Persistent currents in normal metal rings - Auxiliary material

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ADDITIONAL DATA AND CONSISTENCY CHECKS

The most important evidence that the oscillatory signal found in the $R = 0.67$ μm rings does indeed reflect persistent currents is the absence of a similar signal in $R = 1$ μm rings. Data from both kinds of rings are shown in Auxiliary Fig. 1, before and after eliminating the zero field anomaly. Since the latter appeared to vary more from ring to ring for the larger rings, we subtracted a fitted phenomenological model of the form $a_1 \tanh(a_2\Phi_a - a_3 \mathrm{sgn}(d\Phi_a/dt)) + \text{ellipse}$, rather than the mean response from each dataset. Comparison of the datasets in Auxiliary Fig. 1(b) with the same data with the mean response subtracted [Fig. 2(b), datasets 1 to 5 and 10] shows that both procedures give a similar overall picture, even though the specific result for some of the rings changes. The selection of datasets from Fig. 2 is entirely historical and does not reflect an attempt to make the comparison more convincing.

The larger rings are expected to have a slightly larger electron temperature because they are coupled more strongly to the SQUID, and have a smaller E_c of 170 mK. This reduces the expected value of $M\langle I_{h/e}^2\rangle^{1/2}$ to about 0.06 $\mu\Phi_0$, and dephasing effects may lead to a further reduction. Even though the 1 μm rings do not show clear oscillations, we have fitted the curves in Auxiliary Fig. 1(d) with sine curves in order to estimate the amplitude of an oscillatory response that would be consistent with the data. Fits with the period variable and fixed at the expected value of 14 G give $M\langle I_{h/e}^2\rangle^{1/2} = 0.03$ $\mu\Phi_0$ and 0.024 $\mu\Phi_0$, respectively.

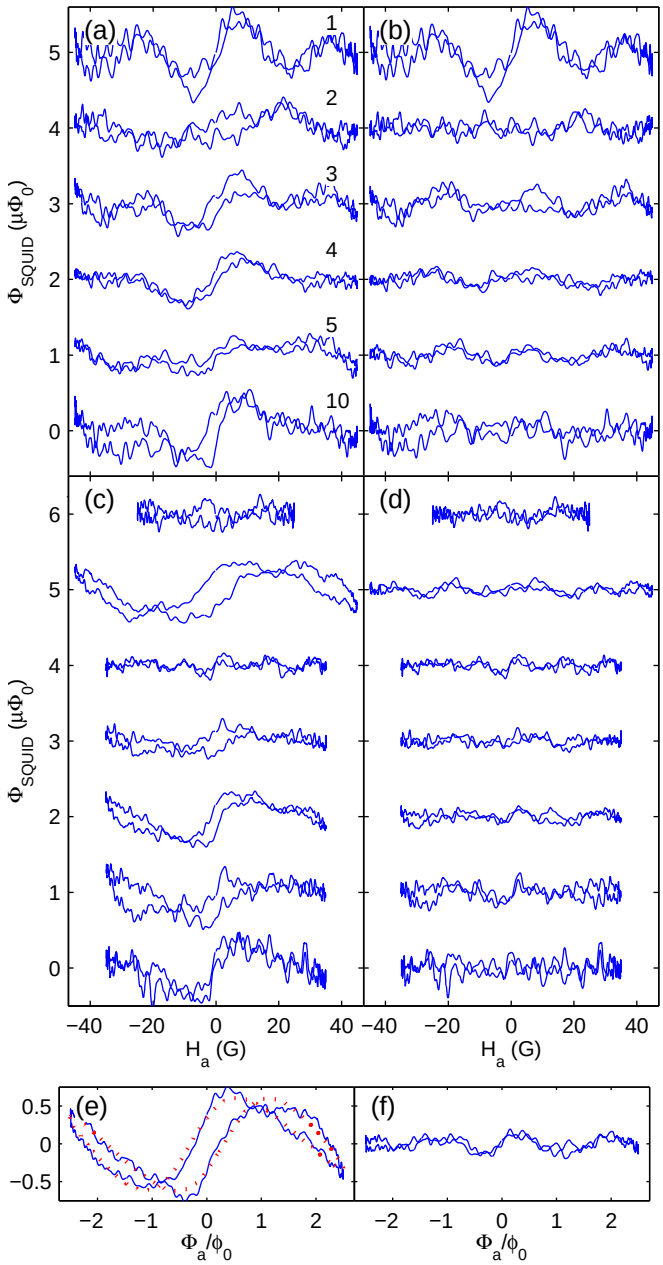
Subtracting fitted steps from the response of four heat-sunk rings results in some aperiodic residuals mostly near zero field, with an amplitude of less than 0.2 $\mu\Phi_0$. Presumably because of the lower electron temperature, the step-feature was more pronounced in those rings than in isolated ones, and it varied considerably from ring to ring. Representative data are shown in Auxiliary Fig. 1(e), (f). The absence of a periodic response might be due to the coherent penetration of the electronic wave functions into the heat sinks, which capture a substantial amount of flux, but do not enclose a well defined area. Thus, both a persistent-current-like orbital response and an imperfect elimination of the step feature may contribute to the aperiodic residual signal.

Auxiliary Fig. 2 shows data from $R = 0.57$ μm rings, which give a similar picture as the $R = 0.67$ μm rings shown in Fig. 2. Fitting sine curves to the data in Auxiliary Fig. 2(b) gives typical amplitudes of 0.07 and 0.06 $\mu\Phi_0$ for variable and fixed period, respectively. The difference from the theoretical value of 0.13 $\mu\Phi_0$ is not statistically significant.

We have repeated the measurements of two $R = 0.67$ μm rings with large oscillatory signals of opposite sign at different field sweep frequencies and amplitudes. The data in Auxiliary Fig. 3(a), (b) show that the response of each ring changes as a function of frequency. The difference between the two responses at the same frequencies on the other hand only show minor variations [Auxiliary Fig. 3(c)]. Computing this difference eliminates any background signal that is the same in both rings. The zero field anomaly is smaller at lower frequency, consistent with the hypothesis that it is a nonequilibrium effect in the spin-response, for which millisecond relaxation times are quite plausible at the low measurement temperatures [1]. The orbital response on the other hand is expected to be governed by the nanosecond-scale electronic relaxation times, and should thus be frequency independent at the much lower measurement frequency. The consistency of the data with those expectations supports the conclusion that the oscillatory component is not related to the zero field anomaly. There may also be a slight amplitude dependence of the step feature, however it is much less pronounced than the frequency dependence, so that we only show the difference data in Auxiliary Fig. 3(d). It is consistent with the response at any given field being independent of the field sweep amplitude.

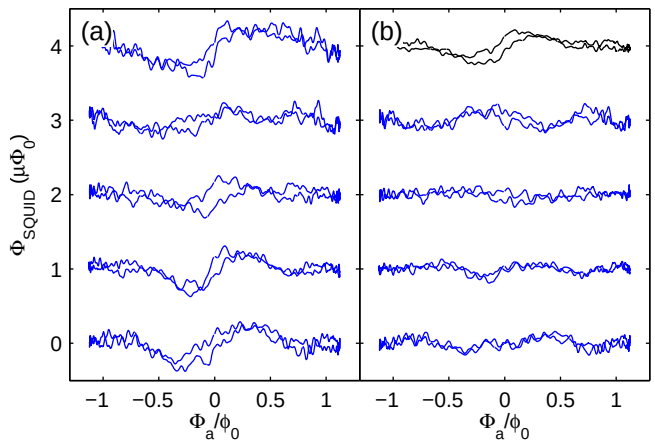
DISCUSSION OF THE ZERO-FIELD ANOMALY

The origin of the step-like feature in the nonlinear response is currently not well understood. It appears that the most likely explanation is a nonequilibrium effect of the spin response. Experimentally, the nonequilibrium nature of the zero field anomaly is supported by the splitting between the up and down sweep, seen for example in Auxiliary Fig. 1(a), (c) and (e), and the frequency dependence discussed in the previous section. While there is no direct evidence for the step-feature being caused by spins, it seems natural to suspect a connection to the observed spin-like susceptibility. The latter had a measurable out-of-phase component [2], which indicates that the linear response is affected by nonequilibrium effects as well.

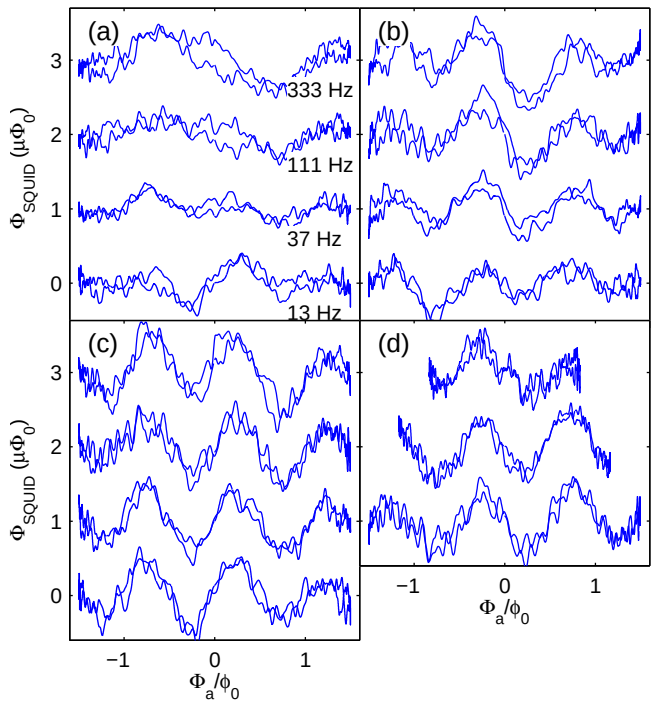


Auxiliary Fig. 1: (a)-(d) Comparison of data from rings with $R = 0.67$ μm [(a), (b)], rings 1 to 5 and 10 from Fig. 2) and $R = 1$ μm [(c), (d)]. The top three datasets in (c) and (d) are from the same rings, but were taken at different field ranges. The left panels (a), (c) show data from which only the linear component (in- and out-of-phase) has been subtracted. The right panels [(b), (d)] show the same data after subtracting the phenomenological step model explained in the text. None of the 1 μm rings shows oscillations comparable to those seen in the 0.67 μm rings. (e) Data from a representative heatsunk ring after subtracting the in- and out-of-phase component of the linear response. (f) Result of subtracting the fitted model (dotted line in (e) from the data in (f).

One may wonder if an orbital response might also contribute to the zero field anomaly. Such an effect could be



Auxiliary Fig. 2: (a) Nonlinear response of five $R = 0.57$ μm rings. (b) Mean of the lower four curves shown in (a) (top, black curve) and results of subtracting their mean from each of them. The topmost dataset from (a) was excluded because it shows a substantially larger zero field anomaly.



Auxiliary Fig. 3: (a), (b) Data from the two $R = 0.67$ μm rings with a strong oscillatory signal at field sweep frequencies 13, 37, 111 and 333 Hz. (c) Difference between data in (a) and (b). While the raw data changes noticeably, the difference remains nearly constant. (d) Difference between the responses of the same two rings at field sweep amplitudes 45, 35 and 25 G, taken at 111 Hz. Note that the fitted linear component that was subtracted is not the same for all datasets and leaves a residual slope for the topmost dataset.

closely related to persistent currents, just as weak localization effects in wires are related to Aharonov-Bohm oscillations in the conductance of mesoscopic rings. While a ring geometry leads to an oscillatory response with a well defined period, the superposition of contributions with a wide range of periods from a geometry without a characteristic area would lead to a total response that is most pronounced near zero field. The mostly consistent sign of the zero-field anomaly would imply that one would have to look for an effect with a finite ensemble average, such as the $h/2e$ component of persistent currents. The relatively large mean amplitude of the zero-field anomaly, which is comparable to the (expected and measured) typical persistent current signal, seems difficult to explain theoretically in terms of an orbital response. However, given the similarly large $\langle I_{h/2e}\rangle$ response seen in previous ensemble measurements [3–6], the current theoretical understanding might be incomplete. An important experimental indication is the presence of hysteresis and a frequency dependence at sub-kHz frequencies. Such effects are rather unlikely for any orbital mechanism, which should be governed by the much faster electronic relaxation times. Nevertheless, it is possible that a nonequilibrium spin response masks a comparable or smaller orbital contribution.

Since both weak localization corrections to the conductance of wires and ensemble averages of persistent currents depend on the Cooperon, it is reasonable to estimate the field range over which an orbital response would decay by comparison with the magnetoresistance due to weak localization. The resistance change of a one dimensional wire (in the absence of spin-orbit coupling) as a function of magnetic field is proportional to $(1+(2\pi L_\phi wB/\sqrt{3}\phi_0)^2)^{-1/2}$. Thus, its characteristic field scale corresponds to about 0.3 ϕ_0 captured in a wire section of length L_ϕ. Replacing L_ϕ with the circumference $L = 4.3$ μm, this corresponds to a field of about 10 G, which is about twice as large as the half-width of the zero field anomaly. Given that the Cooperon may enter the expressions for the weak localization effect and the orbital response in different ways, one cannot quite rule out an orbital contribution based on the field scale of the anomaly.

EFFECT OF JOSEPHSON OSCILLATIONS IN THE SQUID

In order to prevent effects of thermal or technical radiation on the sample, all electrical lines leading into the copper shield containing the sample and scanner assembly were heavily filtered above 10 MHz [7]. However, these filters cannot eliminate the microwave frequency flux coupled into the sample rings from an ac current in the SQUID pickup loop. This oscillating current is generated by Josephson oscillations in the SQUID junctions. For a typical SQUID bias voltage of 20 μV, it would have a frequency of $\omega/2\pi = 20$ μV$/\Phi_0 = 10$ GHz, and its amplitude should be on the order of the critical current, i.e. 10 μA. This corresponds to a flux Φ_{ac} of a few mΦ_0 in a typical sample ring. A simple estimate of the power P dissipated in the ring as $P = (\omega\Phi_{ac})^2/2R_{ring}$, where $R_{ring} \sim 1$ Ω is the resistance of the ring, leads to $P \sim 10^{-14}$ W, which would heat the electrons in the ring to 100 - 200 mK if the cooling is assumed to be limited by electron-phonon coupling, for which one expects $T = (P/wdL\Sigma)^{1/5}$, with $\Sigma \approx 10^9$ W/(m^{3}K^5). Experimentally, the saturation of the spin-response from our isolated rings below about 150 mK [Fig. 1(c)] is consistent with this heating estimate. The different behavior of heatsunk rings indicates that the linear susceptibility does reflect the electron temperature rather than the phonon temperature.

One may also ask whether the high frequency flux can lead to nonequilibrium effects that cannot be accounted for with an elevated electron temperature. Even without causing heating, an ac electric field E at frequency $\omega/2\pi$ can have a dephasing-like effect, which is characterized by a phase randomization rate $\tau_{MW}^{-1} \approx \alpha\omega$ for $\alpha \equiv 2e^2DE^2/\hbar^2\omega^3 \ll 1$ [8, 9]. For typical ring and SQUID parameters, this leads to $\tau_{MW} \sim 1$ μs, which is much longer than the Nyquist noise induced dephasing time of about 3 ns and should thus not be important.

Kravtsov and Altsuler [10] predicted a direct relation between a contribution to $\langle I_{h/2e}\rangle$ from nonequilibrium noise, and the excess dephasing caused by the same noise in long wires. They find that $\langle I_{h/2e}\rangle = C(e/\tau_\phi)e^{-L/2L_\phi}$ with $|C| \approx 1$, and propose that nonequilibrium noise might provide a common explanation for both the large persistent currents seen in experiments and the excess dephasing often observed in weak localization measurements [11]. (In the meantime, the latter has been shown to be due most likely to small concentrations of magnetic impurities [12, 13].) Combining this result with the above estimate of an excess dephasing rate of 1 μs^{-1}, one obtains a sub-pA persistent current caused by the Josephson oscillations, much smaller than the measured values. This further supports the conclusion that effects of the Josephson oscillations other than heating are negligible in our experiment.

VARIATIONS OF THE PERIOD

The variations in the fitted period of the data in Fig. 2 raise the question whether persistent currents in rings with a finite line width could exhibit fluctuations in the period. A simple model based on estimating the variation of the area enclosed by diffusive semi-classical paths around the ring indicates that periods corresponding to effective radii between the inner and the outer radius are indeed quite plausible. A similar effect was observed in

a previous measurement on an ensemble of 30 rings [14], where the oscillation amplitude decreased beyond a few periods from zero applied field. Thus, the deviations from the period corresponding to the mean radius, which may also partly be due to an imperfect background elimination, are consistent with the interpretation of our data as persistent currents.

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[1] R. H. Koch, D. P. DiVincenzo, and J. Clarke, Phys. Rev. Lett. **98**, 267003 (2007).
[2] H. Bluhm, N. C. Koshnick, J. Bert, M. E. Huber, and K. A. Moler, (unpublished).
[3] L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[4] B. Reulet, M. Ramin, H. Bouchiat, and D. Mailly, Phys. Rev. Lett. **75**, 124 (1995).
[5] R. Deblock, Y. Noat, B. Reulet, H. Bouchiat, and D. Mailly, Phys. Rev. B **65**, 075301 (2002).
[6] R. Deblock, R. Bel, B. Reulet, H. Bouchiat, and D. Mailly, Phys. Rev. Lett. **89**, 206803 (2002).
[7] H. Bluhm and K. A. Moler, Rev. Sci. Inst. **79**, 014703 (pages 6) (2008).
[8] B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, JETP Lett. **33**, 94 (1981).
[9] J. Wei, S. Pereverzev, and M. E. Gershenson, Phys. Rev. Lett. **96**, 086801 (2006).
[10] V. E. Kravtsov and B. L. Altshuler, Phys. Rev. Lett. **84**, 3394 (2000).
[11] P. Mohanty and R. A. Webb, Phys. Rev. B **55**, R13452 (1997).
[12] F. Pierre, A. B. Gougam, A. Anthore, H. Pothier, D. Esteve, and N. O. Birge, Phys. Rev. B **68**, 085413 (2003).
[13] N. O. Birge and F. Pierre, Phys. Rev. Lett. **89**, 206804 (2002).
[14] E. M. Q. Jariwala, P. Mohanty, M. B. Ketchen, and R. A. Webb, Phys. Rev. Lett. **86**, 1594 (2001).