core noise, the noise attributed to acoustic sources inside the engine core, has emerged as a growing concern for engine designers. Traditionally, core noise is divided into two categories: direct noise, or acoustic waves emitted by unsteady combustion, and indirect noise, the noise generated by the convection and distortion of turbulent and thermal structures passing from the combustor to the engine exhaust. At operating conditions relevant to gas turbines, it has been shown that indirect noise is likely the dominant contributor to engine core noise [1], and much work has been performed to study this effect. Marble and Candel [2] were the first to identify entropy fluctuations as a source of indirect core noise, and formulated a theory that predicts acoustic emissions associated with fluctuations in entropy. Howe [3] and others have shown that variations in vorticity can also produce indirect noise. Experimentally, there have been considerable efforts to study these indirect noise mechanisms, including dedicated rigs such as the entropy and vorticity wave generators [4,5], as well as more realistic yet complex geometries, such as model gas-turbine burners [4, 6–9]. However, by extending the linear theory of [2], it was demonstrated in [10] that there is a third source of indirect noise, that arises from variations in mixture composition, that, at appropriate operating conditions, can produce an acoustic response of similar order or larger than those described in the traditional core noise literature.

This paper seeks to expand on the analysis of the compositional noise mechanism by more thoroughly investigating the link between chemical composition and indirect noise. The previous work [11] relied on the mixture fraction to parameterize chemical composition and established a link between variations in the mixture fraction and indirect noise. However, these studies were limited to a single fuel (n-dodecane) and did not examine in detail the underlying impact of individual species on the amount of noise produced. This paper provides a detailed analysis of how variations of combustion-product gas composition affect the amount of indirect noise produced.

The next section reviews the theory of how indirect composition noise arises based on a general parameterization of the mixture composition. The following section considers simple inert mixtures of common laboratory gases to identify parameter regimes where the composition noise mechanism can be experimentally verified. The fourth section examines more complex reacting mixtures, comparing the compositional noise generated by the combustion of several common fuels to determine the sensitivity of the compositional noise mechanism to fuel type. The penultimate section provides a detailed examination of the factors that enhance a mixture’s acoustic sensitivity, examining the effect of concentrations of individual species on the noise produced to identify the most acoustically efficient species in common combustion products.

THEORY AND TRANSFER FUNCTIONS

We begin by restating some key results from composition noise theory [10, 11]. The formulation describes an idealized, quasi one-dimensional flow of a mixture of \( N_s \) ideal gases passing through a converging-diverging nozzle. For convenience, the conserved convected scalar \( Z \), or mixture fraction, is introduced to parameterize the chemical composition of the gas such that \( Y_i = f(Z) \) for all \( i = 1, \ldots, N_s \) species. The theory that follows, however, is not dependent on this choice, and \( Z \) could be replaced with any other conserved scalar or vector quantity that describes the mixture composition, including the species mass fractions themselves. It is assumed that the mixture composition is chemically frozen (\( Du = \tau_{\text{con}}/\tau_{\text{chem}} \leq 1 \)). The discussion here is limited to low frequency disturbances such that the acoustic wavelength is much greater than the nozzle length (i.e. \( He = \omega L/\alpha \ll 1 \)), although this assumption is relaxed in [12].

With these assumptions in hand, a theory governing the growth of disturbances passing through the nozzle can be obtained through the linearization of conservation equations across the nozzle. Specifically, the conservation of mass, enthalpy, entropy, and the conserved scalar \( Z \), evaluated at the nozzle entrance state \( a \) and exit state \( b \), combined with linearizations of the definitions of enthalpy and entropy and the equation of state are sufficient to predict the disturbance growth in the case of a subsonic flow through a nozzle. For the case of sonic flow at the nozzle throat, setting the system mass flow rate \( \dot{m} \) equal to the maximum attainable mass flow rate \( \dot{m}^* \) (the choked condition) and linearizing closes the system.

The predicted disturbance growth or decay is presented in the form of transfer functions comparing the magnitude of an imposed disturbance at the nozzle inlet to the resulting disturbance.
at the nozzle outlet. Depending on the Mach number achieved inside the nozzle, not all disturbances at the nozzle inlet can be freely imposed, and typical causality conditions must still be observed. For tractability, these transfer functions are presented in terms of the normalized characteristic decomposition of the gas:

\[ \{ \sigma, \pi^+, \pi^-, \bar{e} \} = \{ s'/c_p, p'/\gamma p + u'/u, p'/\gamma p - u'/u, Z' \} \] (1)

rather than the transported primitive variables \( \{ s', u', p', Z' \} \). Additionally, the chemistry effects are encapsulated in a single term \( \Psi \), a normalized chemical potential, which is written as:

\[ \Psi = \frac{1}{c_p T} \sum_{i=1}^{N} \mu_i Y_i. \] (2)

The meaning and implications of \( \Psi \) will be described in detail in the sections that follow, but it should be noted here that \( \Psi \) is a function of both mean mixture composition \( (Z) \) and mean thermodynamic state \( (T, p) \). In the following, we consider subsonic and supersonic flows. Key results from the transfer function derivation are summarized below:

### 1) Subsonic Case

For subsonic nozzles, the transfer function relating the acoustic pressure wave emerging from the nozzle exit to a unit compositional fluctuation introduced at the nozzle inlet to the resulting pressure wave emerging from the nozzle exit to a unit compositional fluctuation introduced at the nozzle inlet is given by:

\[ \frac{\pi^+_b}{\xi_a} = \left( \frac{\Psi_a - \Psi_b}{(1 + M_b)(M_a + M_b)[2 + (\gamma - 1)M_a M_b]} \right) \frac{2 + (\gamma - 1)M_a^2}{M_a M_b} \]

\[ + \frac{M_b}{[2(\Psi_a - \Psi_b) + (\gamma - 1)\{\Psi_a M_a^2 - \Psi_b M_b^2\}]} \frac{(\gamma - 1)(1 + M_b)(M_a + M_b)[2 + (\gamma - 1)M_a M_b]}{(\gamma - 1)(1 + M_b)(M_a + M_b)[2 + (\gamma - 1)M_a M_b]} \] (3)

where the subscript \( a \) refers to the nozzle entry condition and \( b \) the nozzle exit state.

### 2) Choked Case

For transonic nozzles, the transfer function relating the downstream \((u + a)\) acoustic pressure wave emerging from the nozzle exit to a unit compositional fluctuation imposed at the nozzle entrance is given by:

\[ \frac{\pi^+_b}{\xi_a} = \frac{1}{2(\gamma - 1)} \left[ \frac{2 + (\gamma - 1)M_a^2}{2 + (\gamma - 1)M_a M_b} \Psi_a - \Psi_b \right] \] (4)

### INERT MIXTURES

While the above expressions suggest that compositional fluctuations of sufficient amplitude can give rise to sound in an expansion process, to date, there has been no experimental evidence confirming this effect. With this in mind, the goal of this section is to identify an experimentally realizable test configuration with high sensitivity to compositional indirect noise that can be used to validate the theory. Previous work has focused exclusively on hot combustion products, often at elevated pressures, as these conditions most closely resemble the theory’s likely application to noise produced by a gas turbine. However, from an experimentalist’s perspective, the operating environment associated with aviation engines and hot combustion products is quite hostile to measurement, with high temperatures and elevated pressures rendering it challenging and costly to measure sound accurately and to carefully control the level of inflow perturbations imposed on the flow. Accordingly, the theory is applied here to inert mixtures of noble gases and air at conditions more amenable to experimental probing. Specifically, candidate mixtures of helium-air, argon-air, and krypton-air are studied to identify the mixture and operating conditions with the greatest sensitivity to the compositional noise mechanism. For these calculations, the mixture composition is parameterized by the mass fraction of the noble gas \( Z = Y_{He} = Y_{Ar} = Y_{Kr} \) with air constituting the remainder of the mixture \( (Y_{air} = 1 - Z) \). Thermodynamic properties for the inert gases are taken from the Argonne thermodynamic database [13]. The compositional noise transfer functions from the previous section are evaluated, with the constraint that state \( a \) corresponds to a stagnation condition \( (M_a = 0, p_a = 1 \text{ bar}) \). The results are plotted in Figure 1, along with the chemistry-independent transfer functions for entropy-induced noise and direct noise amplification for comparison. Expressions for the transfer functions for pressure and entropy can be found in [2].

It can be seen that the helium-air and krypton-air mixtures produce comparable acoustic responses, while the argon-air mixture generates noise with approximately 50% less efficiency. Given the substantial difference in cost between the two noble gases, this suggests that a helium-air mixture would be an ideal candidate for testing this mechanism. Additionally, it is observed that, regardless of the noble gas used, the noise produced increases monotonically with exit Mach number, indicating that high-speed experiments will exhibit the greatest sensitivity, and transonic configurations will outperform subsonic ones.

Since all three gases are noble gases, their molar specific heats at constant pressure are well approximated by \( c_p = 5/2R \), and, within the model framework, the only difference between these gases is their molecular weight, \( W \). From this it can be inferred that the acoustic efficiency of the composition wave mechanism is related to differences in molecular weight between a species and the background mixture. The ratio of molecular weights for helium-air, argon-air, and krypton-air are, respectively, \( \{0.137, 1.37, 2.89\} \), which suggests that, in two-component mixtures, the farther this ratio is from unity, the larger the amount of sound produced. This relationship will be explored...
it can be shown that the characteristic composition fluctuation derived from the constraints on the domain of \( t \) must lie on \( 0 \leq \xi \leq \sqrt{Z(1-Z)} \) where \( Z \) is the mean mixture fraction. While the observed value of \( \xi \) can be substantially lower than its upper bound depending on the device and application considered, scaling the transfer function by \( \xi \) provides an estimate of the magnitude of the acoustic response that can be achieved. Therefore, when comparing the noise produced by different mechanisms, the transfer functions must be scaled by the characteristic non-dimensional magnitudes of each fluctuation source. Generally, the effect of scaling the composition responses by \( \xi_{\text{max}} \) is to push the region of maximum response away from highly responsive regions near the bounds of the domain, which may exhibit high sensitivity to forcing but do not allow large fluctuation magnitudes and instead, the maximum response shifts toward the center of the \( Z \)-domain where the sensitivity to forcing may be somewhat lower, but larger forcing values are permitted.

Figure 2 shows the acoustic response to compositional forcing across a range of mixture fractions, scaled by the maximum realizable \( \xi \) to provide an estimate of the magnitude of the dimensionless pressure fluctuations that can be generated. For the noble gas-air mixtures, it can be seen that the region of strongest response lies somewhere between the \( Z \) bound corresponding to the lighter gas and the \( Z = 0.5 \) line indicating an 1:1 mass ratio of noble gas to air. This has implications for experimental design, suggesting that, for the helium-air case, a relatively large quantity of helium is required to obtain the optimal mean background mixture in addition to the helium needed for fluctuations, whereas for the heavier than air gases, only a small amount of background gas is required.

Next, the thermodynamic sensitivity of the helium-air mixture is explored, with Figure 3 demonstrating the sensitivity of the noise produced to the inflow temperature \( T_a \) while pressure is held constant at 1 bar. Similarly, Figure 4 demonstrates the sensitivity to the background pressure \( p_a \) with the inflow temperature held constant at 300 K. While Eqs. 3-4 do not contain an explicit dependence on the stagnation state, the chemical potential term \( \Psi \) is implicitly dependent on \( T_a \) and \( p_a \), resulting in the sensitivities shown. Collectively, these results reveal a moderate sensitivity to inflow temperature and a weak dependence on background pressure. This can be explained thermodynamically, as \( \Psi \) has a strong dependence, both explicit and implicit, on temperature \( (\Psi \sim 1/T) \) and \( \mu = f(T, p) \) and a relatively weak dependence on the pressure. The strong increase in response with temperature occurs because, for a constant Mach number expansion, as \( T_a \) increases, the effective temperature change \( \Delta T = T_b - T_a \) across the nozzle also increases (since the temperature ratio \( T_b / T_a \) is constant). This in turn impacts the \( \Psi \)-term first through its explicit \( 1/T \) dependence, and also through the strong implicit dependence of the chemical potential \( \mu \) on \( T \). The flow exhibits much less sensitivity to changes in \( p_a \), most likely because the mixture chemical potential \( \mu \) is a relatively weak function of pressure over the range of plausible \( p_a \) values. In fact, Figure 4 indicates that the acoustic response decreases as \( p_a \) is increased, indicating that there is actually a disadvantage to using a pressurized rig to detect composition noise.

Taken together, the inert mixture data suggest that the simplest experimental facility needed to demonstrate the composition noise mechanism most likely consists of a mixture with approximately 1:5 ratio by mass of helium to air that is accelerated to a supersonic state. Depending on the level of controlled composition fluctuations that can be attained, pre-heating the mixture may be desirable to increase the degree of acoustic response. There is no appreciable benefit to using a pressurized rig for testing.
Fuel Effects

Turning from the laboratory environment to more practical gas turbine applications, we now consider the indirect noise generated by combustion products in order to assess the impact of the fuel type on noise levels. The theory is used to compare the noise generated by three different fuels - hydrogen, methane, and dodecane - which together are employed over a broad range of combustion applications: hydrogen is commonly used in numerical experiments due to the relative simplicity of its chemical mechanism as well as in rocketry applications and as a component of syngas; methane is a common choice of fuel for laboratory scale combustion studies as well as in stationary power generation; and dodecane is commonly used as a surrogate for kerosene, which is itself representative of common aviation fuels such as Jet-A.

In order to appropriately compare different fuels and introduce a convenient parameterization of the reaction chemistry, a series of one-dimensional flamelets were computed for each fuel using the Flamemaster code [14]. The 11-species Burke mechanism was used to compute the hydrogen flamelets [15], the 53-species GRI-mechanism was used to compute the methane flamelets [16], and a reduced 22-species mechanism [17] was used for the dodecane flamelets. Each flamelet was computed with constant thermal boundary conditions of $T_{\text{fuel}} = T_{\text{ox}} = 300$ K and at ambient pressure of 1 bar; the impacts of preheating and elevated pressures were considered in the previous section. The set of flamelets generated for each fuel can be compared in Figure 5, which shows the “S-shaped” curve for each set of flamelets.
It can be seen that, along the upper, burnt branch, the thermodynamic stoichiometric state of each burnt gas is roughly similar, with maximum temperatures within a few hundred degrees of each other, though the fuels begin to differ more markedly as the strain rate is increased and the flames begin to approach extinction. While it is difficult to completely isolate chemistry effects from the thermodynamic state, the condition corresponding to a strain rate $\chi = 1 \text{s}^{-1}$ was chosen for comparing the fuels, as the stoichiometric temperatures near this point are quite similar and the strain rate is still sufficiently low that strain-induced extinction does not significantly alter the flame. The impact of strain on compositional noise production was studied in detail in [11]. Figure 6 shows the structure of these mildly-strained flamelets as a function of their rescaled mixture fraction $Z^*$, where $Z^* = Z/(Z + Z_{st})$ and $Z_{st}$ is the stoichiometric mixture fraction. The value of $Z_{st}$ for hydrogen-air mixtures is 0.0285, for methane-air mixtures is 0.0552, and for dodecane-air mixtures is 0.0628 $\text{s}^{-1}$. The normalized mixture fraction $Z^*$ was chosen as the ordinate because it divides the realizable domain approximately equally between lean and rich sides with the stoichiometric mixture occurring at $Z^* = 0.5$ for all fuels. The flamelet structures are all relatively similar when compared against the rescaled mixture fraction, suggesting that major differences between the flamelets’ acoustic response will likely be due to chemical effects rather than minor differences in the thermodynamic state.

The transfer functions given by Eqns. 3-4 of the previous section were then evaluated for each fuel over the full range of mixture fraction and a broad range of outflow Mach numbers, using the composition data and stagnation conditions from the flamelet solution. Results are shown in Figure 7. Since the Mach number in most industrial combustion chambers is typically quite low, the transfer functions throughout this work are evaluated at $M_a = 0$ to reduce the dimensionality of the parameter space to be tested. The impact of this constraint was investigated, and it was found that there was little sensitivity in acoustic predictions.

Comparing the acoustic response of the three fuels, it can be seen that the hydrogen-air mixture is predicted to produce substantially more noise than the hydrocarbon fuels, often by as much as an order of magnitude. Between the methane and dodecane mixtures, there is relatively little difference in the amount of predicted noise. However, the response surfaces of all three fuels contain substantial similarities in their structure.

First, it is observed that, consistent with previous results, the noise generally peaks at higher exit Mach numbers. This trend is common not only across fuels, but also to the entropy noise and direct noise mechanisms as well, and can be explained by the Mach number dependence and the $T^{-1}$ scaling of $\Psi$. In fact, freezing the value of the chemical potential, which effectively removes the role of chemistry from the problem, the increase in acoustic response with exit Mach number is still observed. This does not indicate that chemical effects are independent of $M_b$, rather, the dependence of the chemical potential on $M_b$, which occurs through changes in the thermodynamic state $(T_b, p_b)$, also contributes to the observed Mach number dependence, but in a less systematic way than the explicit $M_b$ and $T$ dependence in the transfer functions and $\Psi$.

The second noteworthy feature is the presence of three “silent” zones on the response surface: one on the lean side at subsonic conditions, a second near the rich boundary where pyrolysis occurs for subsonic flows, and a third that begins near the lean edge at $M_b = 1$ and bends toward the stoichiometric condition as $M_b$ is further increased. Similarly, the third common feature is the trend that stoichiometric mixtures are generally loudest at subsonic conditions while lean mixtures produce the most noise once the flow becomes supersonic. Unlike the first shared feature, these two trends are driven largely by combustion chemistry-specific effects as opposed to general features of expanding flows. Evidence of this can be seen by comparing the response surfaces of Figure 7 to the surfaces of noble gas-air mixtures in Figure 1, which do not contain these features. Further support that these features are driven by the chemical composition of the reacting flows and not thermodynamic effects can be seen in Figure 8 which shows the response surface for a methane-air flame with stagnation temperature kept constant at $T_a = 2100\text{K}$ for all values of $Z$, rather than letting $T = T(Z)$ as in Figure 7. In spite of the large change in thermodynamic state occurring on the lean and rich borders of the flame, the response surface is still qualitatively similar to that in Figure 7, with both the “silent” and “loud” regions occurring in the same part of the response surface. Results for the hydrogen and dodecane flamelets were similar to those shown for methane.
The link between these features and combustion chemistry can be more thoroughly explained by exploring the surface of the chemical potential term $\Psi$. The $\Psi$ term is the link between species chemistry and the hydrodynamic equations which govern acoustic behavior; $\Psi$ is what drives the compositional noise mechanism as seen in Eqsns. 3-4. Figure 9 shows the chemical potential term $\Psi$ as a function of Mach number and composition for methane-air flames; the corresponding diagrams for hydrogen-air and dodecane-air mixtures are qualitatively very similar. Taking the limit of Eqns. 3 and 4 as $M_a \to 0$ (and in the subsonic case, linearizing in $M_b$) the transfer functions simplify to the following expressions:

### 1) Subsonic Case

The transfer function relating compositional fluctuations to the pressure disturbance produced in subsonic flows for $M_a = 0$, linearized about small $M_b$, is given by:

$$\frac{\pi^+}{\pi^-} \approx -\frac{(\Psi_b - \Psi_a)}{\gamma - 1} M_b + \frac{(\Psi_b - \Psi_a)}{\gamma - 1} M_b^2 + O(M_b^3) + ...$$ (5)

### 2) Choked Case

The transfer function relating compositional fluctuations to the pressure disturbance produced in transonic flows for $M_a = 0$ is given by:

$$\frac{\pi^+}{\pi^-} = -\frac{(\Psi_b - \Psi_a)}{2\gamma - 1} + \frac{(\gamma - 1)\Psi_a}{2(2\gamma - 1)} M_b$$ (6)
For subsonic conditions, to lowest nonzero order in $M_b$, the acoustic response is proportional to $\Delta \Psi = \Psi_b - \Psi_a$, so regions in which $\partial \Delta \Psi / \partial M_b$ (or more practically, $\partial \Psi_b / \partial T_b$) is negligible are likely to produce “silent” regions of response and regions where this term is large will produce intense responses. Examining the $\Psi$-surface of Figure 9, it can be seen that the near-silent regions at $Z^* = 0.15$ and $Z^* = 0.8$ correspond to nearly vertical isocontours of $\Psi$, while the high subsonic acoustic responses at $Z^* = 0$ and $Z^* = 0.45$ correspond to portions of the $\Psi$ surface where the isocontours exhibit relatively strong curvature. Similarly, the acoustically efficient region at subsonic conditions near stoichiometry is caused by a subtle but sharp increase in $\partial \Psi / \partial T$ near $Z_a$. Additional features and complexity in the subcritical domain are caused by the presence of higher order terms.

At supersonic conditions, the acoustic response to zeroth order scales as $\Delta \Psi$, and to first order, as $\Psi_b M_b$. As in the subsonic case, the high response region for supersonic regions corresponds to $\Psi$ regions with non-vertical isocontours or large values of $\partial \Psi / \partial M_b$, which in this case occur near the lean boundary of the flow. The “silent” low-response region that curves from the lean boundary near $M_b = 1$ to $Z_a$ as $M_b$ increases is caused by an approximate balancing of the zeroth and first order terms in the transfer function. As the scaled value of $\Delta \Psi$ becomes of the same order as the scaled $\Psi_a$ term, the response is minimized.

The complexity of combustion makes it difficult to make general statements about the composition noise produced by all reacting flows. However, the data presented in this section suggest that, over a range of fuels and common operating parameters, there appear to some general trends to the acoustic response. Furthermore, outside this regime, it is shown that analysis of the normalized chemical potential term $\Psi$ can be used to identify trends in acoustic response for a given system.

**SPECIES SENSITIVITY**

While Eqns. 3-4 concisely present predictions for the magnitude of compositional noise generated, they offer little overt explanation as to how differences between species concentrations and properties give rise to indirect noise. This section explores the relationship between the chemical properties of individual species and the amount of indirect noise produced.

The impact of species chemistry is introduced through the term $\Psi$, defined in Eqn. 1 in the transfer functions, which effectively represents the mixture’s normalized chemical potential on a mass basis. The chemical potential $\mu$ can be replaced by the Gibbs free energy of the mixture, resulting in an alternative expression for $\Psi$, given by:

$$
\Psi = \frac{1}{c_p T} \frac{\partial g}{\partial Z} \bigg|_{T_p} = \frac{1}{c_p T} \sum_{i=1}^{N} \gamma_i \frac{\partial Y_i}{\partial Z} 
$$

(7)

The previous section showed that it was differences in the chemical potential term $\Delta \Psi$ that generated pressure fluctuations, so Eqn. 7 indicates that it is largely differences in Gibbs free energy $\Delta g$ and strong species gradients $\partial Y_i / \partial Z$, that generate large changes in $\Psi$ and compositional indirect noise.

To probe the mechanism’s sensitivity to individual species, the case of a single methane-air flamelet is examined in detail. Specifically, three points along the flame solution were chosen for additional analysis, $Z^* = \{0.3, 0.5, 0.7\}$, corresponding to lean, stoichiometric, and rich conditions, respectively. Figure 10 shows the variation of the chemical potential term $\Psi$ as well
as the mixture Gibbs free energy for each operating condition. For the lean case, taken from a high response region, it can be seen that there are strong variations in \(g\) \((\approx 40\%\) from \(M_a = 0\) to \(M_b = 2)\) which contribute to the \(\Delta \Psi\) that drives the composition noise. For the stoichiometric and rich cases, the Gibbs free energy changes less drastically and, correspondingly, both \(\Delta \Psi\) and the acoustic response are much lower.

To relate these changes to individual species, Figure 11 shows the variation of species-specific Gibbs free energy versus temperature for common species. The figure indicates that the major components of air, \(O_2\) and \(N_2\), are relatively acoustically inefficient since their Gibbs free energies are much less sensitive than other common species. Among combustion products, it can be seen that water is substantially more efficient than \(CO_2\) while the fuel is also an efficient species. Radicals such as \(OH\) and \(H\) are also potentially strong sources of acoustic noise; the variation of \(g_{H}\) was so large that it could not be shown on the same scale as the other species.

However, strong gradients in a species’ \(g_i\) alone are not sufficient to produce indirect noise; the relative abundance of these components must also be taken into account. Equation 7 indicates that large values of both \(\partial g_i/\partial M_b\) and strong composition gradients \(\partial Y_i/\partial Z\) must both be present to generate noise. For example, the radical \(H\) has much stronger variation in its mass-specific Gibbs free energy than other molecules, but, due to the chemical instability of the species, it is unlikely to be found at meaningful concentrations regardless of mixture fraction, and therefore cannot generate much indirect noise. Conversely, the Gibbs free energies of \(O_2\) and \(N_2\) vary quite weakly with temperature, but since oxygen is consumed and nitrogen does not diffuse appreciably to the rich side of the flame, the mass fraction gradients of these species, \(\partial Y_i/\partial Z\) are large enough that even their small differences in \(g_i\) over the expansion produce appreciable noise. Thus, to more quantitatively identify each species’ contribution to the indirect noise, Figure 12 shows the weighted contribution of each major species to the chemical potential term \(\Psi\) or \(\partial Y_i/\partial Z \cdot g_i\).

One major feature of Figure 12 is that, in reacting flows, there does not appear to be a single species which dominates the production of indirect noise. Instead, the most acoustically efficient species is a strong function of the mixture stoichiometry just as overall noise production varies strongly with \(Z\). For lean mixtures, both \(O_2\) (which is being rapidly consumed) and \(H_2O\) (which is being rapidly produced) are the strongest components of composition noise, while at rich conditions neither of these molecules produce appreciable noise. Likewise, methane is the dominant noise producer for rich mixtures, but because it is consumed on the lean side of the flame, it has effectively no impact on noise generation near or below stoichiometric values of \(Z\). From this, it can be concluded that the analysis of the composition noise mechanism must account for both a device’s intended fuel and stoichiometry; predictions that only account for fuel will not be accurate over a range of operating conditions.

**CONCLUSIONS**

In summary, chemical effects leading to composition-induced indirect core noise were examined in detail. After describing the underlying theory of the mechanism, several features were investigated in detail. As this core noise mechanism has to date only been studied numerically, the theory was first applied...
to a series of inert candidate gas mixtures to guide experimental design for confirmation of the theory. It was found that a helium-air mixture entering a supercritical nozzle at ambient pressure will produce the highest noise levels, and that preheating would be an effective means to increase the sound amplitude if necessary, depending on the intensity of compositional fluctuation that can be achieved.

Next, the sensitivity of this indirect noise phenomena to fuel selection is examined. It is found that hydrogen-air mixtures were generally more acoustically efficient than hydrocarbon fuels, and there appeared to be no strong sensitivity between different types of hydrocarbon fuels. Comparison of different fuels also revealed that there was a general structure to the composition-induced acoustic response, and reasons for this behavior were explained through linearization of the transfer functions and a detailed examination of the chemical potential. A methane-air flame was examined in detail to identify the individual combustion products that contribute most strongly to the acoustic response. It was found that no single species was responsible for this effect, but, rather the most acoustically efficient compound varied with mixture stoichiometry. Independent of stoichiometry, however, it was seen that species which have high mass fractions, high mass fraction gradients, and whose molecular weight deviates most strongly from that of the background mixture were most acoustically efficient. More generally, it was observed that increasing the nozzle inlet temperature, decreasing the inlet pressure, and increasing the exhaust gas Mach number all generally increased the amount of composition-induced indirect noise produced.

FIGURE 12. CONTRIBUTION OF THE 10 MOST DOMINANT SPECIES TO $\Psi$, I.E. $\partial Y_i / \partial Z$ AT DIFFERENT MEAN MIXTURE FRACTIONS. BLUE BARS CORRESPOND TO THE STAGNATION CONDITION, RED TO AN ISENTROPIC ACCELERATION FROM STAGNATION TO $M_b = 1$, AND MAGENTA BARS TO $M_b = 2.0$. NON-MONOTONICITY IN BAR HEIGHT VS. $M_b$ GENERALLY INDICATES A CHANGE IN SIGN FOR THAT SPECIES’ CONTRIBUTION.

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