Placation and Provocation

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Abstract

This paper relates the seemingly disparate phenomena of industries self-regulating and ide-
alistic revolutionaries aggravating the populace whom they wish to better. In both situations
the agent, be it the industry or the revolutionary, is taking an action that appears to be against
her own interests. This paper models this action as a strategic maneuver against a second player
who has fixed costs to responding. The industry faces the government who has a fixed cost to
enacting legislation and the revolutionaries face a populace that has a fixed cost to organizing.
In subgame perfect equilibrium, the first agent may move the policy such that it just passes
the threshold to dissuade or encourage action from the second agent. Other applications are
discussed and extensions are explored.

1 Introduction

Many industries regulate themselves. The origin of the Motion Picture Association of America
(MPAA) is a typical example. The story of the MPAA begins in the 1920’s. Films during this era
became more lascivious and morally ambiguous. Hollywood was sensationalized by the media as Sin
City. In 1915, the Supreme Court case Mutual Film Corporation v. Industrial Commission of Ohio[1]
ruled that motion pictures were not covered by the First Amendment and that the government had
the power to censor film. The threat of federal regulation loomed.

Hollywood’s solution to this problem was to form their own censorship entity. By self-regulating,
they could restrict themselves minimally while still placating the government. A film history
textbook explains [Mast and Kawin][1992]:

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[1]Citation needed.
Such notoriety brought the film business to the attention of the United States Congress and to the edge of federal censorship—the last thing any producer wanted. The industry decided once again to clean its own house, to serve as its own censorship body . . .

The loose informal advising of the Hays Office in the 1920’s was another in a series of successful Hollywood attempts to keep films out of the hands of government censors.

This paper models this interaction as a game with two agents who have single-peaked preferences over a single dimension. The agents play an extensive-form game where the first agent has a marginal cost associated with moving the policy and the second agent has a fixed cost. Under the right conditions, this structure provides strategic incentives for the first player to move the policy such that it barely dissuades or barely encourages the second player to take action.

In the previous example, Hollywood moves first and the government follows. Congress has fixed costs to establishing a censorship bureau. Both care about the level of “iniquity” in film. A high level of “iniquity” means more profits for the industry (since sex and violence sell) but upsets the sensibilities of those in Congress. Hollywood discourages Congress from taking any action by voluntarily lowering the level of “iniquity”. Ultimately, this leaves the total level of violence and sex higher than if Congress had taken action.

A very similar interaction can be observed between states and political agitators. It is often said that revolutionaries want to make the situation for the people worse in order to spur them to action. There are tremendous fixed costs to organizing and starting a revolution because there is a major collective action problem. The people will not organize unless the situation is sufficiently grim. As Pliny the Elder said, “Reform is the enemy of revolution.” Thus, it may be in the best interest of the vanguard to make the condition for the people worse in order to start a revolt.

This was the philosophy of the Baader-Meinhof Gang of Cold War West Germany. Quoting Huffman (2007), “They felt that the proletariat was enslaved in the temporary comforts of a crippling Capitalist society, and they needed to be educated by an enlightened elite; they clearly felt that this was their role. The way to educate the people was to attack the state, provoke a massive ‘fascist’ response, and have the proletariat ‘learn’ the true nature of the German beast.”

In this example, the revolutionaries move first and the people follow. Both care about the socio-economic condition of the people. Contrary to the Hollywood example, the revolutionaries make the situation worse for people, not to dissuade the people from action but to encourage action.

Paradoxically, both Hollywood and the revolutionaries move the policy in the opposite direction of their interest for strategic reasons. This phenomena is the essence of the model.

In this paper we capture many disparate phenomena and unite them under a simple conceptual framework. We show that when the first player has a higher bias, this may actually help the second player. We show that higher fixed costs for the second player may be a strategic advantage, because it may discourage provocation on the part of the first player. We show that if the first

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2Thanks to Michael Katz for the suggestion.
player factions into multiple first movers, the placation and provocation equilibrium become even more robust.

Maxwell et al. (2000) present a model where polluting firms reduce their emissions to preempt political lobbying on the part of consumers. Demarzo et al. (2005) present a model where an industry forms a self-regulating organization that monitors and enforces policies against fraud. The threat of government enforcement causes the self-regulating organization to improve enforcement just enough to deter entry. The intuition behind self-regulation in these model is very similar to the model in this paper. However, neither model can generate the “revolution” equilibrium where an agent will exacerbate a situation to encourage action.

Acemoglu and Robinson (2001) develop a dynamic infinitely repeated game to model class clash. A government may be either a democracy where the poor choose the tax rate or a nondemocracy where the rich choose the tax rate. The party not in power may choose to pay a fixed cost to revolt thereby becoming the party in power in the next period. They model revolution and equilibrium in terms of placation. The party in power must decide to placate the party not in power to deter a revolution or a coup. They do not interpret revolutionary behavior in the same sense as in this paper, where the vanguard of the revolution actually wants to make the situation worse in order to rise the people to revolt.

The next section sets up the model. Section 3 characterizes the equilibrium regimes and applies them to diverse situations. Section 4 explores the optimal fixed cost for the fixed-cost player. Section 5 examines a behavioral interpretation of the model. Section 6 extends the model to include more than two players, and Section 7 concludes.

2 Model

The game is an extensive-form game with two players, \( i = \{1, 2\} \), who have a single-peaked strictly concave felicity function, \( f_i(\cdot) \), over a single policy dimension \( q \). At the beginning of the game the default policy is \( \bar{q} \). Player 1 acts first and can choose to move the policy an amount, \( m_1 \in \mathbb{R} \). Then Player 2 may move the policy \( m_2 \in \mathbb{R} \). The realized policy \( q = \bar{q} + m_1 + m_2 \) determines the utility of both players.

The bliss point is defined as the realized policy, \( q \), that maximizes the felicity function. Without loss of generality, Player 2 has bliss point 0 and Player 1 has bliss point \( b \), where it is assumed, \( b \geq 0 \). The parameter \( b \) is the bias of Player 1 relative to Player 2. When \( b \) is small, the bliss points of both players are near, and when \( b \) is large the bliss points are distant.

Both players have a weakly convex cost function, that is positive and monotonically increasing in the magnitude of Player \( i \)’s action, \( |m_i| \), and where initial costs and initial marginal costs are zero, \( C_i(0) = 0 \) and \( C_i'(0) = 0 \). Furthermore, we assume the cost function is symmetric so that \( C_i(x) = C_i(-x) \). The main difference between the two utility functions is that Player 2 has a fixed cost of \( e \) to moving the policy a nonzero distance, \( m_2 \neq 0 \). The utility functions are given by:
\[
U_1(m_1, m_2) = f_1(\bar{q} + m_1 + m_2) - C_1(m_1)
\]
\[
U_2(m_1, m_2) = f_2(\bar{q} + m_1 + m_2) - C_2(m_2) - e \ast 1\{m_2 \neq 0\}
\]

Notice that if Player 2 chooses to pay the fixed costs associated with taking an action, then Player 2 will move the realized policy \(q\) such that it maximizes \(U_2(m_1, m_2)\). However, if this gain in utility is not worth the fixed cost of taking an action, then no action will be taken. This fixed cost produces an inaction zone around Player 2’s bliss point. Let \(q_L\) and \(q_U\) denote the lower and upper bounds of the inaction zone respectively. Define the inaction zone as the interval \([q_L, q_U]\) where if \(m_1 + \bar{q} \in [q_L, q_U]\), then Player 2’s optimal action is \(m_2 = 0\). Thus, Player 2’s bliss point in the inaction zone, \(0 \in [q_L, q_U]\). The subgame perfect equilibrium will depend on the location of the two bliss points, the span and location of the inaction zone, and the location of the default policy.

A key assumption in the model is that Player 2 does not have the power to directly penalize Player 1. The government could change the level of “iniquity” in the media, but they cannot directly penalize the industry in this model. This is a realistic assumption in some cases.

Another issue that we should address is the origin of the default policy, \(\bar{q}\). In many political economy models the default policy is considered exogenous. This paper takes the same approach and so follows the literature. Our interpretation is that the location \(\bar{q}\) was determined as the outcome of some other unrelated interaction long ago. The fact that movies had lots of sex and violence in the 1930’s may be a result of many socioeconomic factors. The fact that the populace are oppressed may be a result of the long political economic history of that state. Both of these causes are complex and beyond the scope of the model and so the assumption that \(\bar{q}\) is exogenous is reasonable.

### 3 Equilibrium and Applications

#### 3.1 The Five Equilibrium Regimes

Since this is an extensive game with perfect information, the equilibrium concept we employ is subgame perfect equilibrium. Generically, the parameters uniquely determine the pure-strategy subgame perfect equilibrium. There are five equilibrium regimes that are qualitatively different. These equilibria will be explored in the context of three examples in the following sections. A few of these regimes have multiple interpretations depending on the values of the parameters.

We shall introduce some notation to help characterize the equilibria. Let \(m_2^*(m_1)\) be Player 2’s optimal strategy after observing Player 1’s strategy. Define the set of Player 1’s actions that leave the policy within Player 2’s inaction zone, \(M_1^{In} = \{m_1 : m_1 + \bar{q} \in [q_L, q_U]\}\), and define the set
of Player 1’s actions that leave the policy outside of Player 2’s inaction zone to be \( M_1^\text{Out} = \{ m_1 : m_1 + \bar{q} \notin (q_L, q_U) \} \). Define \( m_1^\text{Out} \in M_1^\text{Out} \) as an action for Player 1 that satisfies \( f'_1(\bar{q} + m_1^\text{Out} + m_2^*(m_1^\text{Out})) \cdot (1 + m_2^*(m_1^\text{Out})) = C'_1(m_1^\text{Out}) \) and define \( m_1^\text{In} \in M_1^\text{In} \) as an action for Player 1 that satisfies \( f'_1(\bar{q} + m_1^\text{In}) = C'_1(m_1^\text{In}) \).

**Lemma 1** If \( C_2(\cdot) \neq 0 \), in subgame perfect equilibrium, the \( m_1^* \) that maximizes \( U_1(m_1, m_2) \) must be \( m_1 \in \{ m_1^\text{Out}, m_1^\text{In}, q_L - \bar{q}, q_U - \bar{q} \} \).

**Proof** We begin using backwards induction by finding Player 2’s optimal strategy as a function of Player 1’s strategy. If \( m_1 \in M_1^\text{In} \), then by construction, Player 2’s fixed cost exceeds any benefit from taking a nonzero action. Thus if \( m_1 \in M_1^\text{In} \), then \( m_2^*(m_1) = 0 \). If \( m_1 \in M_1^\text{Out} \), then by construction, Player 2’s benefit from moving the policy optimally exceeds the fixed cost. It must be the case that the optimal action satisfies the first order condition. Thus \( m_2^*(m_1) \) satisfies \( f'_2(\bar{q} + m_1 + m_2^*(m_1)) = C'_2(m_2^*(m_1)) \). This is a maximum because the second derivative of the utility function is always negative. We now need to show existence of an \( m_2 \) that satisfies this condition. From single-peakedness we know that \( f'_2(x + a) > 0 \) for \( x < -a \), \( f'_2(x + a) = 0 \) for \( x = -a \), and \( f'_2(x + a) < 0 \) for \( x > -a \). From symmetry and the other conditions on the cost function we have \( C'_2(x) \geq 0 \) for \( x \geq 0 \), \( C'_2(x) = 0 \) for \( x = 0 \), and \( C'_2(x) \leq 0 \) for \( x \leq 0 \). Thus \( f'_2(x + a) > 0 \geq C'_2(x) \) for \( x < -a \) and \( x \leq 0 \), and \( f'_2(y + a) < 0 \leq C'_2(y) \) for \( y > -a \) and \( y \geq 0 \). By the Intermediate Value Theorem, there exists a \( z \in [x, y] \) such that \( f'_2(z + a) = C'_2(z) \). This gives us existence. This point is unique by virtue of the fact that both functions are monotonic, specifically \( f'_2(\cdot) < 0 \) and \( C''_2(\cdot) \geq 0 \).

Now that we’ve characterized Player 2’s optimal strategy, we can solve for Player 1’s optimal strategy. First we solve for the optimal \( m_1 \) conditional on \( m_1 \in M_1^\text{In} \). Assume, for the moment, that there exists a maximum in the interior of the domain. At this maximum \( m_1 \) must satisfy the first-order condition and so the optimal strategy will be \( m_1^\text{In} \). Assuming that there is a maximum in the interior, this \( m_1^\text{In} \) exists and is unique (the logic is the same as in the preceding paragraph). If there is no maximum in the interior, it implies that the maximum must be on one of the bounds, \( q_L \), or \( q_U \).

Second, we wish to solve the optimal \( m_1 \) conditional on \( m_1 \in M_1^\text{Out} \). Assume, for the moment, that there exists a maximum in the interior of the domain. At this maximum \( m_1 \) must satisfy the first-order condition and so the optimal strategy will be \( m_1^\text{Out} \). We now wish to show existence. As long as we show that \(-1 < m_2^*(m_1^\text{Out}) < 0 \), we can apply the Intermediate Value Theorem to the first-order condition to guarantee an intersection. Remember that \( m_2^*(m_1) \) satisfies Player 2’s first-order condition, \( f'_2(\bar{q} + m_1 + m_2^*(m_1)) = C'_2(m_2^*(m_1)) \). Keeping \( m_2^*(m_1) \) fixed, as \( m_1 \) increases, the left-hand side decreases and the right-hand side stays constant. This implies that \( m_2^*(m_1) \) must decrease in order to maintain equality. Decreasing \( m_2^*(m_1) \) increases the left-hand side and decreases the right-hand side. Thus \( m_2^*(m_1) < 0 \). However, suppose that \( m_2^*(m_1) < -1 \). Then any increase in \( m_1 \) would both increase the left-hand side and decrease the right-hand side,
clearly violating equality. Therefore \(-1 < m_2^*(m_1)\). This allows us to apply the Intermediate Value Theorem guaranteeing existence conditional on the maximum being in the interior. Now we must show uniqueness. Marginal costs are monotonic. We need to show that marginal benefit is strictly monotonic. If the second derivative of the benefit is negative then the intersection is unique: \(f_2''(\bar{q} + m_1 + m_2^*(m_1)) \cdot [1 + m_2^*(m_1)]^2 + f_2''(\bar{q} + m_1 + m_2^*(m_1)) \cdot m_2''(m_1) < 0\). (Still need to prove this . . .) However, it is still possible that the maximizer is not in the interior. If this is the case then one of the bounds is a maximizer. This gives us all the candidates for our lemma.

**Proposition 1** If \(C_2(\cdot) \neq 0\), the profile \((m_1^*, m_2^*(m_1))\) is the generically unique subgame perfect equilibrium if it is the profile from the set \((m_1^*, m_2^*(m_1)) \in \{(m_1^{Out}, m_2^*(m_1)), (m_1^{In}, m_2^*(m_1)), (q_L - \bar{q}, m_2^*(m_1)), (q_U - \bar{q}, m_2^*(m_1))\}\)

**Proof** From Lemma 1 there are only four candidates for Player 1’s optimal action \(m_1^*\). Generically, one of these four values will give Player 1 higher utility than the others. Player 1 will simply choose the \(m_1\) that maximizes \(U_1(m_1, m_2)\). □

**Corollary 1** If \(C_2(\cdot) = 0\), the profile \((m_1^*, m_2^*(m_1))\) is the generically unique subgame perfect equilibrium if it is the profile from the set \((m_1^*, m_2^*(m_1)) \in \{(0, m_2^*(m_1)), (m_1^{In}, m_2^*(m_1)), (q_L - \bar{q}, m_2^*(m_1)), (q_U - \bar{q}, m_2^*(m_1))\}\).

**Proof** If Player 2’s marginal costs are zero, \(C_2(\cdot) = 0\), and \(m_1 \in M_1^{Out}\), then Player 2’s optimal action is to move the policy directly to her bliss, \(m_2^*(m_1) = -\bar{q} - m_1\). If \(m_1 \in M_1^{In}\) then \(m_2^*(m_1) = 0\). The rest of the proof follows the logic from Lemma 1 and Proposition 1. □

Proposition 1 characterizes the subgame perfect equilibrium regimes of the model. The set contains the five possible equilibrium profiles. The last term \((q_U - \bar{q}, m_2^*(m_1))\) actually represents two profiles because Player 2’s optimal action, \(m_2^*(q_U - \bar{q})\), has two elements, 0 and a nonzero \(m_2\) that satisfies Player 2’s first-order condition. Corollary 1 characterizes the subgame perfect equilibrium for the special case when Player 2 has no marginal costs. The remainder of this section will explore each of these equilibrium regimes in greater detail.

### 3.2 Self-Regulation

#### 3.2.1 Placation

We begin by considering the equilibrium that occurs when Player 1’s bias is rightward of the inaction zone, \(b > q_U\), and the default policy is rightward of the inaction zone, \(\bar{q} \geq q_U\). This is the region in Figure 1 labelled “Sacrificial Placate”. This region results in an equilibrium regime that parallels the story of the MPAA who placates the government by self-regulating just enough to deter government regulation. The industry pushes the policy away from her bliss, reducing
the policy to a level that is acceptable to the government. This leaves the government indifferent between regulating and taking no action. This is illustrated in Figure 1a. The solid arrow indicates where Player 1 moves the policy. We label this a “sacrificial placation” equilibrium because Player 1 moves the policy away from her bliss in order to placate the government.

The resulting equilibrium profile has Player 1, the industry, moving the policy to the upper bound of the inaction zone, \( m_1 = q_U - \bar{q} \), and has Player 2, the government, taking no action, \( m_2 = 0 \). For this to be an equilibrium, Player 1’s equilibrium utility for this action must be greater than for any action that results in having the policy placed outside of the inaction zone, \( U_1(q_U - \bar{q}, 0) \geq U_1(m_1, m_2^*(m_1)) \forall m_1 \in M_{Out}^1 \).

A second region of the parameter space that induces the same equilibrium profile exists when the bias of Player 1 is very large, at least \( b > q_U \), and the default policy begins either within the inaction zone or leftward of the inaction zone, \( \bar{q} < q_U \). This is the centered shaded region in Figure 4 that is labeled “Extraction”. We can explore this parameter space with a counterfactual: suppose the decade of 1920’s began with films that contained only completely unobjectionable content. The possibility of federal censure on Hollywood would be nil. However, the profits of film industry would also be considerably lower since violence and sex sells. In this state of affairs, it would be in the film industry’s best interest to increase the level of sex and violence up to the point where the government is indifferent between censorship and no action. This is the same equilibrium regime, but since we begin in a different region of the parameter space, the interpretation is that the industry is squeezing out as much profit as they can from the government’s immobility. Figure 1c illustrates this last interpretation labeled as the “extraction” equilibrium. The utility conditions to induce this equilibrium are exactly the same as the conditions for the willing placation equilibrium.

Now consider the case when the players preferences are more closely aligned. Specifically, the bias is within the inaction zone \( b \leq q_U \), and the default policy remains rightward of the inaction zone, \( \bar{q} > q_U \). This is the region in Figure 4 labelled “Willing Placate”. This region induces the same equilibrium profile as the sacrificial placation, but the interpretation is slightly different.

Imagine there is a firm that pollutes heavily. The firm wishes to maximize profits but also cares about corporate social responsibility. The manager is willing to sacrifice some profit in order to run a more green business. Suppose that based solely on the manager’s preferences, the manager would choose to reduce the pollution level. However, the manager’s optimal amount of pollution is still far above the government’s optimal amount, and it exceeds the maximum that the government would tolerate without taking action. The manager must clean up even more than her optimum in order to dissuade the government. In this example, the manager is indeed pulling the policy toward her bliss, however, she is pulling the policy so far that her marginal benefit is less than her marginal cost. This action is still a best response because it dissuades Player 2. We call this a “willing placation” equilibrium because Player 1 is moving the policy in the direction of her bliss but she must do so more than she optimally desires in order to placate Player 2. This is illustrated
For this to be an equilibrium it must be the case that the action, \( m_1 \) that satisfies Player 1’s first order condition must move the policy rightward of the inaction zone, \( m_1 + \overline{q} \geq q_U \), and \( b \) must be sufficiently large so that Player 1’s equilibrium utility must be greater than any action that results in having the policy fall out of the inaction zone, \( U_1(q_U - \overline{q}, 0) \geq U_1(m_1, m_2^*(m_1)) \) \( \forall m_1 \in M_1^{Out} \).

Figure 1: Placation and Gravity Equilibria

3.2.2 Gravity

The previous three examples interpreted different regions of the parameter space that induced the same equilibrium profile. Now, we consider a second equilibrium regime. The regions of extreme default policies, far to the left and far to the right where \( \overline{q} \notin (q_L, q_U) \), and where the bias, \( b \), is fairly low, are labeled in Figure 1 as “Gravity”. The parameters might fall into this region for several reasons. If self-regulation were costly because individual firms have commitment problems, or alternatively because the fixed costs for the government are small, the parameters may fall into this region. If the default policy begins as an extreme and the players preferences do not diverge too much the parameters may fall into this region as well. In either case, it may not be worthwhile for the industry to self-regulate. Self-regulation would be too costly relative to letting the government take control. If Congress had strong support for censorship, and if the fixed costs for establishing
a monitoring bureau were cheap, then Hollywood would let Congress regulate.

We can think of this as a “gravity” equilibrium. When Player 2’s (the government’s) marginal costs are nonzero, both players pull the policy toward their blisses. The lower the marginal costs of a player, the more that player will pull the policy toward her bliss (the more gravity that player exerts). In the special case when Player 2’s marginal costs are zero, Player 1 takes no action, \( m_1 = 0 \), and Player 2 moves the policy to her bliss, \( m_2 = -\bar{q} \). This special case is depicted in Figure 1.d. In Figure 1.d, the dotted arrow indicates where Player 2 moves the policy. Specifically, Player 2 moves the policy such that it satisfies her first order condition, \( m_2 = m_2^*(m_1^*) \) and where \( m_1^* \in M_1^{Out} \), and Player 1 moves the policy such that it satisfies her first order condition \( m_1^* = m_1^{Out} \), or Player 1 chooses a corner solution, \( m_1^* \in M_1^{In} \) and \( m_1^* = \min\{q_L - \bar{q}, q_U - \bar{q}\} \). For this to be an equilibrium, Player 1’s utility in this profile must exceed the utility from moving the policy to any location within the inaction zone, \( U_1(m_1^{Out}, m_2^*(m_1^{Out})) \geq U_1(m_1, 0) \forall m_1 \in M_1^{In} \).

There is a second region of the parameter space that will induce this same equilibrium. We can have a gravity equilibrium profile where the default policy begins within the inaction zone, \( \bar{q} \in [q_L, q_U] \). The interpretation is that the current level of iniquity is not enough to encourage regulation, but even if it were, the government would be ineffectual at regulating. Hollywood best responds by producing blockbuster films filled with violence and sex. The industry knows that there will be only minor repercussions for raising the ire of Congress. We can think of this as a “struggle” equilibrium. Player 1 and Player 2 pull the policy toward their respective blisses and in the opposite directions. This can be seen in Figure 2.e. The equilibrium conditions and actions are the same as in the gravity equilibrium. For this to be an equilibrium, rather than having the bias small, it must be very large and the marginal costs of Player 2 must be fairly large as well.

### 3.2.3 Marginal and Fixed Costs

We imagine that in examples of government regulation, most of the costs that the government must incur are fixed costs rather than marginal costs. In the optimal fines literature starting with [Becker (1968)](Becker), we often expect regulators to impose strict fines rather than to invest in monitoring. The level of the fine has no effect on the regulator’s costs. Thus the regulator can set up a more strict fine structure and move the policy a great deal farther without incurring much in the way of extra costs. Perhaps some additional monitoring is also required to reach the desired policy and so this implies some marginal cost but we expect marginal costs are low. Most of the costs are fixed costs incurred from setting up the regulating entity in the first place. Thus after the regulatory board is established, as in the gravity equilibrium, we expect marginal costs to be very low.

What happens when the fixed costs approach zero? When fixed costs are sufficiently small, the default policy and Player 1’s bliss will inevitably fall outside the inaction zone for any \( q \neq 0 \).

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3 The struggle equilibrium cannot result if Player 2’s marginal costs are zero. This equilibrium is not depicted in Figure 4 because the functions used to generate the graph have \( C_2(\cdot) = 0 \).
This implies that the two choices facing the industry are to placate, or a gravity equilibrium. In the latter scenario, both players move the policy toward their blisses (which may or may not be in the same direction relative to the default). As the fixed cost decreases, placation becomes more and more costly until placation implies moving the policy to Player 2’s bliss. When the fixed costs become arbitrarily small, the only equilibrium is gravity.

**Proposition 2** Keeping all else constant, \( \forall \bar{q} \neq 0 \), there exists an \( \hat{e} \) such that for all \( 0 \leq e < \hat{e} \), the only subgame perfect equilibrium is gravity.

Thus a model that ignores fixed costs would also ignore the diversity of all the other equilibrium regimes. Fixed costs are necessary to generate placate and revolution equilibria.

### 3.3 Revolutionaries

All experience hath shown that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed.

-Thomas Jefferson, *The Declaration of Independence*

The region of the parameter space that we now analyze is where the bias of Player 1 is not too large, and the default policy begins in the inaction zone of the proletariat, \( \bar{q} \in [q_L, q_U] \), and must be near one of the two bounds. If \( \bar{q} \) is near \( q_L \) then this parameter space gives rise to a “Revolution by Extremists’ and is labeled as such in Figure 4. If \( \bar{q} \) is near \( q_U \) then this parameter space gives rise to a “Revolution by Moderates” which is also labeled in Figure 4.

The anecdote that accompanies these two equilibrium regimes is the story of the vanguard and the populace. The populace (Player 2) are dissatisfied with the oppressive regime (not a strategic player), but there is a fixed cost to revolt. The cost of organization and the cost of violent clash with the government are fixed and independent of the political goals of people. As long as the “evils are sufferable”, the people will not revolt. The vanguard (Player 1) of the revolution wishes to exacerbate the situation so that the evils are beyond sufferable, and the populace does revolt. Both Players want the policy to move in the same direction. However, Player 1 moves the policy in the opposite direction in order to induce action on the part of Player 2 and then Player 1 free rides.

As presented in the introduction, this was the strategy of the Baader-Meinhof terrorist group who tried to start a Communist revolution in West Germany. It also closely parallels the strategy of al-Zarqawi who aimed to start a civil war in Iraq following the U.S. invasion. By attacking the Shia, al-Zarqawi expected that they would respond with violence which would exacerbate the condition of the Sunni (Hashim, 2006). Our interpretation of al-Zarqawi’s strategy was to make the condition of the Sunni so dire that they would revolt against the Shia and the United States.
There are two different revolution equilibrium regimes. In the first, the revolution by extremists equilibrium, the default policy is less than the populace’s bliss point but within the inaction zone. The vanguard pushes the policy lower, away from their preferred policy in order to induce action on the part of the people. In this scenario the vanguard has more extreme preferences than the populace relative to the default policy. The vanguard moves the policy specifically to the lower bound, $q_L$, of the inaction zone. The people respond by moving the policy toward their bliss. This is illustrated in Figure 2a in the case where the marginal cost of the people is zero, $C_2(\cdot) = 0$. The solid arrow shows where Player 1 moves the policy and the dotted arrow shows where Player 2 moves the policy. When Player 2’s marginal costs are zero, she moves the policy directly to her bliss. In this equilibrium regime, Player 1 chooses $m_1 = q_L - \bar{q}$, and Player 2 best responds by satisfying the first order condition (or moving the policy directly to the bliss if $C_2(\cdot) = 0$), $m_2 = m_2^*(q_L - \bar{q}) \neq 0$. For this to be an equilibrium Player 1’s utility must satisfy $U_1(q_L - \bar{q}, m_2^*(q_L - \bar{q})) \geq U_1(m_1^{In}, 0)$ and $U_1(qL - \bar{q}, m_2^*(qL - \bar{q})) \geq U_1(m_1^{Out}, m_2^{Out}(m_1^{Out}))$ if $m_1^{Out} \in M_1^{Out}$.

In the second equilibrium regime, the vanguard moves the policy to the upper bound, $q_U$, of the inaction zone. Now the default policy is greater than the populace’s bliss. The vanguard actually has more moderate preferences than the populace relative to the default! The vanguard pushes the policy higher, away from their bliss in order to induce the populace to action. This is illustrated in Figure 2b. The vanguard knows that the populace will take the revolution too far from their perspective, but they would rather free ride on the uprising than to do nothing or to try and make the policy better directly. Perhaps the Girondin of the French Revolution fit this description. They were at the forefront at the beginning of the French Revolution even though they were much more moderate than many of the other participating political groups. In this equilibrium $m_1 = q_U - \bar{q}$ and $m_2 = m_2^*(qU - \bar{q}) \neq 0$. The conditions for this equilibrium are analogous to the revolution by extremists.

We imagine that most of the costs to revolution are fixed. The majority of costs are in getting the revolution started: organizing and arming the people. Once this has been achieved, we imagine that the strength of the organized masses easily dominates the strength of the small tyrannical regime. Storming the Bastille is the easy part. Rallying the citizens to march to the stone walls of the fortress is the hard part.

If it were the case that the marginal costs of the populace are high relative to the vanguard’s marginal costs, then the vanguard has little incentive to initiate the revolution. If the marginal costs of the populace are high, then the proletariat will not push the revolution very far. The vanguard will look for another method to improve the policy.

Another option available to the vanguard is to directly improve the condition of the populace through their own actions. The vanguard sets up a soup kitchen rather than igniting a revolt. In this equilibrium regime, the direct improvement does nothing to inspire the populace to take action. This occurs when the preference of the vanguard and the populace are not too distant (b
is sufficiently small), and the default policy begins within the inaction zone, \( \bar{q} \in [q_L, q_U] \), near Player 1’s bliss, 0. This region is labelled as “Soup Kitchen” in Figure 2. In this equilibrium, Player 1 moves the policy within the inaction zone toward her bliss, specifically to the point where the marginal cost of moving the policy equals the marginal benefit. Player 1 chooses \( m_1 = m_1^{in} \) and \( m_2 = 0 \). The final realized policy is \( q = \bar{q} + m_1^* \in [q_L, q_U] \). Since the policy is inside the inaction zone at the start of Player 2’s turn, Player 2 takes no action. Figure 2.c depicts this scenario. This equilibrium occurs when \( U_1(m_1^{in}, 0) \geq U_1(m_1, m_2^*(m_1)) \forall m_1 \in M_1^{Out} \).

When does the vanguard decide that they should set up a soup kitchen rather than organize a revolt? When is direct action to improve the situation more effective than shocking the people to action? In context of the model, does Player 1 prefer moving the policy to the nearest bound of the inaction zone, thereby moving the policy away from her bliss, or does Player 1 prefer to simply move the policy toward her bliss until marginal cost equals marginal benefit?

Let us define \( \hat{q}_E \) (and \( \hat{q}_M \)) as the cutoff values for \( \bar{q} \) at which Player 1, in equilibrium, is indifferent between moving the policy closer to her bliss or moving the policy to the lower bound (or upper bound) of the inaction zone and inducing a “revolution by extremists (moderates)”. When \( \bar{q} = \hat{q}_E \) (\( \bar{q} = \hat{q}_M \)) both a revolution by extremists (moderates) and establishing a soup kitchen are subgame perfect equilibria.

We already have a lower bound of \( \bar{q} \) for which revolution by extremists is possible and that is \( \bar{q} = q_L \). Thus the region of \( \bar{q} \) in which the subgame perfect equilibrium is a revolution by extremists is \( [q_L, \hat{q}_E] \). The equivalent expression for a revolution by moderates is \( [\hat{q}_M, q_U] \). How do these regions change as a function of the divergence in preferences, \( b \), between the two players?
Consider the revolution by extremists where Player 1 prefers to push the policy to $q_L$. As the vanguard’s interests become more extreme relative to the people, on the one hand, the vanguard trust the people less. If there is a revolution, the people are too moderate to push it far enough. This force makes revolution less attractive to the vanguard. On the other hand, as the vanguard becomes more extreme, the status quo appears worse and worse. Thus, the situation gets so bad that establishing a thousand soup kitchens would be unsatisfactory for the vanguard. It would take too much effort, so it is better to ignite the revolution. This force makes revolution more likely. In terms of the revolution by moderates, as the vanguard’s interests become more moderate relative to the people ($b$ increases), the lack of common goal makes revolution less likely.

When the felicity functions and cost functions are quadratic, and preferences are initially identical, the latter force dominates. The interval for values of $\bar{q}$ that results in a revolution by extremists, expands as $b$ increases from 0. The former force, not trusting that the revolution will go far enough, takes over once preferences are very divergent. Thus as $b$ increases further, the span of the interval of $\bar{q}$ that results in a revolution by extremists decreases. This can be observed in Figure 4. In terms of the revolution by moderates, the span of the interval that results in a revolution is strictly decreasing in $b$. When the preferences of the two players are very different, there will be no default states that lead to revolution; building a soup kitchen will always be preferred. When preferences between the populace and the vanguard are far apart, revolution is no longer an appealing option to the vanguard. The vanguard will not trust that the populace will push the revolution to the extent that the vanguard would find acceptable.

**Proposition 3** There exists a bias, $\hat{b}$, such that $\forall b > \hat{b}$, revolution no longer exists as an equilibrium for any $\bar{q}$.

Keep in mind that a revolution is a bad outcome for the people. A revolutionary equilibrium presumes that the default policy begins within the inaction zone and that the vanguard pushes it out of the zone. The people always prefers the vanguard to leave the policy within the inaction zone. In other words, the people always prefers soup kitchens because they do not want to pay the costs of revolution. Thus, there are regions of the default space where the people are actually better off when the vanguard are more extreme because the vanguard switches from agitation to soup kitchens. This result is contrary to other models in which players have single peaked preferences. Crawford and Sobel (1982) use agents with single-peaked preferences but in their model the utility of the second player in the best equilibrium is decreasing in the bias rather than increasing.

Surprisingly, it may also be the case that increasing the fixed costs of revolting may actually increase the proletariat’s utility. Figure 3 shows a graph of Player 2’s utility for some fixed $\bar{q} < 0$ and a fixed $b$. As the fixed costs for Player 2 increase, the equilibrium regime changes from gravity, to revolution by extremists, to soup kitchen. Initially, increasing the fixed costs results in a loss. The intuition is that as the fixed costs grow, there are greater costs for Player 2 to move the policy in a
gravity equilibrium. As the regime changes, increasing $e$ implies that more exacerbation is required by the vanguard to induce a revolution. When these costs become prohibitive, the vanguard decides instead to improve the situation directly. If the populace are ineffective at organizing a revolt, then the vanguard might as well put away their guns and open a soup kitchen. As the fixed costs grow further, the vanguard’s optimal action is unaltered. Once the equilibrium is a soup kitchen regime, increasing the fixed costs further has no effect on the equilibrium nor the utility of either player. We will explore the optimal fixed costs in Section 4 of this paper.

4 Optimal Fixed Costs

What is the optimal fixed cost for Player 2? In the revolution equilibrium regimes Player 1 engages in Pareto damaging behavior that lowers Player 2’s utility, and in a gravity equilibrium Player 1 may shade down her action and free ride on Player 2’s action. It may be in Player 2’s interest to have fixed costs that are large in order to deter Player 1 from revolting or free riding. This would serve as a credible commitment to Player 1 that Player 2 will take no action.

Figure 5 plots the equilibrium regimes in $q$-$e$ space for a fixed positive bias, $b$ (in this graph Player 2 has no marginal costs, $C_2(\cdot) = 0$). Notice that as Player 2’s fixed cost, $e$, increases the region of $\bar{q}$ that results in a soup kitchen equilibrium expands. The region of $\bar{q}$ that results in revolution by extremists expands into the gravity equilibrium region.

In this section we explore situations in which Player 2 can choose her own fixed costs, and in which the default policy, $\bar{q}$ is determined stochastically. First Player 2 chooses her fixed cost, $e$, then nature determines $\bar{q}$, and then the game continues as in the basic model. The interpretation is that Player 2 faces a sequence of the basic game against many different first movers. All of
these first movers have identical and known preferences. However, across these interactions, the default policies vary according to some distribution. Surprisingly, we find that there always exists an interval of $e$ in which the expected utility of Player 2 is increasing in the fixed cost $e$.

4.1 The Model

Let the utility of the two players be given by (1) and (2). Player 2 may choose a fixed cost $e$, from a menu of fixed costs, $[e_L, e_U]$. We assume that $e_L$ is sufficiently large such that $m_1(0) \leq q_u(e_L)$, where $m_1(0)$ satisfies $f_1'(m_1(0)) = C'(m_1(0))$. Let $\bar{q}$ be distributed according to the CDF $H(\bar{q})$, with continuous and atomless PDF $h(\bar{q})$ over the support $[s_L, 0]$. To limit our attention to the relevant region of Figure 5, we set the upper bound of the support to 0, and the lower bound of the support, $s_L$, must satisfy $s_L < \hat{q}(e_L)$. Define $\hat{q}(e)$ to be the value of $\bar{q}$ where both revolution by extremists and soup kitchen are equilibria. This curve is graphed in Figure 5. The timing of the game begins with Player 2 choosing $e$, then nature determines $\bar{q}$ and then the basic game continues.

\footnote{This ensures that the only equilibria that can result from the distribution are gravity, revolution by extremists, and soup kitchen. To focus on the relevant region we ignore the equilibria that result from large values of $\bar{q}$: extraction, placate, revolution of moderates and gravity in the positive region of $\bar{q}$.}

\footnote{This constraint is overly restrictive for our analysis but convenient. The largest upper bound that satisfies our analysis is $s$ where, $m_1(s) = q_u(e_L)$, where $m_1(s)$ satisfies $f_1'(m_1(s)) = C'(m_1(s))$.}

Figure 4: Equilibrium Regions in $\bar{q}$-$b$ space.
as described in Section 2.

When $e$ increases, Player 2 faces a tradeoff. Increasing $e$ expands the soup kitchen regime into the revolution and gravity regimes. Keeping $\bar{q}$ constant benefits Player 2 because Player 2’s utility is higher in a soup kitchen equilibrium, than in either a gravity equilibrium or a revolution equilibrium. Over the region of $\bar{q} \in [s_L, 0]$, in a soup kitchen equilibrium Player 1 will move the policy toward Player 2’s bliss, while in a revolution equilibrium Player 1 pushes the policy away from Player 2’s bliss, and in a gravity equilibrium Player 1 moves the policy towards Player 2’s bliss but shades this movement down (relative to a soup kitchen equilibrium) in order to induce Player 2 to assume more of the costs. Thus expanding the soup kitchen equilibrium relative to revolution and gravity, benefits Player 2. However, the expected cost of increasing $e$ is that when $\bar{q}$ does result in a gravity or a revolution equilibrium, the payoffs will be lower.

**Proposition 4** There always exists a menu of fixed costs $[e_L, e_U]$, where $e_U$ maximizes expected utility. Moreover, there exists an $e^*$ for which all $e^* < e < \hat{q}^{-1}(s_L)$, $\frac{\partial}{\partial e} EU_2(e) > 0$.

Proposition 4 states that there exists a range of $e$ in which the expected utility maximizer is the highest fixed cost. The intuition is that once $\hat{q}(e) = s_L$, the only equilibrium is soup kitchen. By decreasing $e$ slightly, Player 2 is made strictly worse off. The revolution equilibrium expands
into the soup kitchen region which lowers Player 2’s utility, and there is no benefit by lessening the costs of revolution or gravity, since there was zero probability that those equilibria would occur. This is illustrated in Figure 6.

Figure 6: Decreasing the fixed cost from $\hat{q}^{-1}(s_L)$ decreases $EU_2$.

4.2 Examples

4.2.1 Settlers and Native Relations

In American history, settlers and natives have come into conflict. The military must decide how much assistance they will grant to the settlers. The settlers are Player 1 and the military is Player 2.

In a general sense, settlers can choose to pursue relations with the natives in either a cordial manner or an aggressive manner. In our simplified abstraction, settlers care only about access to land and its resources. This may include land ownership, access to water sources, use of pasture land, etc. Let us model this interaction as we have done throughout the paper. Map the settlers’ access to the land onto a single policy dimension. A low value of $\bar{q}$ can be interpreted as natives who grant limited access to the settlers, whereas a high value of $\bar{q}$ can be interpreted as natives who grant wide access to the settlers. Moving the policy rightward toward greater access can be interpreted as having a cordial relation with the natives with the marginal cost of giving tribute. Moving the policy leftward toward lesser access can be interpreted as engaging in aggressive relations with the
marginal cost of instigating skirmishes. Thus it is costly for the settlers to move the default policy.

The military wants the settlers to have a high level of access to the land, but not as high as the settlers want for themselves. The military can attack the natives to improve the settlers’ access but doing so incurs a fixed cost. It requires logistics, recruitment, reconnaissance, etc. Thus the military will only get involved if the settlers’ access to resources is very poor. The settlers are Player 1 and the military is Player 2. If the initial access is horrible, the military will come in to help the settlers by fighting off the natives (gravity equilibrium). If the initial access is bad, this gives the settlers incentive to challenge the natives in order to draw the military into the conflict and to aid them (revolution equilibrium). If the initial access is good, this gives the settlers incentive to have good relations with the natives (soup kitchen equilibrium).

The military faces a large frontier full of many settlements interacting with different native tribes. In some cases, the settlers might have good access, and in other cases the access might be poor. Each interaction is drawn from a distribution. What if the military could control the size of the fixed costs to aiding the settlers? The military might construct forts closer to the frontier to decrease the fixed costs of intervention, or they could choose to construct forts far from the frontier in order to keep the fixed costs high.

The military will choose to build the fort depending on the distribution. If the distribution is heavily weighted toward defaults with very bad access, the military would like to place the fort near the frontier in order to easily intervene. If the distribution is weighted towards poor to mediocre access, then the military should place the fort far from the frontier in order to encourage the settlers to make some effort in extending relations to the natives.

4.2.2 Supervisor and Subordinates

Consider a firm with a hierarchical structure. The supervisor oversees many subordinates, each of whom works on his own project. A subordinate is Player 1 and the supervisor is Player 2. There is an inherent tradeoff between the quality of the final outcome of the project and the time spent on the project. Map this quality/time tradeoff into a single dimension. The subordinates have a bliss point that differs from the supervisor. The supervisor might encourage the subordinates to spend less time in order to complete more projects (or vice-versa). Projects differ in the amount of time it requires to reach a certain level of quality. This translates to a distribution of $\bar{q}$. For example, a low $\bar{q}$ can be interpreted as a project that requires a lot of time to complete it at a decent level of quality, whereas a high $\bar{q}$ can be interpreted as a project that requires little time to complete it at a decent level of quality.

The supervisor may assist the subordinates on their projects. If the supervisor has expertise that the subordinates lack, we might imagine that her marginal costs to working on a project are much lower than the subordinates’ marginal costs. However we may also expect that the supervisor has fixed costs to aiding a subordinate. It may take a large chunk of time for the supervisor to
familiarize herself with the necessary details of a project in order to be of any assistance.

When a project is very difficult (\( \bar{q} \) is very low), it is best for the supervisor to aid the subordinate (gravity equilibrium). And if a project is of easy difficulty (high \( \bar{q} \)) the subordinate finishes the project solo (soup kitchen equilibrium). However, if the project is of medium difficulty (intermediate \( \bar{q} \)), the subordinate may complicate issues or even neglect necessary maintenance in order to allow the quality to become sufficiently low to elicit the aid of the supervisor (revolution equilibrium). If many projects fall under this category, the supervisor would wish to tie her own hands in order to deter this pareto-damaging behavior.

Suppose upper management can choose what kind of supervisor to hire. They can promote someone from within the organization who has lots of detailed knowledge of the projects (low \( e \)), or they can hire an outsider who is competent but has little specific knowledge of any projects (high \( e \)). Furthermore assume upper management’s payoff to be equal to the supervisor’s payoff. If there are many projects of intermediate difficulty, upper management might hire the outsider in order to discourage subordinates from sabotaging the projects. When the subordinates know that the supervisor is incapable of helping them, they will solve their problems on their own.

5 Behavioral Interpretations of the Model

The model can apply to a “multiple selves” model of human behavior. Dynamically inconsistent behavior, as is often modeled using a quasi-hyperbolic discount function (a.k.a. \( \beta/\delta \) model), is a popular model that uses the concept multiple selves. It is possible that one self has marginal costs to change some state of the world while another self has fixed costs. I explore this possibility with three examples.

5.1 The Diet and Messy Friends

Suppose we have a dynamically inconsistent person who has present-biased preferences named George. George is on a diet. On day one he goes food shopping at the market, and on the second day he eats whatever he buys. On day one, George is Player 1 and on day two George is Player 2. George really wants to lose weight but he has self-control problems; he has problems committing to his diet. George knows from past experience that if his meal does not include a dessert, he gets really antsy and will walk down the block to the local convenience store and buy his favorite candy bar. Ideally he would like to only eat fresh vegetables and tofu for his meal the next day. However, he is aware that if he does not provide a dessert for himself, he will probably end up buying an unhealthy candy bar. We assume there is a fixed cost of time and effort associated with the walk down to the local store. If George buys a small square of chocolate, this will probably be sufficient to discourage himself from going to the local store the next day.

This is a placate equilibrium analogous to the regulation example and illustrated in Figure 1.a.
In this example, Player 1 has zero costs ($C_1(m_1) = 0$). The example assumes that George only cares about calories and not about money so choosing what calorie level to consume is costless to day-one George. But on day two there is a fixed cost to changing his meal plan.

We can imagine there are instances in which a person will try to induce a revolution equilibrium on oneself. The structure of the situation must be such that there is an activity that an individual at $t=1$ wants to engage in at time $t=2$, but a fixed cost at time $t=2$ will dissuade action. Furthermore, for the model to apply there must exist some exacerbation that one can impose on one’s future self that causes no immediate harm.

Let us return to George who has procrastination problems. George likes his house to be clean but he rarely musters the inspiration to clean up. He plans to clean tomorrow, but he knows from past experience that he often reneges on his plans when the time comes. Only if the mess is severe will he actually follow through. So George decides to invite his messy friends over for dinner. George does not particularly enjoy the company of these acquaintances but he knows that they will create enough of a mess to serve as a commitment device for George to engage in cleaning tomorrow. This is a revolution equilibrium as depicted in Figure 2.

5.2 Work and Play

I never get enough sleep. I stay up late at night, ‘cause I’m Night Guy. Night Guy wants to stay up late. What about getting up after five hours of sleep? Oh, that’s Morning Guy’s problem. That’s not my problem, I’m Night Guy. I stay up as late as I want. So you get up in the morning, you’re exhausted, groggy–oooh, I hate that Night Guy! See, Night Guy always screws Morning Guy. There’s nothing Morning Guy can do. The only thing Morning Guy can do is to try and oversleep often enough so that Day Guy looses his job, and Night Guy has no money to go out anymore.

-Seinfeld

George’s self-destructive behavior extends beyond cleaning the house. George aims to have an ideal balance between work and leisure over the course of his Fridays and Saturdays. George only cares about the displeasure of effort on the day that he exerts the effort. However, there is an amount of work that George would like to accomplish. Map these preferences onto a single dimension of the amount of work that needs to be finished. A low $\bar{q}$ means that more work needs to be accomplished to give George the same felicity as a higher $\bar{q}$. We can imagine that George has the same bliss points on both days. George has a fixed cost to work on Saturday: he must go out of his way to drive to the office, whereas on a typical weekday this is psychologically a sunk cost. On Friday George is Player 1, and on Saturday George is Player 2.

Various exogenous circumstances affect how much effort is required of George to get his ideal level of work done. On tough weeks (low $\bar{q}$) George works a fair amount on Friday and goes to
the office on Saturday (gravity equilibrium). He knows he will go to the office on Saturday and so he shades down his effort on Friday. On easy weeks (high $\bar{\eta}$), George works on Friday and takes Saturday off (soup kitchen equilibrium). On weeks of moderate difficulty (intermediate $\bar{\eta}$), George knows that he will have trouble committing to going to the office on Saturday due to the commute. He will only go if there is enough work to warrant it. So on Friday, rather than working very hard, George procrastinates and allows the work to pile up. George parties hard on Friday because he knows it is the only way to force himself to work on Saturday. George needs the stress and pressure in order to overcome the fixed cost of commuting to the office on the weekend (revolution equilibrium).

Over the course of George’s job tenure, the work load of the week will be drawn from a distribution. On some weeks George will have to put more effort to get the same felicity as he does in other weeks. Before he starts his job, George can decide how far from work he will live and consequently determine his fixed costs ($e$) to attending the office on the weekend. He must balance out the costs of commuting on Saturday (gravity and revolution) and the downward shading of work on Friday, against having a good commitment device to not work on Saturday (soup kitchen). If the typical work load is moderate, George might choose to live farther away as a commitment device to not attend work on Saturday, thereby encouraging himself to be more productive on Friday. Here George may optimize by separating his work from his leisure.

This anecdote is not here to suggest that present-biased preferences and self-provocation is an important issue for housing choice. Rather, this is an explanation of why a person might want costly barriers between their work and their leisure. These costly barriers make the time at work more productive.

6 Multiple Simultaneous First Movers

In the previous sections, we modeled a whole industry and a group of revolutionaries as single agents. Modeling these entities as single agents may have been a strong assumption. In this section we extend the model to allow for multiple first movers. An industry can now be modeled as an arbitrary number of firms, and a revolutionary vanguard can be modeled as an arbitrary number of factions.

One of the main concepts in this paper is that there are equilibria where the first mover pushes a policy away from her bliss (i.e. placate and revolution equilibrium regimes). In this section we find that these equilibria remain robust when allowing for an arbitrary number of first movers. Furthermore, these equilibria actually become more favorable relative to the equilibria where first movers move the policy toward their blisses. The intuition is that the latter equilibrium regimes, specifically gravity and soup kitchen, exhibit free-riding effects while the former equilibrium regimes, specifically placate and revolution, involve a provision point that eliminates any free riding. All the first movers are pivotal.
6.1 Many Firms

In the self-regulation example, the industry was modeled as a single agent. Alternatively, we might imagine that the industry is composed of \( N \) identical firms, with the same bliss point, that move simultaneously. How does the number of firms, \( N \), affect equilibrium behavior?

Suppose Hollywood is composed of \( N \) identical firms and the federal government is considering censorship. We construct the model to keep the total costs and benefits of the industry invariant to the number of studios. Let \( M = \sum_{k=1}^{N} m_k \) and \( M_{-i} = M - m_i \). The new utility functions are:

\[
U_i(m_i, m_{-i}) = \frac{1}{N} f_F(\bar{q} + M + m_G) - m_i * c_F
\]

(3)

\[
U_G(m_{-G}, m_G) = f_G(\bar{q} + M + m_G) - C_G(m_G) - e * 1\{m_G \neq 0\}
\]

(4)

Notice that \( \sum_{i=1}^{N} U_i(m_i, m_{-i}) \) is equal to the original utility function in (1) except now individual firm cost functions have constant marginal cost. This cost function scales to keep total industry costs invariant to \( N \). Even though constant marginal cost is a break from the basic model, it is considerably more tractable and the analysis does not differ substantially from a model with increasing marginal costs.

In our original model, we compared the placate equilibrium to the gravity equilibrium. When the default policy is very large and outside the inaction zone, self-regulation is too costly and so a gravity equilibrium results. If the default policy were rightward (outside) of the inaction zone but sufficiently low, the industry best responds by self-regulating, thereby deterring government entry. Ceteris paribus, there exists a single value of \( \bar{q} \) for which both are equilibria (as \( b \) changes this single point becomes a curve which is shown separating the gravity region from the placate region in Figure 4).

By introducing multiple firms in this extended model the game acquires an aspect of coordination. Equilibria are no longer generically unique. There may be a region where placation and gravity are equilibria depending on what firms coordinate on. As \( N \) increases all equilibria become more robust. The intuition is that as the number of firms increases (and the total utility of the industry is kept constant), each firm has a smaller influence on the market. A small firm will incur exorbitant costs relative to its size, if it deviates in an attempt to substantially change the final policy.

First we consider the multiple player equivalent of the placate equilibrium. Total action by the first movers moves the default policy to the upper bound of the inaction zone, \( q_U \). Let us consider only a symmetric placate equilibrium. This is the placate equilibrium that can be induced from the largest interval of \( \bar{q} \). Asymmetric placate equilibria can only be induced in a strict subset of the interval. Define the region \([q_U, \bar{q}_P(N)]\) as the region of \( \bar{q} \) where a symmetric placate equilibrium
can be coordinated upon.

Now consider the multiple player equivalent of the gravity equilibrium. Define the region \([\hat{q}_G(N), \infty)\) as the region of \(\hat{q}\) where a gravity equilibrium can be coordinated upon. We wish to know how \(\hat{q}_P(N)\) and \(\hat{q}_G(N)\) change as a function of \(N\). Notice that when there is one firm \(\hat{q}_P(1) = \hat{q}_G(1)\).

**Lemma 2** For all \(N > N', \hat{q}_G(N) > \hat{q}_P(N)\).

Lemma 2 states that when the industry is composed of many firms, the region of \(\hat{q}\) that induces a placate equilibrium, \([q_U, \hat{q}_P]\), overlaps with the region of \(\hat{q}\) that induces a gravity equilibrium, \([\hat{q}_G, \infty)\). The intuition for this is stated above: as \(N\) increases and the total industry utility remains constant, each firm has less influence to affect the final policy. Both equilibrium regimes become more robust as \(N\) grows.

Suppose \(\bar{q} \in [\hat{q}_G, \hat{q}_P]\) and the firms could communicate in advance and coordinate on a subgame perfect equilibrium. Which equilibrium regime would the firms choose? This leads us to our next proposition.

**Definition**

- \(U_{plac}(N) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i})\) in a symmetric placate equilibrium.
- \(U_{grav}(N) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i})\) in a gravity equilibrium.

**Proposition 5** There exists an \(N'\) such that for all \(N < N'\), \(U_{plac}(N) - U_{grav}(N)\) is strictly increasing in \(N\), and for all \(N \geq N'\), \(U_{plac}(N) - U_{grav}(N)\) is constant in \(N\).

Proposition 5 states that equilibrium utility in a symmetric placate equilibrium is monotonically increasing relative to the equilibrium utility in a gravity equilibrium. Thus, if firms could choose on which equilibrium to coordinate, they would tend to prefer to coordinate on the symmetric placate equilibrium when \(N\) is large. Specifically we can say that if \(N = 1\) and \(\bar{q}\) is such that both gravity and placate are equilibria, then as \(N\) increases, the symmetric placate equilibrium becomes Pareto Optimal. Note also that the government does better in this equilibrium as well.

The intuition is that there are positive externalities when a firm moves the policy in a gravity equilibrium. Moving the policy benefits the whole industry, but each firm only internalizes their individual returns. However, when we examine the placate equilibrium, the key is that all players are pivotal and thus if any one firm shirks, the policy will not reach the edge of the inaction zone and then the government will regulate. In the public goods literature we would call this target a provision point.

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6Thanks to John Morgan for drawing my attention to this.
As $N$ increases, the firms internalize less of the benefit of their actions, and hence the positive externality problem worsens in the gravity equilibrium. Since the provision point in the placate equilibrium solves the positive externalities problem, as $N$ increases the industry has a larger incentive to coordinate on a placate strategy. This is Pareto Optimal since it makes all the firms better off, and it unambiguously makes the government better off because it moves the policy toward the government’s bliss without the government incurring any costs.

In the special case where $C_G(m_g) = 0$ this result no longer holds. In a gravity equilibrium the government moves the policy all the way to their bliss and the firms take no action. The resulting utility is invariant to $N$. Thus as $N$ increases, there is no effect on the region of $\bar{q}$ that results in a gravity equilibrium.

6.2 Split Revolutionary Factions

In this subsection we consider analysis similar to the preceding section. We re-examine the question, will the vanguard set up a soup kitchen or revolt? However, this time we posit that there are $N$ identical revolutionary factions, each with the same bliss point and each of which can act independently. They have utility functions given by equation 3. They face a single populace that has a utility function given by equation 4. The default policy, $\bar{q}$ begins within the inaction zone and it is assumed without loss of generality that $b > \bar{q}$. We show that the region of $\bar{q}$ that results in both soup kitchen and revolution equilibrium regimes expands as $N$ increases. We also show that the relative equilibrium utility of the factions in a symmetric revolution equilibrium increases relative to the equilibrium utility of a soup kitchen equilibrium.

As in the previous section, when $N > 1$ the equilibrium requires coordination between the factions. There may be regions of the default space, $\bar{q}$, where both soup kitchen and revolution are equilibrium. Define $[q_L, q_R]$ as the region of $\bar{q}$ where a symmetric revolution by extremists is an equilibrium and define $[q_{sl}, q_{su}]$ as the region of $\bar{q}$ where soup kitchen is an equilibrium.

**Lemma 3** For all $N > N'$, $q_R(N) > q_{sl}(N)$.

This means that as the vanguard splinters into more and more factions, the region of $\bar{q}$ that results in a symmetric revolution of extremists expands and the region of $\bar{q}$ that results in a soup kitchen equilibrium expands as well. The intuition is the same as in the previous subsection. If we keep the total vanguard utility constant, splintering the vanguard into more factions gives each faction less power to affect the final outcome. When $N$ is large, $\bar{q}$ must be extreme for a single deviation to be profitable.

Suppose $\bar{q} \in [q_R, q_{sl}]$ and the firms could communicate in advance and coordinate on a subgame perfect equilibrium. Which equilibrium regime would the firms choose?

**Definition**
• $U_{rev}(N) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i})$ in a symmetric revolution by extremists equilibrium.

• $U_{soup}(N) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i})$ in a soup kitchen equilibrium.

**Proposition 6** There exists an $N'$ such that for all $N < N'$, $U_{rev}(N) - U_{soup}(N)$ is strictly increasing in $N$, and for all $N \geq N'$, $U_{rev}(N) - U_{soup}(N)$ is constant in $N$.

Proposition 6 states that equilibrium utility in a symmetric revolution by extremists equilibrium is monotonically increasing relative to the equilibrium utility in a soup kitchen equilibrium (the same comparison applies to the revolution by moderates equilibrium). Thus, if the factions could choose which equilibrium to coordinate on, they would tend to prefer to coordinate on the symmetric revolution by extremists equilibrium for larger values of $N$. Specifically we can say that if $N = 1$ and $\bar{q}$ is such that both revolution by extremists and soup kitchen are equilibria, then as $N$ increases the revolution equilibrium becomes preferred by all factions. However, the revolution equilibrium is not Pareto Optimal because a revolution equilibrium always makes the people (Player 2) worse off than in a soup kitchen equilibrium.

The intuition for this result is that the edge of the inaction zone, $q_L$, provides a provision point for the factions in the revolution by extremists equilibrium. There is no free-riding since all factions are pivotal. A single deviation would result in a failed revolution. Whereas in a soup kitchen equilibrium, the free-riding problem gets worse as $N$ increases.

In both this subsection and in the previous subsection (on multiple firms), the analysis is predicated on the assumption that there is perfect coordination. There also exist mixed strategy equilibria in which all $N$ firms or factions mix their amount of effort. In this type of equilibrium, there may be a high probability that the effort exerted by the firms or factions falls short of the provision point; revolution rarely occurs and placation is insufficient to deter government regulation. If we assume a mixed equilibrium and compare the default space that maps into equilibria as a function of $N$ we will not get the same result as in the previous analysis.

### Conclusion

A very general phenomenon arises from the interaction between a first mover with marginal costs to affect a policy and a second mover who has fixed costs to affect this policy. This interaction models the self-regulation of industries, revolutionaries who sew the seeds of revolt, and dieters who eat desserts.

The analysis has shown that surprisingly, the second mover might actually be made better off when preferences diverge from the first mover. The intuition is that when preferences are aligned the first mover may exacerbate the policy in order to free ride off of the second mover. When preferences are divergent, free riding will not result in an outcome that is favorable to the first mover, and thus
the first mover instead chooses to directly improve the policy. The implication is that revolutions are less likely when the vanguard have extreme preferences relative to the population.

We show that the second mover may actually choose to increase her fixed costs in order to strategically dissuade the first player from aggravating the policy. There always exists a menu of fixed costs where the optimal fixed cost is the highest fixed cost.

Finally the paper examines comparative statics as the number of first movers increases. As an industry breaks up into more and more firms, self-regulation surprisingly becomes more likely than resistance. The intuition is that resistance has a free-rider effect. If one firm resists, other firms can free ride from the first firm’s effort. However, self-regulation requires a provision point to be reached which makes all firms pivotal. The same logic also implies that as revolutionaries break up into more and more factions, they collectively become more likely to incite violence and less likely to set up a soup kitchen. This simple model yields a rich environment that applies to many situations.

Appendix

Proof of Proposition 2

When $e$ is small the inaction zone shrinks. When $e = 0$, the inaction zone collapses to Player 2’s bliss point, $q_L = 0 = q_U$. When $q \neq 0$, there always exists an $e$ sufficiently small such that $\bar{q} \notin [q_L, q_U]$. This immediately eliminates the revolution by extremists, and revolution by moderates equilibria since these can only occur if the default begins within the inaction zone.

We now wish to eliminate the soup kitchen equilibrium. In a soup kitchen equilibrium, $m_1 = m_1^{In}$. However when $e$ is sufficiently small the $m_1$ that satisfies the first order condition is not in $M_1^{In}$ and so $m_1^{In}$ does not exist. This eliminates the soup kitchen equilibrium.

It remains to be shown that the utility in a gravity equilibrium is greater than the utility in a placate equilibrium. Observe that when $e = 0$ the only equilibrium is a gravity equilibrium. The utility of Player 1 is given by

$$U_1(gravity) = f_1(m_1^*(m_2^* + \bar{q}) + \bar{q}) + C_1(m_1^*).$$  \hspace{1cm} (A.1)

When $e$ is arbitrarily small, the utility in a placate equilibrium is given by

$$U_1(placate) = f_1(0) + C_1(\bar{q}) + \delta$$ \hspace{1cm} (A.2)

$$\text{where it is possible that } \delta > 0. \text{ As } e \text{ decreases toward zero, } \delta \text{ decreases and can be made arbitrarily small.}$$

Since $(m_1, m_2) = (-\bar{q}, 0)$ is in the choice set of Player 1, it is generically true that $f_1(0) +$
$C_1(\bar{q}) \leq U_1(gravity)$. Since $\delta$ can be made arbitrarily small and is decreasing in $\epsilon$, it follows that all $\epsilon < \hat{\epsilon}$, for some $\hat{\epsilon}$, $U_1(\text{placate}) < U_1(\text{gravity})$.

**Proof of Proposition 3**

The utility of Player 1 in revolution by extremists and extract equilibrium respectively are given by

\[ U_1(\text{revolution}) = f_1(q_L + m_2^*) + C_1(\bar{q} - q_L) \tag{A.4} \]
\[ U_1(\text{extraction}) = f_1(q_U) + C_1(q_U - \bar{q}). \tag{A.5} \]

First notice that the costs are invariant to $b$. The difference in felicities, given by $f(q_U) - f_1(q_L + m_2^*)$, is positive when $b$ is large since $q_U > q_L + m_2^*$. Observe that the arguments of the felicity function are both invariant to $b$. Since the felicity function is single peaked, increasing $b$ decreases the felicities. Since the felicity function is concave, increasing $b$ increases the difference between the felicities. Thus this difference can be made arbitrarily large indicating that $U_1(\text{extraction}) > U_1(\text{revolution})$. The same argument can be made for revolution by moderates.

**Proof of Lemma 2**

First we observe that $\hat{q}_P(1) = \hat{q}_G(1)$. When there is only one firm, there is only one default policy in which the firm is indifferent between placate and gravity. If the default policy is just slightly more to the left, placate is preferred, and if slightly more to the right, gravity is preferred. We wish to show that when $N$ is large, the equilibrium regions overlap for more than just a single point.

In order to show this, we must show that $\hat{q}_P(1) > \hat{q}_P(N)$ and $\hat{q}_G(1) < \hat{q}_G(N)$ for all $N > N'$. To prove the first inequality, we specify the condition in which no firm has an incentive to deviate from a placate equilibrium. This condition is given by

\[ f_F(q_U) - (\bar{q} - q_U) * c \geq f_F \left( \frac{N - 1}{N} q_U + \frac{1}{N} \bar{q} + \tilde{m}_i(N) + m_G(\tilde{m}_i(N)) \right) - N * \tilde{m}_i(N) * c \tag{A.6} \]

where $\tilde{m}_i(N)$ is the optimal deviation. This equation holds with equality when $\bar{q} = \hat{q}_P$.

Imagine that this equation holds with equality when $N = 1$, and so $\bar{q} = \hat{q}_P(1)$. When we allow $N > 1$, the left-hand side of equation [A.6] does not change. The right-hand side of the equation, must decrease for two reasons. First, the cost function is now multiplied by $N$. Second, the other firms’ actions place the default policy farther away from the bliss point and closer to $q_U$. Thus for this inequality to continue to hold with equality, it must be the case that $\hat{q}_P(N)$ increases. Increasing $\hat{q}_P(N)$ decreases the equilibrium utility on the left-hand side and increases the deviation utility on the right-hand side.

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*For all the no deviation conditions, both sides have been multiplied by $N$. 27*
Now we show that $\hat{q}_G(1) < \hat{q}_G(N)$ for all $N > N'$. The no deviation condition for a gravity equilibrium is given by

$$f_F(\bar{q} + N \ast m_i(N) + m_G(m_i(N))) - N \ast m_i(N) \ast c \geq f_F(q_U) - N \ast (\bar{q} - q_U + (N - 1)m_i(N)) \ast c$$

(A.7)

where $m_i(N)$ is the optimal action in a gravity equilibrium. Notice that the best deviation is to placate. The equilibrium already specifies the optimal action if the action moves the policy higher. The only way that moving the policy lower can be optimal is if it placates. This equation holds with equality when $\bar{q} = \hat{q}_G(N)$.

Imagine that this equation holds with equality when $N = 1$, and so $\bar{q} = \hat{q}_G(1)$. When $N$ is very large, it must be the case that the right-hand side exceeds the left-hand side. First, observe that $m_i(N) = 0$ for large $N$. The optimal action in gravity, $m_i(N)$ will satisfy Player $i$’s first order condition. Since the marginal benefit is decreasing by $\frac{1}{N}$, there will be an $N$ sufficiently high such that the marginal cost always exceeds the marginal benefit. Thus for very large $N$, $m_i(N) = 0$. This means as $N$ gets large, the left-hand side converges to $f_F(\bar{q} + m_G(0))$. The right-hand side explodes to negative infinity. Thus in order for this equation to hold with equality, it must be the case that $\hat{q}_G(N)$ is decreasing. This lowers the equilibrium utility on the left-hand side and increases the deviation utility on the right hand side.

**Proof of Proposition 5**

Let us more carefully inspect the placate and gravity equilibrium regimes as $N$ increases so that we can determine how the equilibrium utility of these two equilibria change. First observe that the first order condition for each firm in a gravity equilibrium is given by:

$$\frac{1}{N} f_F'(\bar{q} + N \ast m^*_i + m_G) [1 + \frac{\partial}{\partial m_i} m_G] = c_F,$$

where $m^*_i$ is the equilibrium firm action. Notice that as $N$ gets larger, the marginal benefit on the left hand side of the equation decreases while marginal costs are constant. This allows us to say two important things. First, in order for this equation to hold with equality, $m^*_i$ must decrease to zero. There exists an $N'$ such that $\forall N \geq N'$, $m^*_i = 0$. Second, since equilibrium utility is the integral of the difference between marginal benefit and marginal cost, decreasing the marginal benefit curve decreases equilibrium utility. Thus the industry wide equilibrium utility, $U_{grav}(N)$, monotonically decreases with $N$.

Now let us look at equilibrium utility of a single firm in a symmetric placate equilibrium,

$$U_i(m_i, m_{-i}) = \frac{1}{N} f_F(q_U) - \frac{1}{N}(\bar{q} - q_U) \ast c_F.$$
If we sum over all \( N \) firms we find that the total industry utility is invariant to \( N \). This means that \( U_{\text{plac}}(N) \) is invariant in \( N \), but \( U_{\text{grav}}(N) \) is decreasing in \( N \) for \( N < N' \), and is invariant in \( N \) for all \( N \geq N' \).

**Proof of Lemma 3**

First we observe that \( \hat{q}_R(1) = \hat{q}_{sl}(1) \). When there is only one firm, there is only one default policy in which the firm is indifferent between revolution and soup kitchen. If the default policy is just slightly more to the left, revolution is preferred, and if slightly more to the right, soup-kitchen is preferred. We wish to show that when \( N \) is large, the equilibrium regions overlap for more than just a single point.

In order to show this, we must show that \( \hat{q}_{sl}(1) > \hat{q}_{sl}(N) \) and \( \hat{q}_R(1) < \hat{q}_R(N) \) for all \( N > N' \).

To prove the first inequality, we specify the condition in which no firm has an incentive to deviate from a soup kitchen equilibrium. This condition is given by

\[
 f_F (\bar{q} + N \ast m_i(N)) - N \ast m_i(N) \ast c \geq f_F (q_L + m_G) - N \ast (\bar{q} - q_L + (N - 1)m_i(N)) \ast c \quad (A.8)
\]

The logic we use here to show that \( \hat{q}_{sl}(N) \) is increasing is the same as the logic we used when examining the gravity equilibrium. Imagine that this equation holds with equality when \( N = 1 \), and so \( \bar{q} = \hat{q}_{sl}(1) \). When \( N \) is very large the equilibrium utility on the left-hand side converges, while the deviation utility on the right-hand side explodes to negative infinity. In order for this equation to hold with equality it must be the case that, \( \hat{q}_{sl}(N) < \hat{q}_{sl}(1) \) for all \( N < N' \).

Next, we show that \( \hat{q}_R(1) > \hat{q}_R(N) \) for all \( N > N' \). The logic is the same as in the placate equilibrium. The condition for there to be no deviation from a revolution by extremists equilibrium is given by

\[
 f_F (q_L - m_G) - (\bar{q} - q_L) \geq f_F \left( \frac{N - 1}{N} q_L + \frac{1}{N} \bar{q} + \tilde{m}_i(N) \right) - N \ast \tilde{m}_i(N) \ast c \quad (A.9)
\]

where \( \tilde{m}_i(N) \) is the optimal deviation. This deviation will move the policy upward toward the bliss such that marginal costs equal marginal benefit.

Assume this holds with equality for \( N = 1 \), and so \( \bar{q} = \hat{q}_R(1) \). The left-hand side is invariant to \( N \). The right-hand side of the equation, must decrease for two reasons. First, the cost function is now multiplied by \( N \). Second, the other firms’ actions place the default policy farther away from the bliss point and closer to \( q_L \). Thus for this equation to continue to hold, it must be the case that \( \hat{q}_R(N) \) increases. Increasing \( \hat{q}_R(N) \) decreases the equilibrium utility on the left-hand side and increases the deviation utility on the right-hand side.
Proof of Proposition \[6\]

The following analysis parallels the proof for Proposition \[5\]. First observe that the first order condition for each firm in a soup kitchen equilibrium is given by:

\[
\frac{1}{N} f_F'(\bar{q} + M - m_i + m_i) = c_F \tag{A.10}
\]

\[\Rightarrow m_i^* = \max\{f_F^{-1}(N c_F) - \bar{q}, 0\}.\]

where \(m_i^*\) is the equilibrium firm action. Notice that as \(N\) gets larger, the marginal benefit on the left hand side of the equation decreases while marginal costs are constant. This allows us to say three important things. First, in a soup kitchen equilibrium \(m_i^*\) must move the policy toward the \(i\)'s bliss, and since we assumed without loss of generality that \(\bar{q} < b\), then \(m_i^* \geq 0\). Second, since \(m_i^*\) is decreasing with \(N\), there exists an \(N'\) such that \(\forall N \geq N', m_i^* = 0\). Third, since the equilibrium utility is the integral of the difference between marginal benefit and marginal cost, decreasing the marginal benefit curve decreases equilibrium utility. Thus the total vanguard equilibrium utility monotonically decreases with \(N\). Each faction does not internalize the full benefit of their actions and so factions free-ride and shade down their effort.

When we consider the revolutionary equilibria (both extremist and moderates), they have provision points that allow the factions to avoid the free-rider problem. Total vanguard utility in a revolution by extremists equilibrium is given by

\[
\sum_{i=1}^{N} U_i(m_i, m_{-i}) = f_F(q_L - m_G) - (q_L - \bar{q}) * c,
\]

which is invariant to \(N\). Thus increasing \(N\) has no effect on \(U_{revo}(N)\), but \(U_{soup}(N)\) is decreasing in \(N\) for \(N < N'\), and is invariant in \(N\) for all \(N \geq N'\).
Proof of Proposition 4

We will prove Proposition 4 by solving for the expected utility of Player 2 and taking the limit when fixed costs are high.

\[
EU_2(e) = \text{Prob}(\text{gravity}) \ast EU_2(e \mid \text{gravity}) + \text{Prob}(\text{revolution}) \ast EU_2(e \mid \text{revolution})
\]

\[
+ \text{Prob}(\text{soup}) \ast EU_2(e \mid \text{soup})
\]

\[
= \int_{\hat{q}(e)}^{q_L(e)} f_2(\hat{q} + m_1^*(e, \hat{q}) + m_2^*(e, \hat{q})) - C_2(m_2^*(e, \hat{q})) h(\hat{q}) d\hat{q}
\]

\[
+ \int_{q_L(e)}^{\hat{q}(e)} f_2(q_L(e)) - C_2(q_L(e)) h(\hat{q}) d\hat{q}
\]

\[
+ \int_{\hat{q}(e)}^{\hat{q}(e)} f_2(q_L(e) + m_2^*(e, \hat{q})) h(\hat{q}) d\hat{q}
\]

The first term, which is the expected utility when the policy induces a gravity equilibrium, can be broken down into the sum of two expressions.

\[
\text{Prob}(\text{gravity}) \ast EU_2(e \mid \text{gravity}) =
\]

\[
\int_{x(e)}^{q_L(e)} f_2(\hat{q} + m_1^*(\hat{q}) + m_2^*(\hat{q})) - C_2(m_2^*(\hat{q})) h(\hat{q}) d\hat{q}
\]

\[
+ \int_{x(e)}^{\hat{q}(e)} f_2(q_L(e) + m_2^*(\hat{q})) - C_2(q_L(e)) h(\hat{q}) d\hat{q}
\]

The first term is where both players move the policy such that marginal cost equals marginal benefit. The second term is where Player 1 has a corner solution, and she moves the policy to \(q_L(e)\) and Player 2 moves the policy such that marginal cost equals marginal benefit. The region between \([x(e), q_L(e)]\) is where Player 1 has a corner solution. Define

\[
U_{\text{grav}}(\hat{q}, e) \equiv f_2(\hat{q} + m_1^*(e, \hat{q}) + m_2^*(e, \hat{q})) - C_2(m_2^*(e, \hat{q}))
\]

\[
U_{\text{bound}}(\hat{q}, e) \equiv f_2(q_L(e) + m_2^*(\hat{q})) - C_2(q_L(e))
\]

\[
U_{\text{soup}}(\hat{q}, e) \equiv f_2(\hat{q} + m_1^*(e, \hat{q})).
\]
Then expected utility can be rewritten as

\[ EU_2(e) = \int_{s_L}^{\hat{q}(e)} -e \cdot h(\bar{q})d\bar{q} + \int_{s_L}^{x(e)} U_{\text{grav}}(\bar{q}, e) \cdot h(\bar{q})d\bar{q} \]

\[ + \int_{\hat{q}(e)}^{0} U_{\text{bound}}(\bar{q}, e) \cdot h(\bar{q})d\bar{q} + \int_{\hat{q}(e)}^{0} U_{\text{soup}}(\bar{q}, e) \cdot h(\bar{q})d\bar{q}. \]

Now we wish to differentiate the expected utility to see how it varies as a function of \( e \). Using Leibnitz’s Rule we find that,

\[
\frac{\partial}{\partial e} EU_2(e) = -H(\hat{q}(e)) - \hat{q}'(e)h(\hat{q}(e))[U_{\text{soup}}(\hat{q}(e)) - U_{\text{bound}}(\hat{q}(e), e) + e]
\]

\[ + [H(\hat{q}(e)) - H(x(e))] f'(q_L(e) + m_2^2(q_L(e))) \cdot q_L'(e). \]

This is a sum of three terms. First we show that the second term is always positive. Then we show that both the first and third terms can be made arbitrarily small when \( e \) is large.

The difference, \( U_{\text{soup}}(\hat{q}(e)) - U_{\text{bound}}(\hat{q}(e), e) + e \), is the difference between Player 2’s utility in a soup kitchen equilibrium minus Player 2’s utility in an equilibrium in which Player 1 leaves the policy at the lower bound of the inaction zone. This difference is positive because in a soup kitchen equilibrium, Player 1 leaves the policy within the inaction zone which is generically preferred for Player 2. The term \( \hat{q}'(e) \) is negative and so the whole term is positive.

Now as \( e \) gets large, \( \hat{q}(e) \) gets smaller and moves toward the lower bound of the support \( s_L \). When \( e \geq \hat{q}^{-1}(s_L) \), then \( H(\hat{q}(e)) = 0 \). Thus for large \( e \) we can make the first term arbitrarily small. Likewise, when \( e \geq x^{-1}(s_L) \), then \( H(x(e)) = 0 \). This implies that the third term can be made arbitrarily small. Thus for large \( e \), the derivative of Player 2’s expected utility is always positive.

References


