Dispersion of Low Frequency Waves in a Hall Discharge

N. Gascon, E. Chesta, N. B. Meezan and M. A. Cappelli
Mechanical Engineering Department
Stanford University
Stanford, CA

37th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit
8-11 July 2001
Salt Lake City, Utah
Dispersion of Low Frequency Waves in a Hall Discharge

N. Gascon*, E. Chesta, N.B. Meezan*, and M.A. Cappelli†

Mechanical Engineering Department, Stanford University, Stanford, California 94305-3032

Experimental measurements are presented for studies of low frequency (< 100 kHz) oscillations in a cross-field closed-electron drift Hall discharge. Two azimuthally-placed electrostatic probes are used to identify and extract properties of coherent structures associated with the propagation of instabilities within the discharge channel. The azimuthal component phase velocities are determined for a wide range of wave frequencies and over characteristic regimes of operation of these discharges (e.g., from the ionization branch through current saturation). Two analyses of the data are presented. The first is based on conditional sampling of the probe data, either constraining all disturbances to propagate in the Hall direction or allowing bi-directional propagation. The second involves a wavelet analysis, allowing for bi-directional wave propagation. A variety of propagation modes are observed and analyzed, including the possible appearance of an induced mode due to the presence of the probes themselves. While the conditional sampling and wavelet analysis give consistently similar results for the dispersion curves, differences are apparent if the waves are constrained to propagate in the Hall direction, indicating that further experiments should be carried out with multiple probes (>2), to unambiguously resolve the directions of wave propagation.

I. INTRODUCTION

Closed electron-drift Hall discharges are low-pressure (~0.01 – 0.1 Pa), weakly collisional, magnetized plasma sources that generate a relatively high velocity ion beam suitable for use in space propulsion applications. A particular class of Hall plasma thrusters, the so-called “Stationary Plasma Thruster,” has been used in a number of space missions in the former Soviet Union [1]. A variation of this Hall thruster that has an annular (co-axial) discharge channel 100 mm in diameter – the SPT-100, has a high specific impulse (1100 – 2000 sec), operates at moderately high thrust levels (55 – 150 mN), and has an exceptionally high thrust efficiency (40-60%). Because of this performance, this plasma source is now being aggressively developed for use in station keeping applications on western satellites.

In a typical co-axial geometry Hall discharge, the plasma is sustained in imposed orthogonal electric and magnetic fields. The discharge electrons are magnetized, whereas the more massive propellant ions, usually xenon, are not. Consequently, the electrostatic fields established by the retarded electron flow accelerate the ions to high velocities, typically 50-60% of the discharge voltage (~100–300V). The maximum acceleration occurs in the region between the magnetic poles, where the magnetic field is a maximum. In a co-axial geometry, the electrons are constrained to move in the closed, azimuthal \( E \times B \) drift, with cross-field diffusion providing the necessary current to sustain the discharge. An annular ceramic channel confines the discharge. An annular ceramic channel confines the discharge. An annular ceramic channel confines the discharge. An annular ceramic channel confines the discharge. An annular ceramic channel confines the discharge.

It is widely known that the Hall discharge plasmas exhibit a rich spectrum of fluctuations in plasma properties [2]. While it is not yet known if and how these fluctuations can impact the performance of a Hall thruster, it is believed that fluctuations in the bulk plasma properties are partly responsible for anomalous electron transport across the imposed magnetic field [3]. While some studies characterizing the presence and origin of these fluctuations and their possible control were published in the mid 60’s to mid 70’s [3]-[9], they have received increased attention recently [10]-[13], as there is a growing need to extend and enhance the performance of these thrusters for a broader range of space missions.

Our research is motivated by the possibility of enhancing Hall thruster operation by the active control of these fluctuations, or by the passive suppression of the fluctuations in regions of the discharge channel where a reduction in electron current is desired. To do so, it is important that the nature of these fluctuations is adequately understood. In this paper, we present an experimental characterization of the propagation of these disturbances over a range of Hall discharge operation, and demonstrate that it is possible to affect the oscillations within the discharge by the insertion of biased probes.

† Associate Professor, Member AIAA

Copyright© 2001 by Stanford University. Published by the American Institute of Aeronautics and Astronautics, with permission.
II. EXPERIMENT

The Hall discharge plasma source used in this study is a laboratory version of a low-power Hall thruster, and is described in more detail in previous papers [14]-[16]. This particular source is intended to be used as a test bed for studying the discharge physics and not to serve as an operational prototype plasma accelerator, although the principal design is similar to that used in practice. The time-averaged plasma properties within the annular discharge channel have been extensively characterized for a range of operating conditions [14]. The source consists of an annular alumina channel, 90 mm in diameter, 11 mm in width, and 80 mm in length. A magnetic circuit consisting of four outer coils, one inner coil, and three iron plates provides a magnetic field (mostly radial in direction) peaked 5 mm upstream of the exit of the discharge channel. The radial component of the magnetic field is found to drop off by approximately 15% at the inner and outer walls of the acceleration channel, while axially it drops off to 50 Gauss within approximately 15 mm. Details of the two-dimensional magnetic field distribution for magnet currents used here can be found in [16]. A hollow stainless steel ring with 32 holes of 0.5 mm diameter serves both as the anode and the propellant (gas) input of the discharge. A commercial hollow cathode (Ion Tech HCN-252) is used to neutralize the resulting ion beam and provide the necessary electron current to sustain the discharge. The cathode is mounted such that its exit aperture is approximately 2 cm downstream of the plasma source exit. The cathode body was kept at the vacuum chamber ground potential.

For the measurements reported on here, the xenon flow rate was 2 mg/s and a constant solenoid winding current of 250 mA provided a peak magnetic induction of about 100 G at 5 mm upstream of the exit plane (inside the channel). The characteristics of the plasma fluctuations on centerline within the channel were studied at various discharge potentials.

Plasma density fluctuations were detected by two azimuthally-placed low-impedance Langmuir probes biased negatively with respect to the plasma potential to collect the ion saturation current, with the current density given as [17]:

\[ J_i = 0.61 n_e e \sqrt{\frac{kT_e}{m_i}} \]  

(1)

Here, \( T_e \) is the electron temperature, \( n_e \) is the electron number density, \( m_i \) is the xenon ion mass, \( e \) is the electron charge, and \( k \) is the Boltzmann constant. In Eqn. (1), it is assumed that the distant plasma is quasi-neutral \((n_e = n_i)\), and that the ions enter the collisionless sheath at the Bohm velocity. While the fluctuations in collected current can be a result of both fluctuations in electron density and temperature, in prior studies, it has been shown that these low frequency disturbances at low operating potential are largely isothermal [3],[7].

A schematic of the Langmuir probe construction is shown in Figure 1. The exposed part of the probe consisted of a 0.254 mm diameter, 3 mm long tungsten wire. The base of the probe was of a complex design, intended to minimize stray capacitance for extended frequency response. The probe base consisted of an alumina tube directly surrounding the tungsten wire (0.508 mm ID, 1.27 mm OD), followed by a stainless steel tube (1.48 mm ID, 1.58 mm OD), connected to the braided ground of a 50 ohm co-axial cable. The inner tungsten wire was connected to the center pin of the co-axial cable. This outer stainless steel tubing served as a shield to isolate the extended probe base from disturbances other than at the tip of the probe. The stainless steel tubing was then surrounded by another alumina tube (1.6 mm ID, 2.3 mm OD) and the entire inside air spaces were potted with an alumina paste.

Figure 1. Schematic of the Langmuir probes used to detect plasma density fluctuations.

Figure 2. Schematic of the experimental arrangement showing the location and orientation of the Langmuir probes.

A schematic of the Langmuir probe construction is shown in Figure 1. The exposed part of the probe consisted of a 0.254 mm diameter, 3 mm long tungsten wire. The base of the probe was of a complex design, intended to minimize stray capacitance for extended frequency response. The probe base consisted of an alumina tube directly surrounding the tungsten wire (0.508 mm ID, 1.27 mm OD), followed by a stainless steel tube (1.48 mm ID, 1.58 mm OD), connected to the braided ground of a 50 ohm co-axial cable. The inner tungsten wire was connected to the center pin of the co-axial cable. This outer stainless steel tubing served as a shield to isolate the extended probe base from disturbances other than at the tip of the probe. The stainless steel tubing was then surrounded by another alumina tube (1.6 mm ID, 2.3 mm OD) and the entire inside air spaces were potted with an alumina paste.
The probe size was minimized so as to reduce its overall capacitance, and for the studies reported on here, was approximately 10 cm in length. The frequency response of the probe was found to be excellent (gain equal to one, no phase shift, within measuring device accuracy) up to 1 MHz, the 3 dB cutoff being at 4 MHz. Coaxial transmission line feed-throughs provided the transfer of the probe signal through the vacuum chamber to a National Instruments DAQScope 5102 digital oscilloscope card operating at an 800 kHz sample rate. Data was stored on a personal computer for future processing. The probes were terminated with 50 Ω at the input of the oscilloscope card. In this way, the probe tips were negatively biased (close to system ground) with respect to the plasma, which is predominantly at high positive potential, since the anode is maintained at a minimum of 86 V, and since the near exit plane potential is always greater than 40 V due to the cathode fall and finite electron temperature. Occasional scans of the Langmuir probe across the electron retarding regime to regions of high negative bias verified that system ground potential was always in the ion current saturation regime.

The probes were oriented such that the exposed center wire axis was parallel to the axial coordinate of the thruster, and placed midway between the inner and outer insulating walls, as illustrated in Figure 2. Probe translation along the axial direction (the direction of the annular channel) was provided by a translation stage driven by a Slo-Syn stepper motor, powered and controlled by a Compumotor model SX Microstepping Drive/Indexer System. The azimuthal positions of the two probes were manually set before the beginning of each experiment, maintaining an equal angular distance from the two nearest outer magnetic coils. For this study, the probe separation was 30 degrees. In most cases, the axial probe locations varied between a distance of approximately 45 mm from the anode (x = -45 mm) and 10 mm beyond the exit plane (x = 10 mm), with the x = 0 reference taken to be the exit plane of the discharge, and negative positions implying that the probe locations are inside the discharge channel.

A typical current - voltage discharge characteristic, as monitored by digital multi-meters, for the flow rates and magnetic field strength used for these studies is shown in Figure 3. In some cases, the mean values were verified by recording the discharge current oscillations with a differential amplifier placed across a 4-ohm ballast resistor in the anode circuit. At the higher discharge voltages (e.g., -200 V), the probes were seen to clearly affect the mean discharge current, perturbing it by as much as 20-30%. During the collection of the data, the residence time of the probes within the channel was several minutes. Damage to the probes from ion bombardment precluded the collection of data beyond approximately 185 V. While it is difficult to say how the immersion of the probes into the plasma affected the spontaneous instabilities detected, we discuss later in the paper the probable presence of waves that are induced by the presence of the probes themselves within the discharge channel.

III. DATA ANALYSIS

The axial variation in the mean electron density for three operating conditions has been reported previously [14], and is provided again here for reference (Figure 4), as it shows that there is a distinct separation between the peak in the magnetic field strength (x = -5 mm), and the peak in the plasma density (x = -10 mm). Below, we discuss the location of instabilities within the channel, referencing the intensity of the fluctuations to the axial location as measured relative to the exit plan (x = 0 mm).

Single Probe, Power Spectral Densities

The data collected by a single probe was cast to display three-dimensional renderings of the plasma density fluctuations, as shown in Figure 5. The power spectra were estimated using the classical Welch method [18]. In the left column of the figure, the amplitude of the plasma density fluctuations are displayed versus operating voltage, for four axial locations: x = 12.7 mm, 0 mm, -12.7 mm, and -25.4 mm. On the right side of the figure, the spectra are displayed versus axial position, for four discharge voltages \( U_d = 86 \) V, 100 V, 128 V and 184 V. Prominent in these figures is the presence of strong, relatively low frequency disturbances in the range of 5 - 30 kHz, which appears to be strongest very near the exit plane (x = 0 mm), and a broader range of disturbances towards higher frequencies. At low discharge voltages \( U_d = 86 \) V, these higher frequency disturbances seem to be excited downstream of the peak in plasma density.
(x = -10 mm), and at high voltages upstream of the exit plane. Similar spectral maps have been reported for the behavior of discharge current oscillations in the Russian Stationary Plasma Thruster SPT-100 [13].

Figure 4. Axial variation in the mean electron density [14].

Two Probes, Fourier Analysis

A comparison of the response of the two probes located at the same axial position, but separated by some angle on the azimuth, provides additional information on the nature of these disturbances. A cross-spectral analysis of the signals from the two probes provides a measure of their coherence, suggests the direction of propagation, and can be used to estimate the azimuthal phase velocity. The azimuthal phase velocity \( V_y \) of the disturbances can be obtained from:

\[
V_y(f) = 2\pi f \left( \frac{\theta_p R}{\phi_2(f) - \phi_1(f)} \right)
\]

(2)

where \( \theta_p \) is the angular probe separation (in radians), \( R \) is the channel radius, \( f = \omega/2\pi \) is the frequency, and \( \phi_{1,2} \) are the phase shifts of the individual probe signals, determined from:

\[
\phi_{1,2}(f) = \tan^{-1} \left( \frac{\text{Im}(F_{1,2})}{\text{Re}(F_{1,2})} \right)
\]

(3)

with \( \text{Re}(F) \) and \( \text{Im}(F) \) the real and imaginary components of the complex Fourier transform \( F(f) \).

One issue that has to be addressed when using Eq. (2) is the physical interpretation of the sign of the calculated phase velocity. Within a Hall discharge, there is a natural asymmetry in azimuthal direction due to the \( E \times B \) electron drift. In our case, the rotation frequency of the Hall current is of the order of 1 MHz, much higher than the frequencies that are dealt with in this paper. It is reasonable to assume then the possibility that a measured negative phase velocity is not due to a backward propagating disturbance, but rather, from a fast forward one, exciting the second probe first, then propagating all the way around the channel axis to excite the first probe. If this is so, then \( V_y \) would be given by:

\[
V_y(f) = 2\pi f \left( \frac{2\pi - \theta_p R}{2\pi - \phi_2(f) + \phi_1(f)} \right)
\]

(4)

Applying Eq. 4 to the signals from the two probes invokes what we shall refer to as the “unidirectional” assumption, in which negative phase shifts are constrained to be interpreted as forward propagating waves. We shall refer to the relaxed condition, which permits wave propagation in the negative Hall direction as the “bi-directional” assumption. Note that in the bi-directional analysis of the waves, there is no a-priori assumption in the direction of the wave propagation. As shown below, the bi-directional analysis leads to dispersion maps that are very different than those generated assuming unidirectional wave propagation.

The signals from the two probes were used to derive wave dispersion maps, rendered as the phase velocity verses wave frequency. In all cases, conditional sampling is employed, to isolate only the strongest correlated disturbances detected by the two probes. Examples of such dispersion maps are shown in Figure 6, for an axial position of \( x = 0 \) and four different discharge voltages, and in Figure 7, for a relatively low voltage (100V) and four different axial locations. For these dispersion studies, the azimuthal probe separation was \( \theta_p = 30^\circ \). Disturbances were considered only if the amplitude of the Fourier component on the first probe was at least three times its mean, and that on the second probe, within a range of the signal considered on the first probe. The condition imposed on the second probe allows for the possible damping of the wave or loss of intensity due to out-of-plane propagation. Also drawn in the dispersion maps of Figure 6 and Figure 7 are lines corresponding to the azimuthal component phase velocities for the \( m = 1 \) and \( m = 12 \) azimuthal modes of the cylindrical annulus. In general, we can express the azimuthal phase velocity for a tilted azimuthal mode:

\[
V_{\phi_y} = \frac{2\pi \cos(\alpha)}{\sqrt{k_x^2 + k_y^2}} \cdot f = \frac{2\pi \cos(\alpha)}{\sqrt{k_x^2 + \frac{m^2}{R^2}}} \cdot f
\]

(5)
Here, $k_x$ and $k_y$ are the component wavenumbers along the axial and azimuthal directions, respectively. We assume that $k_y = m/R$, and $m$ is the integer azimuthal mode number. For a purely azimuthal wave, the axial wave number component is $k_x = 0$. For $m = 1$ and $k_x = 0$ condition, the oscillation wavelength is equal to the channel circumference, $2\pi R$ whereas it is one-twelfth the circumference for the pure azimuthal and $m = 12$ modes, respectively. For $m = 0$, the wave is longitudinal and the azimuthal phase velocity is infinite. In general, the observed natural disturbances can have both azimuthal and axial components, and can propagate with an angle $\alpha$, defined by:

$$\tan(\alpha) = \frac{k_x}{k_y}$$

(6)

The dispersion of the disturbances can also be displayed as dispersion diagrams of the frequency, $f$, versus the y-component wavenumber, $k_y$, as shown in the left frames of Figure 9 for the 100V discharge case at four axial positions. In these figures, we actually plot $k_y/2\pi$ so that the propagation velocities can be extracted directly (since $f$ is in Hz), but label it as $k_y$ for simplicity, so as to not introduce another variable.

**Two Probes, Wavelet Analysis**

The usual method of studying the dispersion relation of a fluctuating component $B(x,t)$, in some scalar property as detected by two probes consists of calculating its joint power spectrum with a Fourier analysis, assuming that $B(x,t)$ can be decomposed into a superposition of planar waves:

$$B(x,t) = \int B(k,\omega)\exp(-i(\omega t - k.x))dkd\omega$$

(7)

The quantity that describes the wave dispersion is the joint frequency-wavenumber spectrum, $S(k,\omega)$:

$$S(k,\omega) = \langle B^*(k,\omega)B(k,\omega) \rangle$$

(8)

which is evaluated from the Fourier transform of $B(x,t)$, i.e., $B(k,\omega)$. In the wavelet analysis, the assumption of a plane wave superposition, which is often too restrictive [19], is removed, and the Fourier transforms are replaced by the wavelet transform [20]:

$$B(x, a, \tau) = \int B(x,t) \frac{1}{\sqrt{a}} h^*(\frac{t-\tau}{a}) dt$$

(9)

where $h(t)$ is the Morlet wavelet:

$$h(t) = \pi^{-1/4}\sigma^{-1/2}\exp(2\pi i t)\exp(-\frac{t^2}{2\sigma^2})$$

(10)

and $\tau$ is its position (in time), and $a$ its scale ($a = 2\pi/\omega$).

Details on its implementation and on the synthesis of the joint frequency-wavenumber spectrum, $S(k,\omega)$, can be found in Ref. [19]. Examples of the dispersion diagram generated by this wavelet analysis is given in the right frames of Figure 9, also for the 100V discharge case, and for four axial locations.

**IV. RESULTS**

An inspection of the left frames in Figure 6 corresponding to the unidirectional assumption, suggests the presence of a strong $m = 1$ axial mode persisting at conditions corresponding to the highest discharge voltages. Also present at the higher voltages is evidence of an $m = 12$ disturbance, which may be excited by the probes themselves, since they are separated by a distance corresponding to $1/12^{th}$ of the circumference of the channel. Finally, at the higher discharge voltages, there is a locus of points above the $m = 1$ demarcation, which might be associated with $m = 0$ modes propagating in the axial direction. Note that since the probes are separated only along the azimuth (at equal axial locations), an axially propagating $m = 0$ mode would be interpreted as a disturbance that propagates at near infinite velocities.

At lower voltages, e.g., 100 V, the unidirectional analysis indicates that this $m = 1$ disturbance is greatly reduced in frequency, and we see the emergence of activity between the $m = 1$ and $m = 12$ limits, that has a frequency cut-off that is located curiously along the $m = 4$ boundary (the $m = 4$ line is not shown in this figure). We believe that this high frequency activity may be associated with disturbances that are excited and anchored by the four-fold symmetry of the magnetic pole-pieces. At the lowest voltage studied, this $m = 4$ activity seems to be stronger, but is reduced in both frequency and velocity.

The right frames of Figure 6 give the dispersion maps for the same conditions, but allowing for bi-directional wave propagation. In this bi-directional analysis, the most striking feature comparing the left and right frames is the near disappearance of the $m = 1$ mode, concomitant with the appearance of backwards propagating components at relatively low frequency (5 - 30 kHz). While at the highest voltages studied, the $m = 12$ disturbance appears to have a positive phase velocity, there is an emergence of an $m = 12$ disturbance with a negative phase velocity at the lowest discharge voltages shown here.

A comparison is made in Figure 7 of the unidirectional and bi-directional analysis of the probe data, for the 100V discharge case at varying axial positions. It is noteworthy that both analysis show that the $m = 4$ activity is strongest at axial locations beyond the exit plane, and that there are axil “bursts” of $m = 0$ activity upstream of the discharge, apparent from the near vertical
alignment of points at discrete frequency values. However, we once again see that what appears as \( m = 1 \) mode activity in the plots for the downstream locations in the unidirectional case again disappears completely, and emerges as a locus of low frequency waves propagating in the negative Hall direction. Note that the constraint imposed by Eq. 4 can give rise to the collapse of a range of \( m = n \) mode activity propagating in the negative direction to an \( m = 1 \) line if the probe separation is much less than the circumference of the channel.

Figure 8 compares the dispersion profiles determined from the bi-directional conditional sampling results of Figure 6, to the dispersion profiles generated from the wavelet analysis (and artificially unfolded beyond the Nyquist wavenumber limits, as in Ref. [19]). It is apparent that there is excellent quantitative agreement between the two methods. Each one suggests that at the highest voltages studied, there is a favoring of an \( m = 0 \) mode, followed by the emergence of what could be a low order \((m = 0 - 4)\) mode, but propagating with low frequencies, and along the negative Hall direction. The high frequency activity at lower voltages can be a broad mode, but propagating with low \( m = 0 \) or \( 2 \) line (not shown) which would appear as vertical lines at a wavenumber of about \( 4 \, \text{m}^{-1} \), or \( 8 \, \text{m}^{-1} \), respectively. The distinct slanted boundary on the high frequency points in the \( 100 \, \text{V} \) discharge case is due to the sampling rate limit, precluding the measurement of disturbances with velocities in excess of the product of the probe separation times the sampling rate. The activity at large wavenumbers (> \( 40 \, \text{m}^{-1} \)) beyond the Nyquist limit may be a result of the aliasing associated with the strong activity at both low and high frequencies near zero wavenumbers.

A similar agreement is seen between the bi-directional conditional sampling and wavelet analysis when comparing the dispersion plots for the low voltage case (100V) but at varying axial locations (see Figure 9). It is interesting to note that the dispersion plots seen for high voltages near the exit plane are qualitatively similar to those of \( 100 \, \text{V} \), 25 mm upstream of the exit plane (a dominance of an \( m = 0 \) disturbance). Further downstream, there appears to be the emergence of a locus of points centered perhaps about the \( m = 1 \) or \( 2 \) line (albeit with broad scattering), propagating in the positive Hall direction, and the emergence of a strong low frequency mode propagating in the opposite direction.

The Fourier and wavelet analysis allows us to determine the growth rates of the disturbances. Figure 10 gives, in the left panels, the locus of points (or regions, in the case of the wavelet analysis) corresponding to positive growth, for the case of \( 100 \, \text{V} \) and \( x = 0 \, \text{mm} \). The right panels give the locus of points or regions where the disturbances are damped. The top panels are the results of the unidirectional conditional sampling analysis, the middle is that of the bi-directional analysis, and the lower panels correspond to the results of the wavelet analysis. It is noteworthy that again, there is generally good agreement between the bi-directional conditional analysis, and the wavelet analysis, which indicates that the low frequency disturbances propagating along the negative Hall direction are growing, while the broader, higher frequency disturbances propagating in the positive Hall direction are damped (decaying). A comparison between the bi-directional and unidirectional analysis shows excellent agreement for the damped disturbances, and verifies that the discrepancy appears with the disturbances that are seen to be growing (left panels). The unidirectional analysis interprets these to be strong, highly oriented \( m = 1 \) azimuthal waves propagating in the positive Hall direction, where the bi-directional analysis indicates that they are perhaps \( m = 1,4 \) modes that are spatially growing, and propagating in the opposite direction.

V. SUMMARY

Experimental measurements are presented for studies of low frequency (< \( 100 \, \text{kHz} \)) oscillations in a Hall discharge. Two azimuthally placed electrostatic probes are used to identify and extract properties of disturbances associated with the propagation of instabilities within the discharge channel. The azimuthal component phase velocities are determined for a wide range of wave frequencies and over characteristic regimes of operation of these discharges (e.g., from the ionization branch through current saturation). Two analyses of the data are presented. The first is based on conditional sampling of the probe data, either constraining all disturbances to propagation in the Hall direction or allowing bi-directional propagation. The second involves a wavelet analysis, allowing for bi-directional wave propagation. A variety of propagation modes are observed and analyzed, including the possible appearance of an induced mode due to the presence of the probes themselves. While the conditional sampling and wavelet analysis give consistently similar results for the dispersion curves, differences are apparent if the waves are constrained to propagate in the Hall direction. Most noticeably, the bi-directional analysis points to the presence of a low frequency (\( \approx 5-10 \, \text{kHz} \)) mode that may be consistent with an \( m = 1 \) or \( m = 2 \) azimuthal mode, that propagates in the negative Hall direction and exhibits positive growth. This mode is seen at lower discharge voltages, and usually beyond the exit of the channel. A unidirectional analysis indicates this to be a predominantly \( m = 1 \) mode that is highly directional (and perhaps tilted slightly), damped, and propagates in the Hall direction over a range of discharge voltages. Preliminary analysis based on wavelet transforms to determine the joint frequency-wavenumber spectrum is found to give excellent...
agreement with the bi-directional conditional sampling analysis.

Resolving the debate as to which analysis (unidirectional or bi-directional) is correct will require an experimental study involving more than two probes. A multiple probe rake assembly has been designed and fabricated, and the results from this multiple probe study will be presented in future papers.

ACKNOWLEDGEMENTS

This work was supported by the Air Force Office of Scientific Research. Support for E. Chesta was provided by the Politecnico di Torino, Italy, and Ecole Centrale Paris, France. N. Gascon acknowledges support from the French Ministry of Foreign Affairs (Lavoisier Fellowship) and from the European Space Agency. N. Meezan received support from Stanford University through the SGF Fellowship Program. The authors are most thankful to T. Dudock de Wit from CNRS-LPCE (Orleans, France) for providing the wavelet analysis program and for offering numerous and insightful comments.

REFERENCES


7

American Institute of Aeronautics and Astronautics
Figure 5. Power spectra of plasma density fluctuations. Left: different axial positions. Top to bottom: -25.4mm (inside), -12.7mm (inside), 0 (exit plane), +12.7mm (outside) Right: different discharge voltages. Top to bottom: 184V, 128V, 100V, 86V
Figure 6. Dispersion maps in the $V_y$-$f$ plane at constant axial position (0) and different discharge voltages. Left: Unidirectional conditional sampling. Right: bi-directional conditional sampling. Top to bottom: 184V, 128V, 100V, 86V. The solid and dashed lines indicate the $m=1$ and $m=12$ modes, respectively.
Figure 7. Dispersion maps in the $V_x$ - $f$ plane at constant discharge voltage (100 V) and different axial positions. Left: Unidirectional conditional sampling. Right: bi-directional conditional sampling. Top to bottom: -25.4mm, -12.7mm, 0, +12.7mm The solid and dashed lines indicate the $m=1$ and $m=12$ modes, respectively.
Figure 8: Dispersion maps in the f-k plane at constant axial position (0) and different discharge voltages. Left: bidirectional conditional sampling. Right: wavelet analysis. Top to bottom: 184V, 128V, 100V, 86V. The dashed lines indicate the azimuthal Nyquist wavenumber limits -k_N and +k_N.
Figure 9: Dispersion maps in the f-k_y plane at constant discharge voltage (100 V) and different axial positions. Left: bidirectional conditional sampling. Right: wavelet analysis. Top to bottom: -25.4mm, -12.7mm, 0, +12.7mm. The dashed lines indicate the azimuthal Nyquist wavenumber limits -k_N and +k_N.
Figure 10. Regions of positive (left) and negative (right) spatial growth rate in the f-k plane. Top to bottom: unidirectional and bidirectional conditional sampling, wavelet analysis. The dashed lines indicate the azimuthal Nyquist wavenumber limits $-k_N$ and $+k_N$, and the $m=1$ mode ($k_y/6$ in our case).