Integrated analysis of jet-engine core noise using a hybrid modeling approach

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As further reductions in aircraft engine noise are realized, the relative importance of reducing engine-core noise increases. In this work, a representative engine flow-path is considered to examine the mechanisms by which direct and indirect core noise propagate through the engine and affect the far-field sound radiation. The flow-path consists of a model combustor, a single-stage turbine, a converging nozzle, a near-field jet, and far-field acoustic radiation. A combination of high-fidelity simulations and low-order semi-analytic models is used to represent the generation and propagation of disturbances through the flow-path. Particular details are provided for LES calculations of the combustion chamber, nozzle exhaust flow, and jet noise radiation. A one-way coupling procedure is employed for propagating disturbances from one stage of the calculations to the next. Results show substantial increase in far-field sound radiation due to fluctuations generated by the upstream engine core.

I. Introduction

Aircraft noise (including both engine noise and airframe noise) has been identified as a primary obstacle to increasing airport capacity.1 With the number of enplaned passengers in the U.S. alone projected to double over the next two decades,2 noise concerns have the potential to affect growth in commercial aviation. Also, continued interest in supersonic commercial airliners requires significant noise mitigation research. Additionally, airport and military personnel working in close proximity to aircraft can suffer from reduction or loss of hearing and tinnitus due to aviation noise. The Government Accountability Office (GAO) estimates that the Department of Veteran’s Affairs spends over $1B annually on hearing loss;3 a substantial portion of which is attributable to exposure to aviation noise. Accordingly, reducing aircraft noise produces more than mere nuisance mitigation and can unlock substantial economic, physiological, and quality-of-life benefits for those exposed to aircraft and the community at large. Considering that the main source of aircraft noise comes from the engine, reducing noise generated by gas-turbine engines is a major concern for engine manufacturers and the aviation industry today.

Radiated sound from gas-turbine engines (such as modern high bypass-ratio turbofan engines) is often divided into several components for analysis – fan noise, compressor noise, core noise, and jet noise. Specifically, core noise, which will be the focus of this study, is defined as any excess noise generated within the engine core. Its generation is often associated with combustion processes and unsteady convection of high-temperature combusted gas,4–6 and it is characteristically dominated by low frequencies of $O(10^2)$ Hz. Overall, fan noise is a major noise source in both the upstream and downstream directions, while jet noise dominates other sources at high-speed operating conditions such as take-off. Over the past several decades, significant progress has been made in reducing jet noise and fan noise, which has increased the relative importance of core noise.5–7 Furthermore, at low-speed engine conditions where jet noise is less prominent (for example, aircraft during taxi and approach, ground-based gas turbines, and auxiliary power units), core noise can be significant.

In addition, noise generated within the combustor, or direct core noise, is often reflected and scattered through the turbine stages, where it can propagate back into the combustion chamber. This can potentially trigger acoustic resonances which may lead to thermo-acoustic instabilities. In addition to direct core noise...
noise, high-temperature combustion products undergo significant acceleration while propagating through the turbine stages and exhaust nozzle, which can convert some of the entropy and vorticity fluctuations into pressure fluctuations. This process, often termed indirect noise, can significantly impact the sound radiation of gas-turbine engines at high-speed conditions. Since aircraft engines typically operate in this regime (due to the substantial acceleration in the turbine stages and exhaust nozzle), the indirect mechanism is likely to be the dominant source of core noise in aviation gas turbines.

Understanding the fundamental mechanisms of core-noise generation and propagation is an essential step toward further reducing overall noise from gas-turbine engines. It is also important to understand how core noise interacts with engine components, since its generation and propagation can be closely linked with thermo-acoustic instabilities in the combustor.

While high-fidelity simulations based upon large-eddy simulation (LES) have proven useful in modeling high-speed, high-temperature turbulent flows, it is still expensive to directly apply this technique throughout the entire flow-path of gas-turbine engines. Additionally, combining each stage of the flow-path into a single calculation can be prohibitively expensive because of large disparities in time-step restrictions. Thus, it is necessary to use appropriate methods for each stage of the flow-path and then couple the simulations through appropriate boundary conditions. In some portions of the flow-path, there is still a need to use simpler analytic models that offer sufficiently accurate predictions at reduced computational costs, while high-fidelity simulation can be used as required where flow complexities are significant. The appropriate balance of these hybrid techniques can provide insight into the underlying mechanisms of core-noise generation and propagation at sustainable computational costs.

In this study, we investigate fundamental core-noise mechanisms in a modeled gas-turbine flow-path that contains essential components of a commercial jet engine at a cruise condition. To this end, a hybrid modeling approach is used, which combines high-fidelity simulation and lower-order analytic tools. A canonical engine core is designed, consisting of a combustor, a single-stage turbine, a converging nozzle and jet, and far-field acoustic radiation. See et al. studied a similar configuration without a turbine stage. Duran et al. combined a high-fidelity combustor simulation and a turbine stage modeled by Actuator Disk Theory (ADT). The reacting flow within the combustor is modeled using LES based upon a fully compressible formulation of the Navier–Stokes equations supplemented with the flamelet-progress variable approach for chemistry modeling. Acoustic propagation through the turbine stage is modeled using ADT. The nozzle and turbulent jet are modeled using a fully compressible LES formulation, and its acoustic radiation at the far-field is computed using the Ffowcs Williams and Hawkings method. Each calculation is coupled using a one-way coupling procedure which allows disturbances to evolve downstream from one calculation to the next but neglects the effects of any upstream-propagating disturbances. The hybrid model as well as the thermodynamic states of the engine flow-path are illustrated in Figure 1.

![Figure 1: Component models and thermodynamic states for the flow-path of the representative gas-turbine engine.](image)

**II. Combustor and turbine numerical models**

**II.A. Combustor simulation**

The combustor stage is modeled using the in-house code Chris, an LES solver that has previously been validated in similar applications. Chris solves the fully compressible Navier–Stokes equations on unstructured grids. The code is explicit and second-order accurate in time and space on arbitrary grids. The chemical source term is modeled by the flamelet progress variable approach using a three-dimensional chemistry table based on tabulated chemistry for methane–air diffusion flames. The filtered momentum equation is closed using the Vreman model, and the turbulent scalar fluxes are closed using a constant turbulent Schmidt number assumption. The resulting set of governing equations is
fluctuations are normalized for the standard ADT procedure as follows:

\[ \overline{\delta Z} = -\rho \rho_{\delta} \left( \frac{\partial \rho}{\partial x_j} \left( \frac{\partial \rho u_j}{\partial x_j} + \frac{\partial \rho u_j}{\partial x_i} \right) \right) \]

above 20 kHz, corresponding to the upper limit of human hearing. The mean is removed from each signal and associated with the interpolation procedure, a spectral filter is applied to eliminate all frequency contents is run at constant Courant–Friedrichs–Lewy (CFL) number. To eliminate the effects of high frequency noise calculation; each bin is sized to have sufficient support (at least 30 points) such that the averaged data are averaging procedure involves binning the data down to a resolution appropriate for the downstream jet

characteristic length scales are much greater than the chord lengths of the turbine blades. Because of its linearity, the method cannot take into account the “chopping” effect by the turbine blades which can map moderate frequency disturbances to higher frequencies due to blade rotation. However, this effect is not expected to be significant at the low frequencies associated with core noise. The method was first presented by Cumptsy and Marble,12 and has been validated against higher fidelity methods and found to perform satisfactorily for the frequencies of interest in this study.11,14 The set of linearized equations is

\[ \begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_j} &= 0 \\
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} &= -\frac{\partial \rho}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \left( \frac{\partial \rho}{\partial x_j} + \frac{\partial u_i}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) - \frac{2}{3} \rho k \delta_{ij} \\
\frac{\partial \rho \tilde{E}}{\partial t} + \frac{\partial \rho u_j \tilde{E}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \frac{\lambda}{c_p} + \frac{\mu}{\rho \tau_t} \right) \frac{\partial \tilde{h}}{\partial x_j} \right] + \frac{\partial}{\partial x_j} \left( -\tilde{u}_j \rho \tilde{h} + \tilde{u}_j \tilde{h} \right) \\
\frac{\partial \rho \tilde{Z}}{\partial t} + \frac{\partial \rho u_j \tilde{Z}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \overline{\rho D} + \frac{\mu}{S_c} \right) \frac{\partial \tilde{Z}}{\partial x_j} \right] \\
\frac{\partial \rho \tilde{C}}{\partial t} + \frac{\partial \rho u_j \tilde{C}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \overline{\rho D} + \frac{\rho}{S_c} \right) \frac{\partial \tilde{C}}{\partial x_j} \right] + \tilde{w}_c \left( \tilde{Z}, \tilde{Z}'^2, \tilde{C} \right) \\
\frac{\partial \rho \tilde{Q}}{\partial t} + \frac{\partial \rho u_j \tilde{Q}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \overline{\rho D} + \frac{\rho}{S_c} \right) \frac{\partial \tilde{Q}}{\partial x_j} \right] - \left( 2 \overline{\rho D} \frac{\partial \tilde{Z}}{\partial x_j} \frac{\partial \tilde{Z}}{\partial x_j} + \rho \overline{C} \tilde{Z} ' \tilde{Z} \frac{\mu}{S_c \Delta^2} \right)
\end{align*} \]

where \( \tilde{Z} \) is the mixture fraction, \( \tilde{C} \) is the progress variable, \( \tilde{Q} \) is the mixture fraction variance and \( \overline{C} \tilde{Q} \) is a model constant to model the residual scalar dissipation rate.

II.B. Turbine simulation

The turbine stage is modeled using ADT, a low-order analytical method based on a linearization of conservation equations applied across a turbine stage, specifically the conservation of momentum, energy, and entropy as well as the Kutta condition. ADT is best suited for acoustically compact disturbances in which the characteristic length scales are much greater than the chord lengths of the turbine blades. Because of its linearity, the method cannot take into account the “chopping” effect by the turbine blades which can map moderate frequency disturbances to higher frequencies due to blade rotation. However, this effect is not expected to be significant at the low frequencies associated with core noise. The method was first presented by Cumptsy and Marble,12 and has been validated against higher fidelity methods and found to perform satisfactorily for the frequencies of interest in this study.11,14 The set of linearized equations is

\[ s_1^* = s_2^* \]

\[ \frac{p_1^* + \frac{w_1^*}{\text{Ma}_1} - \theta_1^* \tan(\theta_1)}{1 + \frac{1}{2}(\gamma - 1)} \text{Ma}_1^2 = \frac{p_2^* + \frac{w_2^*}{\text{Ma}_2} - \theta_2^* \tan(\theta_2)}{1 + \frac{1}{2}(\gamma - 1)} \text{Ma}_2^2 \]

\[ \frac{1}{1 + \frac{1}{2}(\gamma - 1)} \text{Ma}_1^2 \left( p_1^* + \frac{1}{\gamma - 1} s_1^* + \text{Ma}_1 w_1^* \right) = \frac{1}{1 + \frac{1}{2}(\gamma - 1)} \text{Ma}_2^2 \left( p_2^* + \frac{1}{\gamma - 1} s_2^* + \text{Ma}_2 w_2^* \right) \]

\[ \theta_2^* = 0 \]

To implement the method, instantaneous velocity fields and thermodynamic states of the combustor exhaust gas are recorded on a plane at the combustor exit. The collected data are then azimuthally averaged to increase compactness and reduce the effects of swirl. Since the combustor mesh is unstructured, the averaging procedure involves binning the data down to a resolution appropriate for the downstream jet calculation; each bin is sized to have sufficient support (at least 30 points) such that the averaged data are well converged. Next, the data are interpolated onto a regular temporal grid since the combustor calculation is run at constant Courant–Friedrichs–Lewy (CFL) number. To eliminate the effects of high frequency noise associated with the interpolation procedure, a spectral filter is applied to eliminate all frequency contents above 25 kHz, corresponding to the upper limit of human hearing. The mean is removed from each signal and fluctuations are normalized for the standard ADT procedure as follows: \( \rho^* \equiv \rho^* / \rho_0 \), \( s^* \equiv s^* / c_p \), \( w^* \equiv w^* / a_0 \). The Hanning window is applied to impose periodicity on the signal. Next, proper orthogonal decomposition (POD) is used to construct a basis on which the linear ADT method can be applied. POD is a desirable tool for capturing fluctuations about a base state since the procedure is optimized to capture the variance in the dataset as compactly as possible. However, the procedure involves some subjectivity in the choice of the norm in which to perform the decomposition. For the present calculations, the norm is taken to be the \( L^2 \) norm of the normalized entropy, pressure, and axial velocity fluctuations. Only the fundamental mode is

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retained, and the analysis of the importance of secondary modes on downstream noise is a topic for further research. After this preprocessing is complete, the ADT procedure is applied across both the stator and the rotor stages of the turbine to map the amplitudes of the POD modes through the turbine stage.

III. Combustor and turbine simulation results

The combustor geometry considered is the dual-swirl gas-turbine combustor originally studied by Meier and coworkers.\textsuperscript{15} Previous work has demonstrated the code’s capability to predict the turbulent reacting flow within the same combustor.\textsuperscript{13} To accommodate the full flow-path, the combustor simulation is run at a higher pressure and mass-flow rate and a leaner condition than has been studied experimentally. The mass-flow rate of air is 0.43 kg/s and the global equivalence ratio for methane combustion is 0.40. Air enters the combustor at 215 K and exits at 1010 K. While the global equivalence ratio is below the lean flammability limit for methane-air combustion at these conditions, the partially premixed nature of the configuration allows the flame to stabilize and extinction is not observed. In experimental work, the combustor has demonstrated two distinct behaviors: a flat-flame mode in which the flame remains attached to the wall and a V-flame where it is unattached. At this condition, the V-flame is observed, with a precessing vortex core stabilizing the flame. The V flame structure can be seen in Figure 2, along with the temporally averaged velocity and pressure fields.

![Figure 2: Average flow-field results of normalized a) pressure, b) temperature and c) axial velocity fields in the gas-turbine combustor.](image)

The combustor’s exhaust is sampled at the plane $y = 6.5$ immediately upstream of the calculation’s outflow boundary. The exhaust is characterized by turbulent fluctuation levels on the order of $2 - 5\%$ of the mean velocity and large thermal fluctuations of up to 50K. It should be noted, however, that both hydrodynamic and acoustic perturbations are included in these signals. As Figure 4 indicates, there is a persistent spatial structure most likely attributable to the hydrodynamic field associated with the precessing vortex core. It is found to be a relatively loud operating condition, with sound pressure levels on the order of 140 to 150 dB and substantial spectral content at low frequencies, as seen in Figure 5.
The turbine design considered in this study consists of a single rotor–stator pair taken from the NASA high-pressure turbine design. The pressure drop over the stage is 2.37 and the large jump is used to mimic the thermodynamic effects of a multi-stage turbine. Results from the ADT theory are shown in Figure 3, and the effect of ADT on the pressure spectra is evaluated in Figure 5. Work is underway to assess the adequacy of this low order technique.
IV. Mach 0.9 heated jet and its sound radiation

IV.A. Simulation set-up

High-temperature exhaust gas from the modeled gas-turbine combustor is simulated using LES. Flows from the turbine stage enter into a straight pipe connected to a converging nozzle. The jet exhaust is injected into the ambient air at rest, and its turbulence and sound radiation are simulated.

The converging nozzle is axisymmetric and its radius at the nozzle exit is \( r_J = D_J/2 = 2.54 \text{ cm} \). As shown in Figure 6a, the nozzle cross-sectional area decreases over \( 2.5r_J \) in the axial direction, and the area contraction ratio is \( 4.73 \). The nozzle geometry is generated to meet the ASME standards. The nozzle is connected to the exit of the upstream turbine stage via a straight pipe shown in Figure 6b. The pipe has a diameter of \( 2.17r_J \) and a length of \( 15r_J \).

![Figure 6: (a) Cross-section of the ASME-standard converging nozzle. (b) Three-dimensional geometry of the jet exhaust system.](image)

The nozzle-exit condition is obtained based upon the outflow condition of the turbine stage. It is also the same as the nozzle-exit condition of the test-point number 49 of Tanna.\(^{17} \) The non-dimensional velocity at the nozzle exit with respect to the ambient speed of sound is \( u_J/c_\infty = 1.48 \), and the nozzle-exit temperature relative to the ambient temperature is \( T_J/T_\infty = 2.857 \). The nozzle-exit Mach number is \( M_J = u_J/c_J = 0.876 \), and the Reynolds number based on the nozzle-exit condition is \( Re_J = \rho_J u_J D_J/\mu_J = 2.3 \times 10^6 \). This Reynolds number is lower than the critical Reynolds number \( 4 \times 10^5 \) above which low Reynolds number effects on far-field sound become less significant.\(^{18} \)

The physical domain extends \( 40r_J \) downstream of the nozzle exit and \( 40r_J \) in the radial direction. An absorbing buffer zone is used for \( 20r_J \) downstream of the physical outflow. The center of the nozzle exit is \( (x, r = \sqrt{y^2 + z^2}) = (0, 0) \), which is also the reference point to define the distance \( d \) and the radiation angle \( \varphi \), where \( \varphi = 0^\circ \) corresponds to the downstream jet axis, as illustrated in Figure 7.

The high-fidelity, computational fluid dynamics code, CHARLES\(^{†} \), is used to solve the fully compressible Navier–Stokes equations. The fluid is assumed to be an ideal gas with \( \gamma = c_p/c_v = 1.4 \), where \( c_p \) and \( c_v \) are the specific heats at a constant pressure and volume, respectively. The viscosity is assumed to be a function of temperature only and to follow the power-law relation, \( \mu/\mu_\infty = (T/T_\infty)^n \) where \( n = 0.76 \). The Prandtl number is assumed constant at \( Pr = 0.7 \). The subgrid-scale model of Vreman\(^{19} \) is used with the model constant \( c = 0.07 \) and the constant turbulent Prandtl number \( Pr_t = 0.9 \).

At the nozzle inlet, the total pressure of \( p_0/p_\infty = 1.621 \) and the total temperature of \( T_0/T_\infty = 3.229 \) are prescribed following the test-point number 49 of Tanna.\(^{17} \) At the outflow, an absorbing buffer zone is applied. The solid wall boundary is modeled using the equilibrium wall model\(^{20} \) with zero heat flux. The rest of the domain boundaries are modeled to have the ambient total pressure and the ambient total temperature. There is no co-flow in this flow configuration.

\(^{†} \text{http://www.cascadetechnologies.com/pdf/CHARLES.pdf} \)
The governing equations are discretized in space using a cell-based finite volume formulation. The solutions are time advanced using the standard third-order Runge–Kutta method at a constant time-step size of $\Delta t r_J/c_\infty = 0.00125$, which results in a CFL number of approximately 1.0. The spatial discretization is non-dissipative and formally second-order accurate on arbitrary unstructured grids. In addition, the convective fluxes are combined, depending on local grid quality (for example, element skewness), with fluxes computed by an HLLC-upwind discretization. Khalighi et al.\textsuperscript{21} provides more detailed discussions on the spatial discretization.

A base grid of 0.8 million unstructured control volumes is generated and subsequently refined using ADAPT, the grid-adapting tool in the CHARLES suite of codes. For computational accuracy and efficiency, only hexahedral elements are used. The total number of control volumes for the adapted grid is 25.3 million. Neither turbulent statistics nor sound prediction exhibited appreciable sensitivity upon additional mesh refinement. A cross-section of the computational grid along the plane $z = 0$ is shown in Figures 8a.

Using the same computational code CHARLES, Bres et al.\textsuperscript{22} simulated a Mach 0.9 isothermal jet and its acoustics. Their BL16M_WM is compared for space–time resolution with the current simulation. It should be noted that the nozzle geometry and nozzle-exit condition differ between Bres et al.\textsuperscript{22} (convergent–straight nozzle and Mach 0.9 isothermal jet) and the current set-up (converging nozzle and Mach 0.9 heated jet), and thus the comparison is strictly qualitative. However, it can provide a guideline to assess whether the current resolution is reasonable. To investigate the impact of the turbulent boundary layer on sound radiation, Bres et al.\textsuperscript{22} selectively refined the near-wall regions of the straight nozzle, while, in the current study, the entire region within the nozzle is refined to resolve the fluctuations from the turbine stage (see Figure 8b). For the same reason, the grid within the potential core is refined. When the nozzle interior and the potential core are not refined, the total number of control volumes is approximately 18 million, which is comparable to Bres et al.\textsuperscript{22} Also, the nozzle-exit velocity of the current simulation is 1.64 times faster due to the higher temperature ratio ($T_J/T_\infty = 2.857$ compared to $T_J/T_\infty = 1.0$ of Bres et al.\textsuperscript{22}). As a result, the nozzle-exit boundary layer becomes thinner for the current simulation, and the resolution requirement appears to be higher than Bres et al.\textsuperscript{22} This explains the choice of a smaller time-step size ($\Delta t r_J/c_\infty = 0.00125$ compared to $\Delta t r_J/c_\infty = 0.002$ of Bres et al.\textsuperscript{22}).

Sound radiation at far-field locations is computed using the Ffowcs Williams and Hawkings method.\textsuperscript{23} The integral surface is located within the grid-refined zones, as illustrated in Figure 8a. Its minimum radius is $2.9r_J$ near the nozzle exit and the maximum radius is $8.9r_J$ at the downstream end at $x/r_J = 40$. Testing
Figure 8: Cross-sections of the computational grid on the $x$-$y$ plane for (a) downstream of the nozzle and (b) the nozzle interior. The dashed line in Figure 8a represents parts of the integral surface for the Ffowcs Williams and Hawkings method.

indicated that the acoustic predictions were found to be insensitive to the radial location of the integral surfaces. Following Shur et al., end caps are used to accurately close the Ffowcs Williams and Hawkings surface downstream. Sixteen end caps are used for $30 \leq x/r_j \leq 40$. The upstream end is closed by the nozzle wall and thus no end caps are required. Solutions are sampled on the integral surface at a rate of $St_D \approx 21.6$. The maximum resolved frequency is estimated as $St_D \approx 1.54$ by computing the grid Strouhal number.

IV.B. Baseline clean-nozzle simulation

Figure 9 shows the comparisons of time-averaged axial velocity and fluctuating axial velocity rms with the particle image velocimetry (PIV) measurement along the jet centerline and the nozzle lipline, respectively. The “consensus” dataset (i.e. a weighted average of the as-measured PIV data to account for uncertainties) is compared with the current LES prediction. Also shown in Figure 9a is the measurement data obtained for the the Acoustic Research Nozzle (ARN) 2. The ARN2 nozzle is a converging nozzle with the same exit diameter as the current ASME-standard nozzle. The contracting section is 3 times longer and the area contraction ratio is 2 times larger than the current nozzle. Also, the internal contour is slightly different. Agreement with both PIV datasets is good along the jet centerline and the nozzle lipline. Flows within the current ASME-standard nozzle undergo a strong acceleration and the boundary layer is likely to be relaminarized (see, for example, Mi et al.). This is also the case for the ARN-type nozzles, which can presumably explain its slightly better agreement than the consensus dataset in Figure 9a. As a result, velocity fluctuations at the current nozzle exit are much lower compared to the experiment, as shown in Figure 9a. This causes the nozzle-exit boundary layer to undergo rapid transition to turbulence, substantiated by a peak in $u'_x \text{rms}$ near the nozzle exit in Figure 9b. Artificial inflow turbulence based upon the digital filtering technique was tested to improve the agreement with the fluctuation levels at the nozzle exit; however, due to the strong contraction of the current nozzle, even unrealistically strong fluctuations showed negligible improvement.

Radial velocity profiles at several streamwise locations compare well with the consensus PIV measurement, as illustrated in Figures 10a and b. At the nozzle exit, time-averaged velocity has a nearly top-hat profile and the computed boundary-layer thickness corresponds to 7 cells. As shown in Figure 10b, fluctuation levels at the nozzle exit are as low as 0.3% of $u_J$ and their maximum value is less than 5% near the nozzle wall.

Figure 11a shows sound directivity at $72D_f$ from the nozzle exit. Also shown are the measurement data of Tanna and Brown & Bridges. Agreement is good within 2 dB over the angles considered in this study. At sideline and upstream angles ($\varphi \gtrsim 90^\circ$), the measurement of Tanna shows overprediction by 2 to 3 dB, which is attributed to the very large ($\approx 36$) area contraction ratio of the nozzle. Figure 11b shows sound pressure levels (SPL) at three radiation angles. Only the results up to the estimated grid Strouhal
number $St_D \approx 1.54$ are shown. At $\varphi = 30^\circ$, SPL is well predicted over the computed frequency range. As $\varphi$ increases in the upstream direction, sound at higher frequencies becomes overpredicted by 2 to 3 dB at $St_D \gtrsim 0.5$. It was confirmed that varying the radial locations of Ffowcs Williams and Hawkings surfaces gives no improvement in Figures 11a and b.

Presumably, the overprediction at the sideline and upstream angles in Figure 11b is due to insufficient grid resolution near the nozzle exit. As stated before, the ASME-standard nozzle used in this study has a rapid area contraction and thus, the nozzle-exit boundary layer becomes very thin and presumably laminar. As illustrated in Figure 10a, the nozzle-exit velocity profile is similar with a top-hat profile and the computed boundary layer thickness is resolved by only 7 cells. The equilibrium wall model appears to be less effective in reducing the resolution requirement due to the local boundary-layer state. As a result, the shear layer downstream of the nozzle exit is essentially that of a plug flow, which becomes unstable and undergoes rapid transition to turbulence, as discussed in the context of Figure 9b. It is well known that abrupt transition to turbulence in the shear layer can generate significant sound (see Zaman and Bogey & Bailly, for example). Figure 12 shows evidence that successive vortex merging occurs at $x/r_J \approx 1$ and transition to turbulence takes place downstream.
In addition, it is worthwhile mentioning the work of Viswanathan & Clark, who studied the effects of nozzle internal contour. Their finding was that among three different converging nozzles having the same nozzle-exit diameter and the same operating condition, the ASME-standard nozzle (also used in this study) generates more sound. They observed an increased SPL (∼ 3 dB) at the sideline and upstream radiation angles, especially at higher frequencies than the spectral peaks. This trend becomes more pronounced for heated jets. They argued that different nozzle internal contours produce distinctly different boundary-layer characteristics at the nozzle exit, thereby affecting the early development of the mixing layer. Their Figure 12 shows similar 2 to 3 dB more sound for $St_D \gtrsim 0.5$ at the sideline and upstream angles as the current jet (see Figure 11b). Although the current overprediction in SPL may not be entirely accounted for based upon their argument, it is expected that the current jet is likely to be louder than Brown & Bridges at sideline and upstream angles. An additional grid-refinement study near the nozzle exit will be able to resolve this issue.

IV.C. Forced jet simulation

In this section, the response of the baseline jet to the fluctuations from the turbine stage is investigated. This study is intended to demonstrate that the overall coupling procedure works, and a more quantitative study of core-noise generation and propagation mechanisms will be conducted in a future study.

The modally-decomposed fluctuations from the turbine state are prescribed as a reference state for the
buffer zone near the nozzle inlet. Instantaneous temperature contours for the baseline, clean nozzle and forced jet simulations are compared in Figures 13a and b. The prescribed fluctuations convect and decay to some extent within the pipe. Downstream of the nozzle exit, the forced jet spreads faster than the baseline unforced jet, and its potential core breaks down earlier.

Figure 14a shows that, along the centerline of the jet, the streamwise velocity of the forced jet starts to decay slightly earlier than the baseline jet. Fluctuation levels within the potential core are substantially enhanced and comparable with the experiment at the nozzle exit. Along the nozzle lipline, no significant changes are observed, as shown in Figure 14b.

The impact of the forcing is more significant on sound radiation. Figure 15 shows that the forcing amplifies sound nearly uniformly by 2 dB over the radiation angles, while OASPL for $\varphi \lesssim 35^\circ$ and $60^\circ \lesssim \varphi \lesssim 80^\circ$ shows an increase by approximately 5 dB. Sound pressure levels at several radiation angles are shown in Figures 16a through 16f. At the downstream angles ($\varphi < 90^\circ$), an increase in SPL over all frequencies is observed. At $\varphi = 30^\circ$, a more than 10 dB increase is obtained for $0.2 \lesssim St_D \lesssim 1.0$. The SPL amplifications are not broadbanded and rather tonal, which are presumably associated with the characteristics of the forcing shown in Figure 3. The maximum amplification is 17 dB at $St_D = 0.7$. This suggests that core-noise
propagation strongly affects jet-noise generating mechanisms. It is known that directly transmitted pressure fluctuations from the engine core are refracted and most intense at $50^\circ \lesssim \varphi \lesssim 80^\circ$, which can also be seen in Figures 15 and 16c. Thus, the amplified sound at $\varphi = 30^\circ$ in Figure 16a is more likely to be caused by stronger jet-noise generation mechanisms, rather than core-noise propagation. This is, in fact, not surprising considering the substantial magnitude of the inflow forcing demonstrated in section III.

In Figures 16a through d, it is interesting to observe consistent increases in SPL by 6 to 10 dB at $St_D \lesssim 0.03$ ($\approx 300$ Hz), which is not found in the upstream direction (see Figures 16e and f). This observation appears to be aligned with the conventional definition of core-noise propagation and will be examined in a future study.

![Figure 15: Sound directivity at $d/D_J = 72$ for unforced and forced jets.](image)

V. Summary and future work

A hybrid modeling approach is proposed to predict the engine core noise from a modeled gas-turbine engine and to assess its receptivity to modeled fluctuations. The modeled core-noise system consists of combustor, single-stage turbine, converging nozzle, and free-field radiation to the acoustic far field. The computational strategy for the generation and propagation of turbulent fluctuations from the combustor to the nozzle exhaust is developed. In this paper, modeling tools for the individual components are separately developed and tested with a simple one-way coupling procedure.

A compressible reacting large-eddy simulation (LES) is performed for flows within the modeled gas-turbine combustor. Fluctuation data are collected at the combustor exit. Proper orthogonal decomposition (POD) is applied to decompose the fluctuation data for an efficient downstream coupling with the turbine stage. The effects of the turbine stage on the fluctuating fields are simulated using the semi-analytic actuator disk theory (ADT) technique to estimate the fluctuations that would be seen at the exhaust-nozzle entrance. These fluctuations are then used to force a subsonic heated jet and to assess the changes induced by the upstream perturbations on far-field acoustic radiation.

High-fidelity simulation of the high-temperature jet exhaust flow is conducted using the combined LES and acoustic analogy based upon the Ffowcs Williams and Hawkings formulation. Good agreement is obtained for jet turbulence and sound radiation. Some discrepancies remain for the upstream propagating sound at higher frequencies. The strongly contracting converging nozzle and less-than-ideal grid resolution for the nozzle-exit boundary layer appear to cause overprediction in SPL. Additional grid refinement study is expected to resolve the discrepancies.
The baseline jet is perturbed by the fluctuations generated by the POD–ADT technique to model the effects of the turbine stage. Fluctuation levels at the nozzle exit are comparable with experiments. The forced jet decays faster than the baseline jet, and far-field sound is increased almost uniformly over the radiation angles. Maximum amplifications are obtained at $\varphi = 30^\circ$ where jet noise is dominant and $\varphi = 70^\circ$ where direct transmission is significant. Sound spectra show that the prescribed forcing significantly enhances jet noise radiation.

Future work will focus on more detailed analysis of the effects of upstream perturbations on downstream noise, including attempts to assess the relative importance of direct and indirect core noise. More sophisticated coupling techniques and a higher fidelity model for the turbine stage will also be pursued.

Acknowledgments

The first author acknowledges the Stanford Graduate Fellowship program for continued support of this work. The authors are grateful to Prof. Sanjiva Lele for useful discussions. The authors also thank Dr. James Bridges at the NASA Glenn Research Center for sharing the PIV and acoustic measurement data for validation.

The authors acknowledge the following award for providing computing and visualization resources that have contributed to the research results reported within this paper: MRI-R2: Acquisition of a Hybrid CPU/GPU and Visualization Cluster for Multidisciplinary Studies in Transport Physics with Uncertainty Quantification (http://www.nsf.gov/awardsearch/showAward.do?AwardNumber=0960306). This award is funded under the American Recovery and Reinvestment Act of 2009 (Public Law 111-5). Additional computing resources were provided by the Argonne National Laboratory through the ASCR Leadership Computing Challenge.

References


Figure 16: Sound pressure levels at (a) $\varphi = 30^\circ$, (b) $50^\circ$, (c) $70^\circ$, (d) $90^\circ$, (e) $110^\circ$, and (f) $130^\circ$. Measurement is made at $d/D_J = 72$. 