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Single particle simulations of electron transport in the near-field of Hall thrusters

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Abstract
The results of 3D, single particle electron trajectory calculations are presented for the near-field of a laboratory $E \times B$ Hall plasma thruster. For a prescribed static magnetic and electric field distribution, single electrons are launched and tracked from a simulated cathode. Collisions with external thruster surfaces are accounted for; however, field fluctuations are disregarded. Bulk statistics including the channel to beam electron current ratio, electron lifetimes and spatial distributions of the number density, mean energy, energy distributions, velocity distributions and velocity component ratios are catalogued. For conditions typical of a moderate power Hall thruster, the mean lifetime of electrons in the domain of axial scale length, $L \approx 0.3 \text{ m}$, is approximately 120 ns. Electrons which eventually enter the channel are found to strike the thruster $\sim 10^3$ times as frequently as electrons which exit the domain in the plume. For the static $E$ and $B$ field distributions used in this study, the channel to beam current ratio is found to be on the order of 0.1 and the velocity ratio, $V_{E \times B}/V_B$, over the channel has a mean of $\sim 0.5$, with higher values driven largely by collisions with the thruster indicating the importance of such events in driving transport into the channel.

(Some figures in this article are in colour only in the electronic version)

1. Motivation
The behaviour of the electron current emitted from the cathode of Hall plasma accelerators is still poorly understood. In a typical coaxial $E \times B$ Hall thruster, such as that illustrated schematically in figure 1, approximately 90\% of the electron current leaving the cathode serves to neutralize the ion beam, while the remaining 10\% migrates into the thruster to service the discharge. How electrons migrate across the magnetic field in the near-field (the region between the cathode plane and the discharge channel exit) is the subject of much research. In simulating this region, a Bohm-model for the electron mobility is often assumed \cite{1, 2} although no direct evidence exists linking transport to plasma fluctuations. A possible alternate explanation for the anomalous transport in this region is the role played by strong field non-uniformities (in both $E$ and $B$) and collisions with the thruster front-face. Electron transport inside the thruster is sometimes attributed to the so-called near-wall conductivity \cite{3}, and a similar mechanism may influence near-field transport due to the possibility of collisions with the front-face of the thruster. There are no other plasma-confining walls in the downstream region between the exit plane of the thruster and the external cathode. An understanding of the extent to which each of these factors drives transport in the near field requires an examination of the dynamics of electrons in the presence of the non-uniform fields typically found near the channel exit. This paper provides initial insight into this behaviour and shows that the non-uniform field structure (e.g. the convergence of $B$-field lines along the central axis) combined with collisions with the thruster face may play a significant role in driving the observed cross-field transport seen in this region of these plasma devices.

2. Background
Among the most frequently utilized tools for simulating the discharge and near-field region of Hall thrusters are...
or equivalently, a constant effective Hall parameter, is anomalous (i.e. cannot be accounted for by classical electron mobility observed inside the discharge channel and near-field cross-field electron current. Experiments suggest that the electron velocity, assuming some constitutive relation for the electric potential, derived from an equation for the average quasi-neutral hybrid-particle-in-cell (PIC) methods in which the use of ad hoc transport descriptions such as an electron mobility that scales as $B^{-1}$, in accordance with the model of Bohm [10], or equivalently, a constant effective Hall parameter, $\omega_e \tau_{\text{eff}}$. While, in some cases, one can surmise that secondary electron emission from the dielectric (typically boron nitride—BN) walls of the discharge channels may impact the axial transport of electrons to the anode, in the near-field, as mentioned above, the front-face of the thruster is the only confining wall, and the importance of collisions with this surface remains to be investigated fully.

Here, we show that anomalous transport in the near-field can be accounted for by the behaviour of electron motion due to the strongly non-uniform electric and magnetic fields and collisions with the thruster front-face, without invoking field fluctuations. Electrons which eventually leave the domain in the channel are found to suffer $\sim 10^3$ times as many collisions as electrons which leave the domain in the plume. Plume-bound electrons rarely suffer a collision with the thruster, leaving the domain as a result of their initial velocity and the character of the $E$ and $B$ fields. Additional particle-background gas collisions are found to have a relatively minor effect on the bulk transport, even when allowed to occur at artificially high rates. While the simulations presented herein are for a particular laboratory thruster, the field non-uniformities and collisions considered are common to the discharges of all modern Hall thruster sources. Though this particular simulation utilizes fields which are assumed to be static and axisymmetric, future studies will forgo these assumptions and consider fluctuating, 3D fields, such as those measured by our laboratory recently [11].

3. Simulation

Our study focuses on computing discrete particle trajectories of single electrons in practical Hall thruster configurations. While, in principle, full PIC simulations can be carried out with the simultaneous dynamical tracking of a large number of super-particles (electrons and ions) allowing for the electric potential to be solved for self-consistently; such simulations are still intractable for the full geometry of a typical Hall thruster. Instead, we prescribe the electric field distribution, allowing for a relatively smaller number of particles to be considered. At present, some of our simulations include collisions between the electrons and background gas by assuming that they result in elastic collisions at a prescribed rate. Electron scattering events with external thruster walls are accounted for, and the location and frequency of these events are catalogued. A uniform sheath is assumed on the external surfaces of the thruster exit of thickness 1 mm, and potential drop of 15 V. This value is an estimate motivated by the simplified analysis presented by Bittencourt [12], which gives the wall potential relative to the bulk plasma (the sheath potential drop) as $\Delta \phi = -kT \ln(m_i/m_e)/(4e)$, where $k$ is Boltzmann’s constant, $T$ is the effective temperature of the plasma (assuming the electrons and ion are in thermodynamic equilibrium at the same temperature, $T$), $e$ is the fundamental charge and $m_i$ and $m_e$ are the ion and electron masses, respectively. For a xenon plasma, $\ln(m_i/m_e) \sim 12.4$. The 15 V sheath potential is obtained by assuming that the equilibrium temperature in the near-field is of the order of 5 eV, giving $\Delta \phi = 15.5V$. A small additional subset of simulations investigate how the wall-collision model (e.g. diffuse versus specular) impacts near-field transport.

The results presented here are dependent on knowledge of the spatial variation in the electric field in the near-field of the thruster. One avenue for obtaining this field is from hybrid-PIC simulations which extend into the near-field region, such as those presented by Capitelli et al [13]. Some measurements of the plasma potential in the near-field and plume of Hall thrusters are available [14–18]. At this time, however, sufficiently high resolution information of the near-field plasma potential for the particular thruster simulated here is not yet available. Our simulations therefore use an electric potential that is adapted to our thruster geometry from the hybrid simulations of a slightly different thruster, described in [13, 18]. The potential of [13, 18] is scaled to match that measured at the exit of our thruster, which is an 82 mm channel diameter laboratory Hall thruster operating at a discharge voltage and current of 200 V and 2.6 A, respectively [8, 18]. We also treat the fields (both $E$ and $B$) as axisymmetric, although this constraint will be lifted in future simulations. A portion of the resulting plasma potential distribution used in our simulation is shown in figure 2. Here, the radial coordinate is $r = (x^2 + y^2)^{1/2}$.

The near-field magnetic field distribution used in the simulations is expected to be dominated by the externally

![Figure 1. Schematic of a coaxial Hall thruster.](image-url)
applied field of the thruster’s magnetic circuit. Induced magnetic fields (steady or fluctuating) generated by currents within the plasma are neglected. We use Finite Element Method Magnetics (FEMM [19]), a finite element magnetic solver in which we have built a model of the magnetic circuit of the thruster studied here, to obtain this externally applied magnetic field distribution, as shown in figure 3.

The canonical angular momentum, $\vec{K}$, of a charged particle in electromagnetic fields is given by

$$\vec{K} = \vec{r} \times (\vec{p} - q\vec{A}),$$  

where $\vec{p}$ is the standard mechanical momentum ($\vec{p} = m\vec{V}$), $m$ is the particle mass, $q$ is the particle charge and $\vec{A}$ is the vector potential which is related to the magnetic field, $\vec{B}$, by

$$\vec{B} = \nabla \times \vec{A}.$$  

In axisymmetric fields such as those utilized in this study, the canonical azimuthal angular momentum (i.e. the $z$-component of $\vec{K}$) should be conserved by integrators of charged particle motion in electromagnetic fields [19]. In fact, conservation of the canonical angular momentum is often used as a constraint for such problems as magnetic detachment of particles in plasma thrusters [20]. In this study, the particles are tracked using a fourth-order Runge–Kutta (RK4) integrator [21] for the equations of motion. While it is known that the RK4 method does not conserve canonical momentum exactly [22], we have compared the method with other traditional particle-tracking methods [23] that do conserve this quantity (such as the popular method of Boris [24]). We have found that, for the time-steps (i.e. 1% of the local cyclotron period) and lifetimes of particles (i.e. $O(10^{-7})$ s) studied herein the differences between the two methods with regard to momentum conservation (both mechanical momentum and canonical angular momentum) are small.

In the simulation, electrons released from a point-source cathode are tracked until they exit the simulation domain, which extends axially ($z$) from the exit plane to 300 mm (over six times the radius of the outer channel) and radially ($r$) from the central axis to 300 mm. Periodically, the position and velocity components of the particles are recorded to establish spatially resolved maps of the relative electron density, mean energy, energy distributions and velocity distributions. In this paper, the simulation is evaluated for its ability to yield information concerning the electron transport in the near-field while matching typical values for the ratio of channel to beam current found experimentally.

4. Results

For all of the results of computations shown below, the cathode was placed in the domain at $x = 80$ mm, $y = 0$ mm and $z = 20$ mm as this is the actual experimental placement for the Hall thruster simulated. Researchers have measured the electron temperature upstream and downstream of the cathode tip [25, 26]. Frequently, the electron temperature is of the order 1 eV inside the cathode, increasing to >5 eV downstream [25]. In the present simulation, the detailed physics of the cathode discharge are not modelled. Electrons are released from the cathode with their initial speed obtained by sampling a 1000 K Maxwellian speed distribution, and their initial direction from a Gaussian distribution (with the most-likely angle being along the cathode centreline defined by $\theta$, see figure 1). We take $\theta = 135^\circ$ and the standard deviation of the angular spread of the electrons to be 30°. This distribution was based on density measurements [25] which indicate a distribution which is neither a narrow beam nor a very diffuse cloud. Electrons emitted from the cathode are given an additional energy ‘kick’ along their initial direction with a magnitude equal to the plasma potential at the point of emission (in the present simulation, the kick is of order 10 eV). While this does result
in a small energy dispersion relative to the mean in this study, future studies will treat the cathode physics in more detail.

Collisions with the thruster surfaces are generally treated by re-emitting colliding electrons from the point of collision with a random direction and a speed sampled from a 500 K Maxwellian speed distribution, although a small subset of simulations considers the effects of specular collisions of varying degree (see section 4.5). Initially, electron-background gas collisions are not included, but the effect of such collisions is investigated later.

In this paper, we take the x- and y-axes to be parallel to the exit plane of the thruster and the z-axis parallel to the central axis (see figure 1). The point \((x, y, z) = (0, 0, 0)\) mm is located at the intersection of the central axis with the exit plane. An adaptive time-step is used, with the time-step selected to be 1% of the local electron-cyclotron orbital period, unless otherwise noted. This time-step affords a high degree of accuracy in regions of strong magnetic field, while greatly enhancing the speed of the calculation when the electrons travel away from the exit plane where the magnetic field strength drops rapidly. The \(E\) and \(B\) fields were prescribed on spatial grid points every 1 mm giving 300 axial points and 300 radial points for a total of 90,000 points. As the particles move through the domain, at each time-step the fields at the exact location of the particles are interpolated from the four surrounding grid points (two points bracketing the location radially and two axially). The trajectories of \(10^6\) super-particles are studied throughout their lifetimes in the domain. For our discharge conditions, each superparticle represents approximately 2 \(\times\) \(10^{12}\) electrons.

### 4.1. Trajectories, reflections and current ratio

Figure 4 shows two sample trajectories for electrons emitted from the cathode (one which leaves the domain in the plume and one which leaves the domain at the channel). The plume-bound electron leaves the domain along the central axis where the \(B\)-field lines converge. The inset shows the trajectories as viewed looking down the \(z\)-axis towards the channel. Although the domain extends 30 cm in every direction, only a subset is shown here for clarity. The channel and thruster boundaries are indicated by black circles in the \(z = 0\) plane. The open symbols show the electrons’ positions at 1 ns intervals.

Both of the particles shown in figure 4 strike the thruster; the plume-bound strikes only once while the channel-bound electron strikes twice. The additional apparent reflections occur within the sheath and further downstream. Figures 5(a), (b) and (c) show the spatial distribution of particle flux (collisions mm\(^{-2}\) s\(^{-1}\), on a logarithmic scale) resulting from physical collisions with the thruster, mirroring events in the sheath and exit locations for electrons leaving the domain in the \(z = 0\) plane, respectively. The channel and outer thruster boundary are indicated by black circles.

The distribution of the physical collisions shows that a large number of electrons strike the thruster near the central axis, with additional collisions occurring primarily in a single band on the centre pole and \(\sim 5\) bands on the outer portion of the thruster. On average, each electron collides with the thruster once every \(\sim 10^{-6}\) s, with some electrons suffering multiple collisions while others suffer no collisions. The distribution of sheath reflections shows that virtually no electrons reflect off the sheath along the central axis (figure 5(a) shows that they instead penetrate the sheath and physically strike the thruster). On average, each electron is mirrored within the sheath once every \(1.1 \times 10^{-6}\) s, a rate slightly lower than the rate of physical collisions. The exit locations shown in figure 5(c) indicate that, with some exceptions, electrons frequently reach the channel near one of the edges of the annulus (e.g. near the outer channel boundary for \(x < 0\)). This is due to the fact that electrons are striking the thruster with relatively high energy but are re-emitted with relatively low energy (as their post-collision speed is sampled from a 500 K Maxwellian speed distribution). These low-energy electrons suffer repeated collisions with the thruster before trickling into the channel. The prominent band of electron exit locations which crosses the channel on an angle near \(x = 0\) mm is a continuation of the arc within which many electrons are reflected from the sheath and off the thruster. Some of these electrons are funnelled directly into the thruster through the combined effects of their momentum and the \(E\) and \(B\) fields present in this region, while others first suffer a collision with the thruster before reaching the channel.

The time that each electron spends within the domain is tracked, and the overall distribution is shown in figure 6. The separate distribution for those electrons which end up in the channel is shown with circular data markers (those that end up in the plume make up the remainder). Once electrons traverse the domain boundary, it is assumed that they do not return into the computational domain. The mean lifetimes of electrons exiting the domain at the channel and in the plume are \(\sim 35\) ns and \(\sim 130\) ns, respectively, with an overall mean of \(\sim 120\) ns. Given these lifetimes and the collision rates discussed above, it is apparent that, on average, only one in nine electrons suffers a collision during its lifetime. However, closer inspection of the collisional events reveals that \(\sim 71\%\) of collisions are suffered by electrons bound for the channel, with the remaining 29% suffered by plume-bound electrons. Taking this disparity into account, on average three out of four electrons bound for the channel suffer a collision, with a mean time between collisions of 45 ns. In contrast, only 4% of plume-bound electrons collide during their residence in the domain, with 3.3 \(\mu\)s elapsing between collisions, on average.
Figure 5. Spatial distribution of fluxes (collisions mm$^{-2}$s$^{-1}$, on a logarithmic scale) of (a) physical collisions between electrons and the thruster, (b) mirroring events in the sheath and (c) exiting electrons in the $z = 0$ plane.

Figure 6. Distribution of electron lifetimes (total is shown with square symbols, subset for electrons reaching the channel is shown with circular symbols).

Therefore channel-bound electrons collide with the thruster nearly $10^3$ times more frequently than plume-bound electrons. The ratio of the beam current (electrons leaving in the plume) to channel current (electrons which reach the channel) is 11.8%. These results are found to be robust to changes in time-step, varying by less than 4 ns and 2% for the mean lifetime and current ratio, respectively, until the time-step exceeds 10% of the local cyclotron period.

4.2. Position, energy and identity distributions

As the simulations were carried out, the position and velocity components of the electrons in the domain were catalogued once every $10^{-9}$ s. This data are used to reconstruct spatial maps of electron density (i.e. the frequency of electrons present in a given volume) and mean energy and to identify regions where present electrons are generally destined to leave the domain in the plume (tagged with a numerical value of 2) or channel (tagged with a value of 1). This later property is referred to here as the electron ‘identity’. After the simulation, the exit locations of all of the electrons are checked, and the catalogued positions and velocities are subsequently tagged with this identity. Results are presented in ten planar slices of the domain parallel to the exit plane. These regions have an axial thickness of 1 cm (with boundaries at 1 cm intervals between $z = 0$, 10 cm). The regions are composed of cells 1 cm on a side, extending in both the $x$- and $y$-directions between ±10 cm (giving a total of 400 cells per layer). Figures 7(a), (b) and 6 show the electron number density (in cm$^{-3}$, on a logarithmic scale), average energy (in eV) and mean identity in these spatial layers. White areas indicate regions where no electrons were catalogued.

Figure 7(a) reveals that the distribution of electrons in the domain is not smooth, but rather strongly affected by the location of the cathode (located at $(x, y, z) = (8, 0, 2)$ cm). The distribution is somewhat more homogeneous nearer to the thruster (in part due to the collisions with the thruster occurring in this region) than just downstream of the cathode (although for $z > 8$ cm the distribution becomes somewhat more uniform). The effect of particle-background gas collisions on smoothing this distribution is discussed later in section 4.

Figure 7(b) indicates that the electrons possess elevated energy near the exit plane over the channel and further downstream along the central axis. This is somewhat expected, as the regions of plasma over the channel and along the central axis for $z > \sim 2$ cm are at elevated potential. These regions coincide approximately with regions of the plasma that are highly luminous in the laboratory discharge simulated. The high-energy beam along the central axis indicates that few electrons which have suffered collisions (and thus an energy-shift) reach this region. The region directly over the channel shows a greater mix of relative energy values due to the mixed population of electrons (some suffering collisions, others not) in this region.

Figure 7(c) confirms that few channel-bound electrons (which suffer more collisions, on average) are found for $z > \sim 4$ cm. Interestingly, there are regions of plasma directly above the channel within which all electrons present eventually leave the domain in the plume (i.e. for $x = \sim 0$ and $y < \sim -5$ cm). Electrons which reach the central axis region invariably ended up reaching the plume. The nature of the magnetic field lines (see figure 2) in this region is conducive to such behaviour since the field lines are converging here and are oriented axially (i.e. along $z$).
Figure 7. Near-field distributions of (a) scaled electron number density (cm\(^{-3}\), logarithmic scale), (b) mean electron energy (eV) and (c) average identity (values closer to 1 indicate areas rich in electrons which will eventually reach the channel).

4.3. Spatially resolved electron energy distribution

This simulation also facilitates investigation of the electron energy distribution functions (EEDFs) anywhere within the computational domain. In figure 8, the energy distribution for electrons in a 1 cm\(^3\) volume just downstream of the channel (3.5 cm \(\leq x \leq 4.5\) cm, \(-0.5\) cm \(\leq y \leq 0.5\) cm and 0.0 \(\leq z \leq 1.0\) cm) is shown. The total distribution is indicated with a solid line, with the contributions from channel-bound and plume-bound electrons indicated by lines with square symbols and circular symbols, respectively.

The EEDF exhibits a broad distribution with multiple peaks. The spread in energy at this location is due to the range of plasma potential within this 1 cm\(^3\) region (between 47 and 90 V). Without suffering any collisions, particles are expected to possess this range of energy. The dispersion to low energy, particularly below 45 eV, is due to collisions with thruster surfaces as these most often reduce the energy of a particle. Wall collisions are also responsible in part for the multi-peaked distribution at higher energies. Particles which reach the channel without suffering a collision are expected to possess about 90 eV of energy. If an electron suffers a single collision in a region of elevated potential before entering the channel, it will possess considerably lower energy since less change in potential is possible between the collision/re-emission location and the channel. Electrons which suffer a collision in a region of relatively low potential will gain considerable energy before reaching the channel, but those which collide with the thruster very near the channel will not gain much energy before reaching the channel. The fringe-like pattern of concentric arcs seen in the wall-collision map (see figure 5(a)) and the plasma potential distribution (see figure 2) correlates well with the energy distribution of electrons over the channel. Other researchers have noted that the experimentally measured EEDFs taken near the exit plane of a small Hall-type discharge often possess multiple maxima as well [30]. A more extensive investigation of the EEDFs throughout the near-field will follow in a future paper.

4.4. Velocity distributions and ratios

The relative rate of transport of electrons along \(E\), \(B\) and \(E \times B\) lends insight into the transport mechanisms dominating in the domain. The velocity components in the \(E\), \(B\) and \(E \times B\)-direction are referred to as \(V_E\), \(V_B\) and \(V_{E\times B}\), respectively. The local mean value is represented by angled brackets, with a subscript of \(\theta\) when the mean is extended over azimuthal position (e.g. \(\langle V_E \rangle\) and \(\langle V_E \rangle_\theta\) represent the local mean of \(V_E\) and the azimuthally averaged mean of \(V_E\), respectively). Figures 9(a), (b) and (c) show the spatial variation of the local values (within each 1 cm\(^3\) volume) in the 0.0 \(\leq z \leq 1.0\) cm plane (i.e. just downstream of the exit) of \(\langle V_E \rangle\), \(\langle V_B \rangle\) and
the imposed electric field (with $\langle V_{E,B}\rangle_0 = -4.8 \times 10^5 \text{ m s}^{-1}$) and are drifting in the $E \times B$ direction (with $\langle V_{E,B}\rangle_0 = 1.5 \times 10^6 \text{ m s}^{-1}$). The mean velocity along $B$, $\langle V_B \rangle_0 = 9.9 \times 10^4 \text{ m s}^{-1}$. Plume-bound and channel-bound electrons are distinct in their respective contributions to all of the velocity distributions in addition to the $\beta$ distribution. In the cases of the velocity distributions, the plume-bound electrons tend to have more-positive contributions with $\langle V_{E,\text{plane}} \rangle_0 = 8.1 \times 10^5 \text{ m s}^{-1}$, $\langle V_{B,\text{plane}} \rangle_0 = 1.7 \times 10^6 \text{ m s}^{-1}$ and $\langle V_{E,B,\text{plane}} \rangle_0 = 3.3 \times 10^6 \text{ m s}^{-1}$ compared with the channel-bound values of $\langle V_{E,\text{channel}} \rangle_0 = -1.0 \times 10^6 \text{ m s}^{-1}$, $\langle V_{B,\text{channel}} \rangle_0 = 7.1 \times 10^5 \text{ m s}^{-1}$ and $\langle V_{E,B,\text{channel}} \rangle_0 = 7.8 \times 10^5 \text{ m s}^{-1}$. One plausible explanation for the distinct $E \times B$ distributions stems from the fact that channel-bound electrons suffer approximately 10$^5$ times as many collisions as plume-bound electrons (essentially resetting the $E \times B$ velocity to $0$). In addition, these collisions may allow electrons to escape the near-exit-plane region due to the strong gradients in $E$ and $B$. However, this issue needs further investigation. The expected drift velocity near the exit plane (which figure 10(c) addresses) has a value of $|E|/|B| \sim 3 \times 10^6 \text{ m s}^{-1}$ (which the infrequently colliding, plume-bound electrons possess). Due to the electrons’ gyromotion, fluctuations on the order of $1 \times 10^6 \text{ m s}^{-1}$ are expected and apparent in both the distributions.

The mean value of $\beta$ in this region is, $\langle \beta \rangle_0 = -0.44$ (negative because the majority of electrons are travelling towards the thruster, in the direction of $-E$, and are drifting in the positive $E \times B$ direction) with a standard deviation of 2.32. The distribution has a primary peak at $\beta = 0.20$. The channel-bound electrons are the primary source of the large-magnitude values of $\beta$ with $\langle \beta_{\text{channel}} \rangle_0 = -0.79$, and the standard deviation is 3.16. The plume-bound electrons have $\langle \beta_{\text{plume}} \rangle_0 = 0.31$ with a standard deviation of 0.79. These statistics, along with the finding that channel-bound electrons suffer collisions more than 10$^3$ times as frequently as plume-bound electrons (see section 4), indicates that collisions with the thruster are responsible for the largest values of the velocity ratio, $\beta = V_E/V_{E,B}$.

Figure 11 shows the azimuthally averaged values, $\langle V_E \rangle_0$, $\langle V_B \rangle_0$, $\langle V_{E,B} \rangle_0$ and $\langle \beta \rangle_0$, in the region of space defined by $3.6 \text{ cm} \leq r \leq 4.6 \text{ cm}$ at a variety of axial positions. In the figure, the centre of the axial range considered is shown on the horizontal axis. We find that $\langle \beta \rangle_0$ varies between a maximum of $\langle \beta \rangle_0 = 6.0$ for $1.0 \text{ cm} \leq z \leq 2.0 \text{ cm}$ and a minimum of $\langle \beta \rangle_0 = -1.3$ for $4.0 \text{ cm} \leq z \leq 5.0 \text{ cm}$ with an overall mean and standard deviation within the cylinder limits.
Figure 10. The distribution of (a) $V_E$ (in m s$^{-1}$), (b) $V_B$ (in m s$^{-1}$), (c) $V_{E \times B}$ (in m s$^{-1}$) and (d) $\beta = V_E / V_{E \times B}$, in the region of space bounded by $3.6 \text{ cm} \leq r \leq 4.6 \text{ cm}$ and $0.0 \text{ cm} \leq z \leq 1.0 \text{ cm}$.

Figure 11. $\langle V_E \rangle_\theta$, $\langle V_B \rangle_\theta$, $\langle V_{E \times B} \rangle_\theta$ and $\langle \beta \rangle_\theta$ in the ring of space bounded by $3.6 \text{ cm} \leq r \leq 4.6 \text{ cm}$ at various axial positions (midpoint of $z$ is shown).

4.5. Collisions

4.5.1. Character of collisions with walls. Wall collisions are a necessary driver for transport as already evidenced in the velocity distributions and dynamics in channel-exit regions (see section 4). The collisions in all of the results described above have been treated as diffuse; however, a subset of simulations was also carried out to ascertain how sensitive the results are to the collision treatment. In one extreme limit, we consider specular collisions (i.e. colliding electrons were re-emitted with their $z$-component of momentum reversed). Other sets of simulations considered collisions in which some fraction, $\xi < 1$ of the electrons undergo specular collisions, with the remainder diffuse. We generally find that the ratio of channel to plume current was found to be rather insensitive to the wall collision treatment with increasing fraction of specular collisions, maintaining a value near 0.1 until $\xi \geq 0.8$, at which point the current ratio rapidly dropped to below 0.04. Very few electrons seem to reach the channel when the collisions are all specular since many electrons reflect with sufficient energy to overcome $E$ and exit the simulation domain. The electrons’ lifetimes in the domain were less sensitive to the choice of $\xi$, increasing slightly from $\sim 120 \text{ ns}$ (diffuse) to $\sim 125 \text{ ns}$ (fully specular).

4.5.2. Background gas collisions. Up to this point, gas-phase particle (electron-neutral) collisions have not been considered. However, in order to understand the effect that such events may have on transport in the near-field, simulations were carried out in which scattering is taken to occur at a prescribed position-independent rate, $\nu_e$ (with a statistical spread). Assuming $\nu_e = n \sigma V_e$ ($n$ is the background neutral particle number density, taken to be independent of position, $\sigma$ is the momentum-transfer collision cross-section and $V_e$ is the mean electron speed), then for typical conditions near the exit plane of a Hall thruster ($n \approx 10^{18} \text{ m}^{-3}$, $\sigma \approx 10^{-20} \text{ m}^2$ and $V_e \approx 10^6 \text{ m s}^{-1}$) we expect $\nu_e \approx 10^4 \text{ s}^{-1}$. We find that the mean time between such collisions is $\sim 100 \mu$s. Distances further away from the exit plane should have a lower collision frequency (due to a diminishing background gas density), and so use of this value...
over the entire domain should be seen as an upper limit on the effect of scattering. Since the mean lifetime of electrons in the simulation is on the order of 0.1 µs, this collision rate is not expected to significantly impact the results. Simulations were carried out with a much larger rate, to qualitatively ascertain what effect such an artificially high rate of collisions will have on transport. The mean time between collisions is taken to be 100 ns (a rate nearly $10^3$ times greater than that expected) with a standard deviation of 30 ns.

The additional gas-phase collisions, albeit at this exaggerated rate, are found to increase the fraction of electrons reaching the channel from 11.8% to a mere 13.6% while the mean residence time in the domain increased from 119 to 144 ns. The increased lifetime by the addition of collisions seems contradictory at first, but is due to the fact that only plume-bound electrons (with a mean lifetime of 167 ns) generally suffer gas-phase collisions since channel-bound electrons have an average lifetime that is much shorter (35 ns) than the assumed time between collisions. We find that plume-bound electrons suffer a collision during their migration out of the domain and the effect of a collision is likely to cause it to scatter back into the domain, thus increasing the lifetime. The additional collisions do slightly change the azimuthally averaged velocity ratio, $(\beta)_0$ in the region $3.6 \, \text{cm} \leq r \leq 4.6 \, \text{cm}$ and $0 \, \text{cm} \leq z \leq 10 \, \text{cm}$ from an overall mean value of 0.41 (without electron-background collisions) to −0.42. These collisions also lead to a slightly smoother density distribution; however, the distribution of electrons remains significantly asymmetric, even with the artificially high gas-phase collision frequencies considered. We surmise that with a realistic collision frequency, the effects would have been negligible. We conclude therefore that an additional mechanism may be needed to homogenize the plasma density (and other properties), since time-average measurements of plasma properties seem to be generally independent of azimuthal position [27]. One mechanism that is likely to influence the azimuthal uniformity and to affect transport in the near-field is electron scattering from field fluctuations. Fluctuations in plasma properties have been found to occur in the near-field of Hall thrusters [16] will be presented in a future publication.

5. Discussion and summary

Despite ignoring field fluctuations in carrying out these simulations, the observed ratio of electron channel to plume current is 0.118, consistent with typical experimental values of $O(0.1)$. The distribution of electrons as well as their average energy has been catalogued, and together these results show that, for the field distribution investigated, the central axis of the near-field region tends to be occupied by electrons of high energy. The distribution of high-energy electrons is qualitatively consistent with the luminous regions seen in the near-field plasma of Hall thrusters. Electrons near the central axis rarely reach the channel of the thruster; instead they are ejected into the plume. Collisions between the electrons and the thruster are concentrated along the central axis and in several arcs sweeping outward from the outer channel boundary. The distribution of the electrons in the domain is influenced by the point-source cathode position, with little homogenization from $\mathbf{E} \times \mathbf{B}$ drift. Physical collisions with the thruster and mirroring events in the sheath lead to some azimuthal homogenization near the exit plane. The spatially resolved electron energy distribution functions indicate that in some regions of the near-field, the distributions are multi-peaked, though more dramatically so than those indicated in the probe measurements of Fedotov et al [30]. The effect of potential fluctuations on smoothing these distributions will be the subject of future research.

Collisions with the front-face of the thruster are found to be very important to the near-field transport with 75% of channel-bound electrons suffering a collision during their lifetime (once every 45 ns, on average). This rate is nearly $10^3$ times greater than the rate that plume-bound electrons strike the thruster (once every ~3.3 µs, on average). For those electrons which collide with the thruster, we can estimate a random-walk diffusion coefficient as $D_{\text{rand}} = r V^2$. Considering only those electrons which suffer a collision (which are almost exclusively those which reach the channel), $D_{\text{rand}}$ has a mean value of $8.8 \times 10^3 \, \text{m}^2 \, \text{s}^{-1}$ (where $V_e$ is taken to be the mean velocity of the electron in the axial direction over the entire extent of its lifetime), with a peak in the distribution at $7.7 \times 10^3 \, \text{m}^2 \, \text{s}^{-1}$. In simulating the near-field region, hybrid-PIC models [4] often assume a Bohm diffusion coefficient which may be expressed as $D_{\text{Bohm}} = k T_e/(16 e |B|)$. In the near field, $k T_e/e$ is approximately 20 V, and the magnetic field strength is on the order of 20 G. Thus, $D_{\text{Bohm}}$ has a value on the order of $6 \times 10^2 \, \text{m}^2 \, \text{s}^{-1}$ (though some regions in the near-field may have values below $10^2 \, \text{m}^2 \, \text{s}^{-1}$ or exceeding $10^3 \, \text{m}^2 \, \text{s}^{-1}$). Measurements [8] made on the thruster simulated in this work indicate that the diffusion coefficient is between $k T_e/(100 e |B|) \sim 1 \times 10^2 \, \text{m}^2 \, \text{s}^{-1}$ and $k T_e/e |B| \sim 1 \times 10^3 \, \text{m}^2 \, \text{s}^{-1}$ very near the exit plane. Therefore, the random-walk diffusion coefficient predicted by these simulations for channel-bound electrons is greater than the expected Bohm diffusion coefficient by about an order of magnitude and is bounded by the experimental measurements. In other words, the random-walk diffusion can easily account for the transport towards the channel seen in the near-field. Collisions are less important for transport away from the thruster as only 4% of these electrons suffer collisions with the thruster. We note that the finite domain studied here must be taken into consideration as some of the ‘plume-bound’ electrons could return in a large-radius gyro-orbit, depending on their momentum. Also, our simulations do not account for the possibility for the return of electrons that exit the domain into channel.

Over the channel (i.e. $3.6 \, \text{cm} \leq r \leq 4.6 \, \text{cm}$) the azimuthally averaged value of the ratio of $\beta = V_e/V_{E \times B}$ i.e., $(\beta)_0$, varies between a maximum of 6.0 when $1.0 \, \text{cm} \leq z \leq 2.0 \, \text{cm}$ and a minimum of −1.3 for $4.0 \, \text{cm} \leq z \leq 5.0 \, \text{cm}$ with an overall average of 0.41 and standard deviation of 2.16. Taking the ratio of averages of $V_e$ and $V_{E \times B}$ (i.e. $(\beta) = (V_e)/(V_{E \times B})$, in contrast to $(\beta) = (V_e)/(V_{E \times B})$, the azimuthally averaged value, $(\beta)_0$, varies from a maximum
of 38.0 for 7 cm $\leq z \leq 8$ cm to a minimum of $-10.8$ for 3 cm $\leq z \leq 4$ cm with an overall mean of 6.2 and standard deviation of 14.4. Closer to the exit plane of the thruster, $(\beta')_a = -0.32$ for 0 cm $\leq z \leq 1$ cm and $(\beta')_a = -0.28$ for 1 cm $\leq z \leq 2$ cm. In this region, over the channel, particularly near the exit plane where $E$ and $B$ are orthogonal, $(\beta')_a$ approaches the inverse Hall parameter. While $(\beta')_a$ in this region is larger than the $1/16$ value of the effective inverse Hall parameter predicted by Bohm diffusion [10], channel-bound electrons are largely responsible for the largest values near the exit plane (see figure 10(d) for the distribution of $\beta'$, and compare the values of $V_E$ and $V_{E\times B}$ in figures 9, 10(a) and 11). Also, our simulations do not account for the possibility of the return of electrons that exit the domain into channel. The relatively large value of $(\beta')_a$ near the exit plane where it may be compared with the inverse Hall parameter is encouraging as preliminary results using two-dimensional hybrid simulations indicate that stability/convergence of a hybrid-PIC model, and the ability to capture properties seen experimentally, cannot be achieved without having an inverse Hall parameter, $(\omega_{ce}\tau_d)^{-1}$, in the near field of about 0.2 to 0.33 [31]. Since channel-bound electrons were found to suffer $10^3$ times as many collisions as plume-bound electrons, it is clear that collisions with the thruster are responsible for the largest values of $(\beta')_a$ near the exit plane (and the large effective inverse Hall parameter) and thus may be responsible for inducing non-local cross-field transport in the near field between the cathode and thruster channel.

The results are moderately sensitive to the character of these collisions (specular versus diffuse), and specular collisions result in a $>50\%$ reduction in the current reaching the channel compared with that assuming diffuse collisions. The effects of varying the sheath thickness and potential drop were not studied here, but may also have a similar effect on transport. Accounting for electron-gas phase collisions, even when considered at a rate $10^7$ times that expected at the exit plane, did not change the results appreciably. The electron density remained azimuthally non-uniform. However, additional homogenization from azimuthal fluctuations often seen in coaxial Hall discharges [11, 28, 29] may lead to a more uniform plasma density. While sufficient cross-field transport of electrons can be achieved without invoking field fluctuations, future simulations will include these effects to better understand the potential role played by fluctuations in this near-field region.

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