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Enhancing Superradiance in Spectrally Inhomogeneous Cavity QED Systems with Dynamic Modulation

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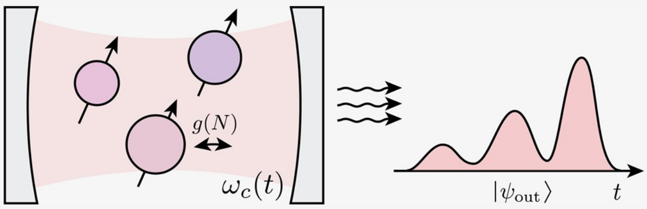
ABSTRACT: The Tavis−Cummings system, wherein emitters collectively interact with the same optical mode, has been a long-standing model to study collective photon emission phenomena such as superradiance. However, disorder in the resonant frequencies of the quantum emitters can perturb these effects. In this paper, we study the impact of dynamic modulation on the interplay between superradiance and spectral disorder. Through numerical simulations and analytical calculations, we provide evidence that the effective cooperativity of the superradiant mode can be multiplicatively enhanced with a quantum control protocol modulating the resonant frequency of the optical mode. Our results are relevant to experimental demonstration of superradiance in solid-state quantum optical systems, wherein the spectral disorder is a signficant technological impediment toward achieving photon-mediated emitter−emitter couplings.



KEYWORDS: *quantum optics, cavity quantum electrodynamics, light−matter interaction, superradiance, optimization*

INTRODUCTION

Coherent interaction between a collection of quantum emitters and an optical mode, theoretically described by the Tavis−Cummings model,[1] has been a topic of intense theoretical interest since the conception of quantum optics. Multiple emitters coupling strongly to the same optical mode are known to exhibit cooperative effects, the most prominent of which is the formation of a superradiant state.[2−5] Superradiant states are fully symmetric collective excitations of the multiple emitters whose interaction with the optical mode is enhanced due to a constructive interference of the individually emitted photons. These states underlie the physical phenomena of collective spontaneous emission (Dicke superradiance)[5−8] and superradiant phase transitions.[9−11] Furthermore, the enhancement of light−matter interaction in such systems has implications for design and implementation of a number of quantum information processing blocks, such as transducers,[12] memories,[13] and nonclassical light sources.[14]

However, technologically relevant experimental systems that can potentially demonstrate and use superradiance, such as solid-state quantum optical systems like quantum dots,[15,16] color centers,[17] and rare-earth ions,[18] often suffer from spectral disorder among the quantum emitters. Since spectral disorder can disrupt interactions between emitters, it competes with the collective emitter−optical mode interaction and can prevent the formation of the superradiant state. This interplay between disorder and coherent interaction has been extensively studied in time-independent quantum systems arising in many-body physics[19−26] as well as quantum optics.[27−38] There has been recent interest in understanding the impact of dynamical modulation, applied globally on all the emitters, on the properties of such models. Such modulation schemes are easily experimentally accessible, for example, in quantum optical systems with a dynamically modulated optical mode (such as modulation of the resonant frequency of a cavity[39]) or with collectively modulated emitters (through simultaneously laser driving or stark shifting the resonant frequencies of all emitters[40,41]). These global quantum controls designed with off-the-shelf optimization techniques[42,43] have been used to potentially compensate for, or in some cases exploit, the spectral disorder[44,45] for building quantum information processing hardware such as quantum transducers and memories. However, an understanding of their effectiveness in compensating disorder, specially in the thermodynamic limit of a large number of emitters, is less well understood.

In this paper we study the interplay of superradiance and spectral disorder in a dynamically modulated Tavis−Cummings model. Within the single-photon subspace, the all-to-all coupling between the emitters enables the formation of a superradiant state over an extensive number of emitters, irrespective of the extent of disorder in the system. We study the impact of the dynamical modulation, designed as a quantum control to compensate for the spectral disorder in the

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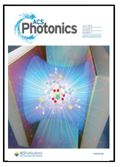

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system, on the formation of this superradiant state. We demonstrate that this dynamical modulation can achieve a multiplicative enhancement in the cooperativity of the superradiant state, even in the limit of a large number of emitters. Finally, we provide evidence that this control pulse, designed by only considering single-photon dynamics, also multiplicatively enhances superradiance in the multiphoton subspaces of the Tavis−Cummings model.

SYSTEM SETUP

We consider N emitters coupled to a cavity with the following Hamiltonian (Figure 1a):

$$H = \omega_c(t) a^\dagger a + H_e + H_c, \quad \text{where } H_e = \sum_{i=1}^{N} \omega_i \sigma_i^\dagger \sigma_i \quad \text{and}$$
$$H_c = g(N) \sum_{i=1}^{N} (a^\dagger \sigma_i + \sigma_i^\dagger a) \tag{1}$$

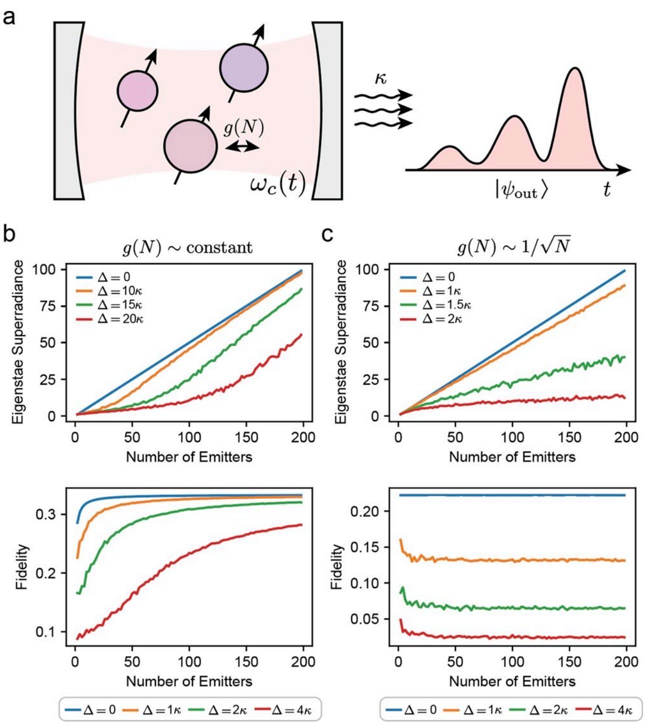

Figure 1. Setup and unmodulated system scaling. (a) We consider N emitters with frequencies ω_i coupled to a cavity with rate $g(N)$. The cavity is driven by a pulse that imparts a frequency shift $\omega_c(t)$ and is coupled to a waveguide with rate κ. (b) Single photon eigenstate superradiance and fidelity of the unmodulated ($\omega_c(t) = 0$) system in (a), with coupling strength $g = \kappa$ held constant. Single photon emission fidelity is calculated in the presence of external decay rate $\gamma = 2\kappa$. (c) Eigenstate superradiance and fidelity with g scaled as $g = \kappa/\sqrt{N}$. All plots show the average value from 25 different emitter ensembles with frequencies sampled from a uniform distribution.

where $\omega_c(t)$ is the time-dependent frequency of the cavity, ω_i is the resonant frequency of the ith emitter, $a^\dagger$ is the raising operator of the cavity, $\sigma_i^\dagger$ is the excitation operator of the ith emitter, and $g(N)$ is the emitter−cavity coupling strength (which we allow to depend on the number of emitters). Furthermore, we assume that the cavity emits into an output channel with decay rate κ, and the emitters, in addition to coupling to the cavity, individually decay with decay rate γ. We will denote the state of this model with no photons in the cavity and all emitters in their individual ground state, by $|G\rangle$.

To quantify the superradiant behavior of this system, we use two metrics: eigenstate superradiance and photon generation fidelity. The eigenstate superradiance (ES) is calculated as

$$\mu_{ES}[\omega_c(t)] = \max_{|\phi\rangle} \left| \langle G | \sum_{i=1}^{N} \sigma_i |\phi\rangle \right|^2 \tag{2}$$

where the maximization is done over all $|\phi\rangle$, the single-photon Floquet eigenstates of $H(t)$. This metric can be interpreted as a measure of the coupling between the (most) superradiant state and the ground state $|G\rangle$ induced by H_c. We point out that in the absence of disorder and with the emitters being on-resonance with the cavity mode, $\mu_{ES} = N/2$. Additionally, we calculate the photon generation fidelity; we initialize the emitters to an initial symmetric state (i.e., $\sum_{i=1}^{N} \sigma_i^\dagger |G\rangle/\sqrt{N}$), allow it to decay into the output channel through the cavity, and compute the probability of a photon being emitted into the output channel:

$$\mu_{FID}[\omega_c(t)] = \kappa \int_0^\infty \langle a^\dagger(t) a(t) \rangle \, dt \tag{3}$$

The eigenstate superradiance is an eigenstate property that captures the similarity of the dynamics of the system under study and a superradiant system and may be relevant to other phenomena involving superradiance, not necessarily just photon emission. On the other hand, the photon generation fidelity is a more practically relevant quantity of superradiance that is possible to measure experimentally.

RESULTS

We begin by studying the behavior of an unmodulated system ($\omega_c(t) = 0$) within the single-photon subspace as a function of N. Figure 1b considers a setting where the coupling strength between the cavity and emitters is independent of N (i.e., $g(N) \sim$ constant); in this case, within the single-photon subspace, $\|H_c\| \sim g(N)\sqrt{N} \sim \sqrt{N}$ and $\|H_e\| \sim$ constant. Hence, for large N, the dynamics of this system is completely dominated by H_c, and spectral disorder does not play any role. This is evidenced both by the eigenvalue superradiance approaching $N/2$ as $N \to \infty$ and the single-photon generation fidelity approaching the fidelity of the homogeneous system (Figure 1b). A more interesting setting, studied in Figure 1c, is where $g(N) \sim 1/\sqrt{N}$ so as to make the norms $\|H_c\|$ and $\|H_e\|$ comparable at large N. While it might seem unphysical to scale g down with N, typical g's and N's satisfy this condition (where $g\sqrt{N}$ is of the order of other frequency scales in the problem, like decay rate or disorder), so this setting is relevant. For example, experiments have demonstrated 10^0 to 10^1 color centers coupled to a cavity with a $g/\kappa \approx 10^{-1}$,[46] and 10^5 to 10^6 rare earth ions coupled to a cavity with $g/\kappa \approx 10^{-3}$.[47] In this case, while superradiance is not completely recovered as $N \to \infty$, we find that the eigenstate superradiance still scales as N; consequently, a superradiant state is still formed between an extensive number (but not all) of emitters. Likewise, the single photon generation fidelity approaches a nonzero constant as $N \to \infty$; since the coupling constant vanishes in this limit, this is only possible if an extensive number of emitters are cooperatively emitting into the cavity mode. The formation of a superradiant state over an extensive number of emitters stems from the all-to-all coupling between the emitters mediated by the cavity mode, making it fundamentally

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different from the impact of disorder in models with local interactions, wherein the number of emitters cooperatively interacting with each other would grow at most logarithmically with N.[19,48]

We next consider the impact of the dynamical modulation of cavity resonance $\omega_c(t)$ on superradiance. To find a modulation signal that best compensates for the spectral disorder, we maximize the single-photon generation fidelity (eq 3) with respect to $\omega_c(t)$. The optimized modulation signal is computed by using time-dependent scattering theory[49,50] together with adjoint-sensitivity analysis[51] (see Supporting Information for details and examples) to solve the maximization problem. Figure 2a shows the impact that applying this modulation has

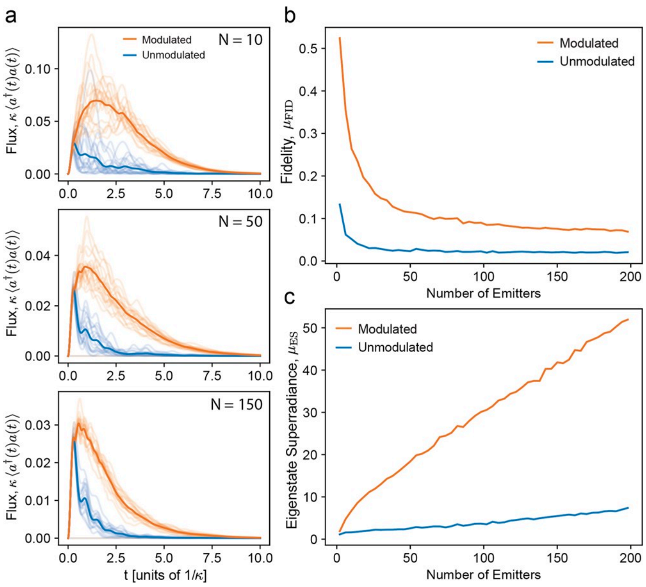

Figure 2. Pulse optimization. (a) Output photon flux before and after optimization for $N = 10$, 50, and 150, with $g = \kappa/\sqrt{N}$, $\Delta = 10\kappa$, and a constant external decay rate of 0.5κ. Transparent lines show the photon flux for each ensemble and solid lines show the average. (b) Optimized single photon emission fidelity scaling with N for $g = \kappa/\sqrt{N}$, $\Delta = 10\kappa$, and constant external decay rate $\gamma = 0.5\kappa$. (c) Optimized eigenstate superradiance scaling with N for $g = \kappa/\sqrt{N}$ and $\Delta = 10\kappa$. With these parameters, a homogeneous ensemble would exhibit an eigenstate superradiance of $N/2$. Plots show the average value from 25 different emitter ensembles drawn uniformly from $[-\Delta, \Delta]$.

on the photon emission rate ($\kappa\langle a^\dagger(t)a(t)\rangle$) into the output channel; we clearly see a significant sustained and consistent enhancement of photon flux due to the modulating signal. Furthermore, as is seen in Figure 2b,c, both the single-photon generation fidelity as well as the eigenstate superradiance (now computed with the eigenstate of the propagator corresponding to the duration for which the pulse is applied) are multiplicatively enhanced when compared to the unmodulated system. This occurs even in the presence of inhomogeneity in emitter coupling strength g (Supporting Information, Figure S2).

Surprisingly, a constant enhancement in the superradiance metrics persists, even in the limit of a large number of emitters, and improves with an increase in the spectral disorder. To confirm and provide an explanation of this behavior, we derive an effective analytical model for directly capturing the dynamics in the thermodynamic limit ($N \to \infty$). Assuming a Lorentzian distribution of the emitter frequency $[p(\omega_i) = \Delta_0/\pi(\omega_i^2 + \Delta_0^2)]$ and an external decay rate $\gamma = 0$, the single-excitation dynamics in the limit of $N \to \infty$ can be captured by a coupled oscillator model (Figure 3a):

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{\beta}(t) \end{bmatrix} = \begin{bmatrix} -(i\omega_c(t) + \kappa/2) & -G \\ G & -\Delta_0 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} - iG \begin{bmatrix} e^{-\Delta_0 t} \\ 0 \end{bmatrix} \quad (4)$$

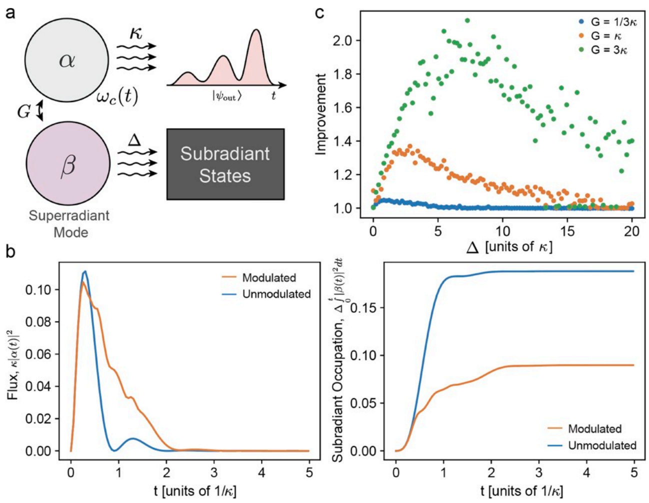

Figure 3. Large N model. (a) Effective model for the system presented in Figure 1a when emitter frequencies are drawn from a Lorentzian distribution and $N \to \infty$. (b) Output photon flux and subradiant occupation for the optimized large N model with $G = 3\kappa$ and $\Delta = 5\kappa$. (c) Large N optimization improvement scaling with Δ for $G = 1/3\kappa$, 1κ, and 3κ.

Here, $G = \lim_{N\to\infty} g(N)\sqrt{N}$, $\alpha(t)$ is the amplitude of the dynamically modulated cavity mode, and $\beta'(t) = \beta(t) + iS_{in}(t)$ is the amplitude of the superradiant mode, which directly couples to the cavity mode. A detailed derivation of these dynamics can be found in the Supporting Information. Note that the inhomogeneous broadening, Δ_0, effectively induces a decay in the superradiant mode due to its coupling to the subradiant states due to the inhomogeniety in the emitter frequencies. By optimizing the single photon generation fidelity with this effective model, we find an enhancement in superradiance (Figure 3b), consistent with the finite N results. We clearly see that the application of the modulation reduces the number of photons lost to the subradiant states and can be viewed as dynamically decoupling the subradiant states from the superradiant mode.[52] Figure 3c shows the dependence of the improvement in the photon-generation fidelity achieved by the modulation on the coupling strength G and the broadening Δ; as is intuitively expected, higher improvement in superradiance is seen for more strongly coupled systems. Furthermore, the improvement in single-photon generation initially grows with Δ, but tends back toward 1 as $\Delta \gg G$ as the decay into the subradiant bath dominates the system dynamics.

Finally, we study how superradiance within the multiphoton subspaces is impacted by the dynamical modulation. While it is in principle possible to recompute the modulation signal $\omega_c(t)$ by simulating an N excitation problem, where all the emitters are initially excited (demonstrated in the Supporting Information), the cost of performing this simulation increases exponentially with N. However, physical intuition suggests that

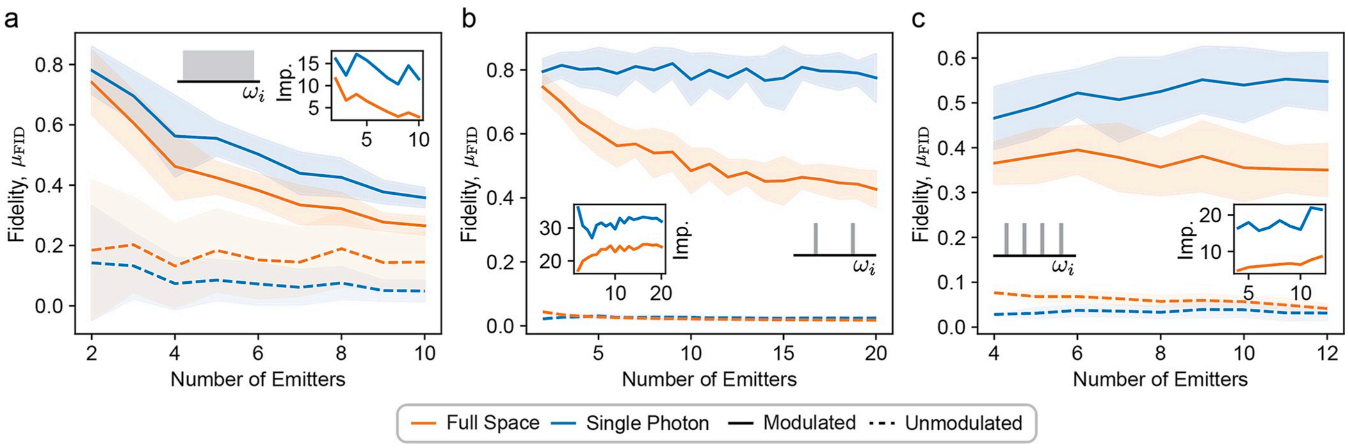

Figure 4. Multiphoton optimization. (a) Photon emission fidelity normalized to a homogeneous ensemble for the single-photon subspace and full space with emitter frequencies drawn from a uniform distribution. Modulation in both the single photon and full space are optimized in the single-photon subspace. Here $g = \kappa/\sqrt{N}$, $\Delta = 10\kappa$, and $\gamma = 0.5\kappa$. Filled area represents the standard deviation across 25 ensembles drawn uniformly from $[-\Delta, \Delta]$. Inset shows the average improvement (ratio of μ_{FID} with and without modulation averaged across ensembles) by modulating the system with the optimized pulse. (b) Photon emission fidelity and improvement (ratio of μ_{FID} with and without modulation), with emitter frequencies drawn from 2 bins and with standard deviation the same as in (a). (c) Photon emission fidelity and improvement with emitter frequencies drawn from 4 bins, with standard deviation the same as in (a).

the pulse obtained by maximizing the single-photon superradiance facilitates the transfer of excitations between the disordered ensemble of emitters and the cavity mode and, hence, should also enhance superradiance within the full $N-$excitation subspace. We see this effect in the photon emission fidelity in Figure 4a; we point out that the enhancement obtained for the $N-$excitation problem is smaller than that obtained for the single-particle problem, since the pulse designed within the single-photon space is conceivably suboptimal for the $N-$excitation problem.

An important question that arises for the $N-$excitation superradiance enhancement is whether it survives in the limit of large N. Since numerical simulations of the large N model become prohibitively expensive, we instead consider the emitter frequencies to be chosen randomly only from a discrete set (Figure 4b,c) as opposed to a continuous distribution. Exploiting the permutational invariance of emitters at the same frequency, this system can be simulated with a cost that scales polynomially in the number of emitters, but exponentially in the number of frequencies.[53−57] We expect the discrete probability distribution to at least qualitatively capture the properties of the continuous probability distribution, since it should, in principle, be possible to discretize a continuous probability distribution over the emitter frequencies into bins whose widths depend on the line width of the system. While we do not have rigorous proof of this statement, we numerically verify this for the problem of computing the fidelity metric in the Supporting Information. Figure 4b,c shows the dependence of the fidelity on N for 2 and 4 frequency problems; we find that the enhancement in superradiance on applying a pulse optimized in the single-photon subspace remains nearly constant with N in both cases, indicating that a multiplicative enhancement is possible even in $N-$excitation superradiance.

■ CONCLUSION

In conclusion, we have studied a dynamically modulated Tavis−Cummings Hamiltonian with spectral disorder and presented an analysis of the emergence of superradiance in this model. Our conclusions indicate that superradiance can persist and be potentially technologically useful, even in the limit of large spectral disorder, and that global quantum controls can be used to enhance it. These results are relevant to a number of ongoing experimental efforts in studying and scaling-up solid-state quantum optical systems. One of the most important and interesting problems left open in our work is scaling quantum control designs with experimentally realistic local or quasi-local controls to multiexcitation subspaces of systems with a large number of emitters. While we explored how controls designed with low excitation number subspaces can provide some improvement, even in the high excitation number subspaces, there might be the potential to design controls within the low entanglement spaces of the Hilbert space (i.e., states described by low bond dimension matrix product states), which have been investigated in the context of waveguide QED systems with spatial disorder.[33]

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acsphotonics.2c00581.

Numerical techniques, adjoint optimization, derivation of large-N model, and additional multiphoton optimization (PDF)

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Notes

The authors declare no competing financial interest.

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■ REFERENCES

(1) Tavis, M.; Cummings, F. W. Exact solution for an N-molecule—radiation-field Hamiltonian. *Phys. Rev.* **1968**, *170*, 379.

(2) Gross, M.; Haroche, S. Superradiance: An essay on the theory of collective spontaneous emission. *Physics reports* **1982**, *93*, 301−396.

(3) Rehler, N. E.; Eberly, J. H. Superradiance. *Phys. Rev. A* **1971**, *3*, 1735.

(4) Scully, M. O.; Svidzinsky, A. A. The super of superradiance. *Science* **2009**, *325*, 1510−1511.

(5) Dicke, R. H. Coherence in spontaneous radiation processes. *Physical review* **1954**, *93*, 99.

(6) Andreev, A. V.; Emel'yanov, V. I.; Il'inskiĭ, Y. A. Collective spontaneous emission (Dicke superradiance). *Soviet Physics Uspekhi* **1980**, *23*, 493.

(7) Clemens, J.; Horvath, L.; Sanders, B.; Carmichael, H. Collective spontaneous emission from a line of atoms. *Phys. Rev. A* **2003**, *68*, 023809.

(8) Eleuch, H.; Rotter, I. Open quantum systems and Dicke superradiance. *European Physical Journal D* **2014**, *68*, 1−16.

(9) Hepp, K.; Lieb, E. H. On the superradiant phase transition for molecules in a quantized radiation field: the Dicke maser model. *Annals of Physics* **1973**, *76*, 360−404.

(10) Lambert, N.; Emary, C.; Brandes, T. Entanglement and the phase transition in single-mode superradiance. *Physical review letters* **2004**, *92*, 073602.

(11) Bamba, M.; Inomata, K.; Nakamura, Y. Superradiant phase transition in a superconducting circuit in thermal equilibrium. *Physical review letters* **2016**, *117*, 173601.

(12) Casabone, B.; Friebe, K.; Brandstätter, B.; Schüppert, K.; Blatt, R.; Northup, T. Enhanced quantum interface with collective ion-cavity coupling. *Physical review letters* **2015**, *114*, 023602.

(13) Ortiz-Gutiérrez, L.; Muñoz-Martínez, L.; Barros, D.; Morales, J.; Moreira, R.; Alves, N.; Tieco, A.; Saldanha, P.; Felinto, D. Experimental fock-state superradiance. *Physical review letters* **2018**, *120*, 083603.

(14) Jahnke, F.; Gies, C.; Aßmann, M.; Bayer, M.; Leymann, H.; Foerster, A.; Wiersig, J.; Schneider, C.; Kamp, M.; Höfling, S. Giant photon bunching, superradiant pulse emission and excitation trapping in quantum-dot nanolasers. *Nat. Commun.* **2016**, *7*, 1−7.

(15) Scheibner, M.; Schmidt, T.; Worschech, L.; Forchel, A.; Bacher, G.; Passow, T.; Hommel, D. Superradiance of quantum dots. *Nat. Phys.* **2007**, *3*, 106−110.

(16) Grim, J. Q.; Bracker, A. S.; Zalalutdinov, M.; Carter, S. G.; Kozen, A. C.; Kim, M.; Kim, C. S.; Mlack, J. T.; Yakes, M.; Lee, B.; et al. Scalable in operando strain tuning in nanophotonic waveguides enabling three-quantum-dot superradiance. *Nature materials* **2019**, *18*, 963−969.

(17) Angerer, A.; Streltsov, K.; Astner, T.; Putz, S.; Sumiya, H.; Onoda, S.; Isoya, J.; Munro, W. J.; Nemoto, K.; Schmiedmayer, J.; et al. Superradiant emission from colour centres in diamond. *Nat. Phys.* **2018**, *14*, 1168−1172.

(18) Zhong, T.; Kindem, J. M.; Rochman, J.; Faraon, A. Interfacing broadband photonic qubits to on-chip cavity-protected rare-earth ensembles. *Nat. Commun.* **2017**, *8*, 1−7.

(19) Anderson, P. W. Absence of diffusion in certain random lattices. *Physical review* **1958**, *109*, 1492.

(20) Lee, P. A.; Fisher, D. S. Anderson localization in two dimensions. *Phys. Rev. Lett.* **1981**, *47*, 882.

(21) Alet, F.; Laflorencie, N. Many-body localization: An introduction and selected topics. *Comptes Rendus Physique* **2018**, *19*, 498−525.

(22) Nandkishore, R.; Huse, D. A. Many-body localization and thermalization in quantum statistical mechanics. *Annu. Rev. Condens. Matter Phys.* **2015**, *6*, 15−38.

(23) Pal, A.; Huse, D. A. Many-body localization phase transition. *Physical review b* **2010**, *82*, 174411.

(24) Nandkishore, R. M.; Sondhi, S. L. Many-body localization with long-range interactions. *Physical Review X* **2017**, *7*, 041021.

(25) Nag, S.; Garg, A. Many-body localization in the presence of long-range interactions and long-range hopping. *Phys. Rev. B* **2019**, *99*, 224203.

(26) Smith, J.; Lee, A.; Richerme, P.; Neyenhuis, B.; Hess, P. W.; Hauke, P.; Heyl, M.; Huse, D. A.; Monroe, C. Many-body localization in a quantum simulator with programmable random disorder. *Nat. Phys.* **2016**, *12*, 907−911.

(27) Moreira, N. A.; Kaiser, R.; Bachelard, R. Localization vs. subradiance in three-dimensional scattering of light. *EPL (Europhysics Letters)* **2019**, *127*, 54003.

(28) Akkermans, E.; Gero, A.; Kaiser, R. Photon localization and Dicke superradiance in atomic gases. *Physical review letters* **2008**, *101*, 103602.

(29) Ashhab, S.; Semba, K. Superradiance phase transition in the presence of parameter fluctuations. *Phys. Rev. A* **2017**, *95*, 053833.

(30) Biella, A.; Borgonovi, F.; Kaiser, R.; Celardo, G. Subradiant hybrid states in the open 3D Anderson-Dicke model. *EPL (Europhysics Letters)* **2013**, *103*, 57009.

(31) Cottier, F.; Kaiser, R.; Bachelard, R. Role of disorder in super- and subradiance of cold atomic clouds. *Phys. Rev. A* **2018**, *98*, 013622.

(32) Kelly, S. P.; Rey, A. M.; Marino, J. Effect of Active Photons on Dynamical Frustration in Cavity QED. *Phys. Rev. Lett.* **2021**, *126*, 133603.

(33) Manzoni, M. T.; Chang, D. E.; Douglas, J. S. Simulating quantum light propagation through atomic ensembles using matrix product states. *Nat. Commun.* **2017**, *8*, 1−11.

(34) Fayard, N.; Henriet, L.; Asenjo-Garcia, A.; Chang, D. Many-body localization in waveguide quantum electrodynamics. *Physical Review Research* **2021**, *3*, 033233.

(35) Temnov, V. V.; Woggon, U. Superradiance and subradiance in an inhomogeneously broadened ensemble of two-level systems coupled to a low-Q cavity. *Physical review letters* **2005**, *95*, 243602.

(36) Trivedi, R.; Radulaski, M.; Fischer, K. A.; Fan, S.; Vučković, J. Photon blockade in weakly driven cavity quantum electrodynamics systems with many emitters. *Physical review letters* **2019**, *122*, 243602.

(37) Mivehvar, F.; Piazza, F.; Ritsch, H. Disorder-driven density and spin self-ordering of a bose-einstein condensate in a cavity. *Physical review letters* **2017**, *119*, 063602.

(38) Debnath, K.; Zhang, Y.; Mølmer, K. Collective dynamics of inhomogeneously broadened emitters coupled to an optical cavity with narrow linewidth. *Phys. Rev. A* **2019**, *100*, 053821.

(39) Zhang, M.; Wang, C.; Hu, Y.; Shams-Ansari, A.; Ren, T.; Fan, S.; Lončar, M. Electronically programmable photonic molecule. *Nat. Photonics* **2019**, *13*, 36−40.

(40) Sun, S.; Zhang, J. L.; Fischer, K. A.; Burek, M. J.; Dory, C.; Lagoudakis, K. G.; Tzeng, Y.-K.; Radulaski, M.; Kelaita, Y.; Safavi-Naeini, A.; et al. Cavity-enhanced Raman emission from a single color center in a solid. *Physical review letters* **2018**, *121*, 083601.
(41) Lukin, D. M.; White, A. D.; Trivedi, R.; Guidry, M. A.; Morioka, N.; Babin, C.; Soykal, O. O.; Ul-Hassan, J.; Son, N. T.; Ohshima, T.; Vasireddy, P. K.; Nasr, M. H.; Sun, S.; MacLean, J.-P. W.; Dory, C.; Nanni, E. A.; Wrachtrup, J.; Kaiser, F.; Vuckovic, J.; et al. Spectrally reconfigurable quantum emitters enabled by optimized fast modulation. *npj Quantum Information* **2020**, *6*, 1−9.
(42) De Fouquieres, P.; Schirmer, S.; Glaser, S.; Kuprov, I. Second order gradient ascent pulse engineering. *J. Magn. Reson.* **2011**, *212*, 412−417.
(43) d'Alessandro, D. *Introduction to Quantum Control and Dynamics*; Chapman and Hall/CRC, 2007.
(44) Julsgaard, B.; Grezes, C.; Bertet, P.; Mølmer, K. Quantum memory for microwave photons in an inhomogeneously broadened spin ensemble. *Physical review letters* **2013**, *110*, 250503.
(45) Gorshkov, A. V.; Calarco, T.; Lukin, M. D.; Sørensen, A. S. Photon storage in Λ-type optically dense atomic media. IV. Optimal control using gradient ascent. *Phys. Rev. A* **2008**, *77*, 043806.
(46) Lukin, D. M.; Guidry, M. A.; Yang, J.; Ghezellou, M.; Mishra, S. D.; Abe, H.; Ohshima, T.; Ul-Hassan, J.; Vučković, J. Optical superradiance of a pair of color centers in an integrated silicon-carbide-on-insulator microresonator. *arXiv:2202.04845 [quant-ph]* **2022**, na.
(47) Kindem, J. M.; Ruskuc, A.; Bartholomew, J. G.; Rochman, J.; Huan, Y. Q.; Faraon, A. Control and single-shot readout of an ion embedded in a nanophotonic cavity. *Nature* **2020**, *580*, 201−204.
(48) Aizenman, M.; Molchanov, S. Localization at large disorder and at extreme energies: An elementary derivations. *Communications in Mathematical Physics* **1993**, *157*, 245−278.
(49) Trivedi, R.; Fischer, K.; Xu, S.; Fan, S.; Vuckovic, J. Few-photon scattering and emission from low-dimensional quantum systems. *Phys. Rev. B* **2018**, *98*, 144112.
(50) Fischer, K. A.; Trivedi, R.; Ramasesh, V.; Siddiqi, I.; Vučković, J. Scattering into one-dimensional waveguides from a coherently-driven quantum-optical system. *Quantum* **2018**, *2*, 69.
(51) Cao, Y.; Li, S.; Petzold, L.; Serban, R. Adjoint sensitivity analysis for differential-algebraic equations: The adjoint DAE system and its numerical solution. *SIAM journal on scientific computing* **2003**, *24*, 1076−1089.
(52) Viola, L.; Knill, E.; Lloyd, S. Dynamical decoupling of open quantum systems. *Phys. Rev. Lett.* **1999**, *82*, 2417.
(53) Chase, B. A.; Geremia, J. Collective processes of an ensemble of spin-1/ 2 particles. *Phys. Rev. A* **2008**, *78*, 052101.
(54) Xu, M.; Tieri, D.; Holland, M. Simulating open quantum systems by applying SU (4) to quantum master equations. *Phys. Rev. A* **2013**, *87*, 062101.
(55) Richter, M.; Gegg, M.; Theuerholz, T. S.; Knorr, A. Numerically exact solution of the many emitter-cavity laser problem: Application to the fully quantized spaser emission. *Phys. Rev. B* **2015**, *91*, 035306.
(56) Damanet, F.; Braun, D.; Martin, J. Cooperative spontaneous emission from indistinguishable atoms in arbitrary motional quantum states. *Phys. Rev. A* **2016**, *94*, 033838.
(57) Shammah, N.; Ahmed, S.; Lambert, N.; de Liberato, S.; Nori, F. Open quantum systems with local and collective incoherent processes: Efficient numerical simulations using permutational invariance. *Phys. Rev. A* **2018**, *98*, 063815.