REVIEWS OF GEOPHYSICS, SUPPLEMENT, PAGES 1103-1109, JULY 1995
U.S. NATIONAL REPORT TO INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS 1991-1994

Recent advances in geostatistical inference on hydrogeological variables

Peter K. Kitanidis

Department of Civil Engineering, Stanford University, California

Introduction

Among the challenges facing hydrogeologists are to (a) predict where the contaminants are and where they are going and (b) to design effective remediation schemes. To meet these challenges, they must analyze data in order to estimate the parameters of geologic formations that affect the flow of water or the transport of chemicals. However, because these parameters vary in complicated and insufficiently understood ways, estimates are usually approximate and hard to obtain. Statistical methods are helpful in selecting the "best" possible estimate given the information and in evaluating the confidence bounds associated with this estimate. Furthermore, they can be used to evaluate the prediction uncertainty and are thus valuable in assessing management strategies and monitoring plans.

Geostatistics started (Matheron, 1971) with simple and practical linear methods for the estimation of spatial functions from scattered observations. Many of the original applications were in the mining industry. By the early nineties, geostatistics has been applied in most areas of earth sciences. Hydrogeology has possibly been the most productive field of application where geostatistics has been combined with groundwater models, adjusted for geophysical methods, used in solving upscaling problems, and been the cornerstone in analyzing prediction uncertainty. Furthermore, geostatistics now consists of more than one schools. Some researchers are introducing methods for the improvement of linear geostatistics while others question the premises of linear geostatistics and seek alternative methods.

The focus of this review is on geostatistical inference in hydrogeology, thus excluding problems of upscaling ("effective parameters") or network design, unless they involve data analysis. The reader may want to consult also the following recent review papers. Rouhani *et al.* (1990a, 1990b) review geostatistics in geohydrology and include tutorial sections as well as extensive literature reviews. Loaiciga et al. (1992) review the literature on groundwater quality monitoring network design, including geostatistical methods. Kitanidis (1993b) presents linear geostatistical practices. Among new books, the recent book of Cressie (1991) has extensive coverage of geostatistics and Gelhar (1993) covers the stochastic perturbation-spectral approach in groundwater modeling.

Copyright 1995 by the American Geophysical Union.

Paper number 95RG00183.
8755-1209/95/95RG-00183$15.00

Methodologies

While originally the attitude in geostatistics used to be "all that matters is the experimental variogram (the average square difference between measured values as a function of the distance separating the measurements)", most practitioners now accept that the experimental variogram is an estimate of the underlying "model" variogram and that this estimate is in fact fraught with error. Shafer and Varljen (1990) describe a resampling method to approximate the confidence limits on variograms obtained experimentally as done in practice. Data resampling schemes, such as the bootstrap and the jackknife, are conceptually simple but computationally intensive methods that have the advantage over classical parametric methods that they use directly the experimental distribution. Demonstrating rigorously that the method accomplishes the set objective may be rather tricky. In this particular application, it is not immediately clear what effect the spatial correlation in the observations would have on the representativeness of the error bounds computed in the proposed method. However, the authors confirm through multiple-realization Monte Carlo experiments that the method they proposed give accurate estimates for the error bounds of the variogram even for a single realization. Farrel et al. (1994) utilized this approach in a geostatistical analysis of the Borden data (Sudicky, 1976).

While the classical approach has been to infer variograms from the experimental variogram, an alternative approach has been to determine variograms from cross-validation, least squares, or restricted maximum likelihood. Orthonormal residuals (Kitanidis, 1991) are differences between kriging estimates and observed values that are useful in variogram estimation and in diagnostic checking of the model. Kriging is an algorithm to obtain an estimate of a spatial variable at a point from other observations and the structure of the spatial variable (typically represented by the variogram). The same work showed the relation between the cross-validation method based on the orthonormal residuals and the restricted maximum likelihood parameter esti-

mation method. Another work (Van der Linde, 1993) showed theoretically the relation between the intuitive least squares estimation approach (Delfiner, 1976) with the more general quadratic unbiased estimation method (Kitanidis, 1985a) and recommended the latter as a method for estimation of parameters of generalized covariance functions. Dietrich and Osborn (1991) presented a method for the calculation of variogram parameter estimates using the restricted maximum likelihood approach.

On the topic of diagnostic checking, Chistensen et al. (1990b, 1992, 1993) adapted techniques from linear statistical inference to determine influential observations. Validation or diagnostic checking, defined as "systematic checking of the fitted model in its relation to the data" with intent to reveal model inadequacies or influential data, has been neglected for too long in geostatistical inference. Consequently, research in this area is important and should continue in order to bring geostatistics in step with other fields of applied statistical modeling.

The relation between generalized and actual covariance functions was discussed in Kitanidis (1993). The author concluded that variogram analysis of detrended data is appropriate as long as it is clear that the objective is to determine the generalized covariance function (i.e., the part of the second moment that is relevant in Best Linear Unbiased Estimation.) On a related issue, it was confirmed (Christensen, 1990a) that "universal kriging" and "kriging with generalized covariance functions" is effectively the same procedure.

Rajaram and McLaughlin (1990) discuss a procedure in which the large-scale variability is described with a different spectrum (or covariance function) than the small-scale variability and the objective is to identify the large-scale trend.

There were relatively few papers employing Bayesian methods. (In this approach to inference, Bayes rule plays an important role and unknowns are looked upon as random variables, see Savage, 1962.) Van Tonder (1990) discussed the use of prior information in the estimation of water levels using Bayesian techniques. Butchet et al. (1991) proposed a parametric empirical Bayes method for the estimation of spatial functions. The method allows the utilization of prior information, such as from other "similar" sites.

Whereas the emphasis of geostatistics has beenon spatial variability, hydrogeologic variables may fluctuate in time as well as in space. Rouhani and Myers (1990) and Rouhani and Wackernagel (1990) discussed how to model variables that vary in time as well as in space, within the classical "geostatistical" framework using empirically derived variograms.

The concept of entropy has started receiving a modicum of the attention it deserves in the context of geostatistical inference. Woodbury and Ulrych (1993) discussed the principle of maximum entropy and (in its less known generalization "minimum relative entropy") as a standard for assigning prior probability distributions to unknown parameters. Similar ideas were presented for the inverse problem of groundwater modeling in Kitanidis (1985b). Journel and Deutsch (1993) discussed entropy of a spatial function as a measure of spatial disorder and used it as a medium to discuss limitations of the model of the random field with "multi-Gaussian" distribution. They argued that realization of such a random field are characterized by a high degree of disorder, because the maximum-entropy distribution for a given covariance function is the Gaussian. In particular, they illustrated that realizations from the Gaussian distribution lack connectivity at high or low values and then conjectured that the Gaussian model is inappropriate because it cannot describe preferential flow paths or barriers. The authors did not discuss the effect that accounting for observations might have on blurring the distinction between the predictions made by Gaussian and non-Gaussian models.

Applications

Woodbury and Sudicky (1991) reanalyzed the hydraulic conductivity data at the Borden aquifer (Sudicky, 1986) in Ontario, Canada. They examined how various assumptions and methodologies affect the values of inferred parameters. They concluded that the data cannot discredit either the assumption of a normal distribution or the assumption of an exponential distribution for the log-conductivity. This conclusion was based on the analysis of a small sample of 100 observations randomly drawn from a data set of 1188 and neglecting whatever spatial dependence there might have been. They used the classical method to construct the experimental variogram, after removal of outliers (low logconductivity values). Then, through nonlinear least squares, they fitted to the experimental variogram a parametric expression for the variogram model and used an ad hoc method to evaluate the covariance matrix of estimation errors of the parameters. They found what they considered significantly different parameter estimates from those of Sudicky (1986). They used these parameters to predict how the tracer spatial moments change over time, using the stochastic macrodispersion theory of Dagan (1982, 1987). An important conclusion was that uncertainty in the values of geostatistical parameters can lead to substantial uncertainty in the prediction of macrodispersion in a heterogeneous aquifer. They concluded that even large-scale simulations using macroscopic parameters should be treated in a stochastic framework, due to inability to estimate with certainty the parameters that control the effective ("macroscopic") parameters.

Desbarats and Srivastava (1991) took a transmissivity function artificially generated over a domain without the Gaussian assumption and then solved the steady groundwater flow equations to obtain the distribution of head and specific discharge. The results of these sim-

ulations were compared with theoretical predictions using a second moment characterization of the conductivity field. The simulated field had a log-transmissivity variance of around 1, but embodied features that are perceived as "non-Gaussian", such as spatial continuity of extreme values, a bimodal histogram, strong directional anisotropy, and nested scales of heterogeneity. The results of Desbarats and Srivastava (1991) showed reasonable agreement with the predictions of "ensemble theoretic models based on perturbation developments" with respect to head covariance (Mizell et al., 1982, Rubin and Dagan, 1988), cross covariance between head and log-transmissivity (Dagan, 1984, 1985b, Rubin and Dagan, 1988), effective transmissivity (Gelhar and Axness, 1983), and transport (Dagan, 1982, 1984, Gelhar and Axness, 1983, Neuman et al., 1987).

Desbarats and Bachu (1994) presented a geostatistical analysis of hydraulic conductivity from the core scale to the well scale and to the basin scale, for a sandstone hydrostratigraphic unit in Canada. The study provides insights into the spatial structure of hydraulic conductivity in a hierarchy of scales. It also studies upscaling from local conductivity to local transmissivity (upscaling through integration over the vertical direction) and from local transmissivity to global or macroscopic transmissivity.

Keidser and Rosbjerg (1991) presented a comparison of four inverse approaches on a synthetically generated case. The four methods, which differ in the parameterization, were evaluated with respect to prediction accuracy of simulated flow and transport. It is hard to draw conclusions from this study, however, because the results may be a consequence of some unconventional steps taken in the implementation of these procedures. For example, in one of the methods, a quadratic drift with many parameters was used to represent the large-scale variability in logtransmissivity, despite the scarcity of data and the lack of any supporting evidence that such a drift would be a successful representation of spatial variability. As is unfortunately not uncommon, diagnostic checking was neglected and the principle of parsimony or Occam's razor ("use the simplest empirical model consistent with the observations", see Tuckey, 1961) was overlooked.

Hoeksema and Clapp (1990) used the stochastic approach to the inverse problem but, in order to derive second moments of the head, they used the method of Monte Carlo simulations instead of first-order analysis (Hoeksema and Kitanidis, 1984). The linear estimator was still used. Monte Carlo simulations, if properly carried out, is a more accurate method for the derivation of the second moments. As the cost of computations drops, its usefulness is bound to increase.

Sun and Yeh (1992) presented a method for the solution of the inverse problem (estimation of transmissivity using the hydraulic head) under transient conditions. In the determination of covariance functions, the adjoint state equation of the differential equation satisfied by the head perturbation was implemented. (The adjoint state equations provide a computationally efficient meathod to compute sensitivity coefficients, see Neuman, 1980.) They concluded that the utilization of head observations simultaneously at all observation times produced much better results than a previously used approach in which parameters were estimated separately at each time period and then results were averaged. The method of Sun and Yeh (1992) is more efficient in data utilization but has a high computational cost, associated with inverting large measurement covariance matrices. A more efficient computationally method might have been to decompose the problem in stages, as done in the Kalman filtering literature (Schweppe, 1973). The paper also presents a first-order (linearized) method to predict the variance of prediction errors.

Lee and Kitanidis (1991) had to solve a similar estimation problem in developing a method for the optimization of monitoring and remediation at a contaminated site. The method combined first-order analysis and sequential estimation (Kalman filtering) which is in principle more efficient computationally than simultaneous estimation. However, a very coarse grid was used in the simulations. Van Geer et al. (1991) implement a Kalman filtering as a method to update update groundwater flow predictions as observations of head become available over time. The updating in Kalman filtering is the same as the condictioning or kriging in ordinary geostatistics but it is done sequentially in "real time". Zhou et al. (1991) applied this model in the design of a monitoring network.

LaVenue and Pickens (1992) presented a methodology for the solution of the inverse problem following the pilots points approach. In this, transmissivity values at selected points, called pilot points, were fixed to match head and other data. The pilot points were then treated as observation points. The authors found the effect of changes in the transmissivity through the adjoint-state equations. The locations that are most influential on the data were selected as the pilot points. Kriging was used to condition on all real or pilot-points transmissivity observations. This interesting method of solving the inverse problem appears quite general, not limited by small-perturbation or probabilistic assumptions. However, it is ad hoc and seems that it effectively uses a large number of adjustable parameters (the transmissivities at pilot points) to reproduce the head data. It is not clear in what sense the mean square estimation errors computed in this method are realistic.

Rubin (1991a, 1991b, 1991c) described methods for using solute concentration observations to predict concentrations at other other points and times. Harvey and Gorelick (1994) presented a method for the sequential conditioning on observations of local conductivity, hydraulic head, and solute arrival time.

Most studies use hydrogeologic data obtained in wells or in laboratory tests. However, geophysical exploration

methods hold promise as valuable tools for site characterization. Geophysical exploration data, such as seismic data, must somehow be integrated with other data to yield better estimates of the parameters that affect groundwater flow and mass transport. Geostatistics provides an ideal framework and some important steps have already been made. Rubin et al. (1992) used seismic velocities to compute maximum likelihood conductivity estimates and demonstrated the potential benefits from the use of seismic data. Copty et al. (1993) presented a Bayesian updating procedure to infer conductivity distributions from conductivity, pore pressure, and seismic velocity data.

There is now considerable excitement in the hydrogeologic community about the use of non-Gaussian stochastic approaches. Although I find that most arguments against "Gaussian" methods are flimsy, there are some good reasons for seeking alternatives to Gaussian techniques. Geological information or intuition is not described well through covariance functions or variograms and the realizations of Gaussian random fields may look "too fuzzy" and "too disordered". Rubin and Journel (1991) review indicator kriging and provide an example where they examine the effects on solute transport. Suro-Perez and Journel (1991) presented a method that combines elements from indicator kriging and principal component analysis.

In McKinney and Loucks (1992), the kriging variance of conductivity estimates was used as a measure of the parameter uncertainty in a first-order second-moment calculation of the uncertainty in model predicted contaminant concentrations. The focus of this research was to improve the design of a monitoring network. Russo and Bouton (1992) presented a geostatistical analysis of spatial variability in unsaturated flow parameters.

Brannan and Hazelow (1993) presented a method for generating realizations of a conductivity field that can capture characteristics at two different scales, such as laboratory and field scales. The method is about representing large- and small-scale statistical characteristics of the conductivity defined over the smaller scale and the issue of the relation between conductivities defined at different scales ("upscaling problem") is apparently not addressed.

Several innovative methods have been proposed. Rizzo and Dougherty (1994) treated the estimation of a spatial function as a problem of "pattern completion" that they approached through artificial neural networks. They called their approach "neural kriging". This method does not require the explicit estimation of a covariance function, but the analogous phase of training the neural network making it capable of predictions. The authors provided an example that compared results from neural kriging to the results from ordinary kriging using quantized measurements. There is clearly need for additional tests but the presented results were encouraging and the authors envisioned extensions.

Conclusions

In the last few years, the hydrogeologic community has been exploring the use of geostatistics. As is to be expected in any new field, there have been examples of misapplication of methodologies and the reader often encounters in the literature conflicting statements and claims. There is no scarcity of sophisticated techniques but their use is not always tempered with a basic understanding of statistical methodology and the use of plain common sense. Widespread slips are using models with too many parameters, confusing fitting with prediction, confusing simulation with prediction, using an unnecessarily complex empirical model, neglecting to check whether the empirical model is consistent with the data, using a method outside of its intended scope, and adopting inconsistent assumptions.

Nevertheless, significant progress has been made and hydrogeologists now have a better grasp of the scope of geostatistics as well as of statistics in general than ever before. In fact, not only is geostatistical research vibrant and exciting but hydrogeology is leading the way on many fronts by proposing and testing innovative methodologies. Summarizing, some of the most important areas of current and needed research are:

- Improvement of linear geostatistics, which include adjustments of methods from analysis of variance, time series, regression, and Kalman filtering. The trend is to recognize gradually that many successful methods that appear under different names in different fields are basically the same. At the same time, it is hoped that geostatistics will remain pragmatic and focused on the analysis of geophysical data.

- Development of nonparametric and resampling estimation methods. Such methods could be quite useful because they would allow the analysis to proceed without hypotheses concerning distributions or specific values of statistical parameters. So far, such methods have been found useful in relatively simple problems but this is an important area of current research in statistics [see Lall, this volume].

- Development of Bayesian methods. Usually, the error associated with parameter estimation is quite large. In some cases, the uncertainty in the parameters affects the predictions significantly. Bayesian methods can be used to account for the effect of parameter uncertainty and to incorporate information from other sources.

- Development of methods that utilize geological and other information. Important steps have been made but the emphasis so far has been on stochastic simulation and less on estimation, i.e., how to

extract the information from the data. Problems of parameter estimation and model validation are much more challenging than in linear geostatistics.

- Development of nonlinear and nonGaussian estimators in the utilization of the groundwater flow and transport equations. The mathematical problems are quite challenging but estimation can be improved by considering the mechanics of flow and transport. The real challenge will be how to strike the right balance between practicality and exactitude.
- Combination of information from all sources of data, including geophysical and tracer data, principles of flow and tranport, and geological understanding.
- There is a need for amalgamation of geostatistical estimation methods with "upscaling" methods which recognize the differences between parameters or obervations that are defined at different scales.

Acknowledgments. Funding for this work was provided by the office of Research and Development, U.S. Environmental Protection Agency, under agreement R-815738-01 through the Western Region Hazardous Substance Research Center. The content of this study does not necessarily represent the views of the agency. I thank Keerthi Angammana for his help in conducting the bibliographic search. This review is not meant to be comprehensive but rather an evaluation of the overall progress and of remaining problems in the field.

References

Bates,B. C., Improved Methodology for Parameter Inference in Nonlinear Hydrologic Regression Models, *Water Resour. Res.*, 28(1):89-97, 1992.

Brannan, J. R., and J. S. Haselow, Compound Random Field Models of Multiple Scale Hydraulic Conductivity, *Water Resour. Res.*, 29(2):365-372, 1993.

Butcher, J. B., M. A. Medina, and C. M. Marin, Empirical Bayes Regionalization Methods for Spatial Stochastic Processes, *Water Resour. Res.*, 27(1):7-15, 1991.

Campell, C., Bootstrapped methods for intrinsic random functions, *Mathematical Geology*, 20(6), 699-715, 1988.

Christensen, R., The equivalence of predictions from universal kriging and intrinsic random function kriging, *Mathematical Geology*, 22:655-664, 1990a.

Christensen,R., L. M. Pearson, and W. Johnson, Case deletion diagnostics for mixed models, *Technometrics*, 34:38-45, 1990b.

Christensen,R., L. M. Pearson, and W. Johnson, Prediction diagnostics for spatial linear models, *Biometrica*, 79:583-591, 1992.

Christensen, R., W. Johnson, L. M. Pearson, Covariance function diagnostics for spatial linear models, *Mathematical Geology*, 25(2):145-160, 1993.

Copty, N., Y. Rubin, and G. Mavko, Geophysical-Hydrological Identification of Field Permeabilities Through Bayesian Updating, *Water Resour. Res.*, 29(8):2813-2825, 1993.

Cressie, N., *Statistics for spatial data*, Wiley, New York, 1991.

Dagan, G., Stochastic modeling of groundwater flow by unconditional and conditional probabilities, 2, The solute transport, *Water Resour. Res.*, 18(4):835-848, 1982.

Dagan, G., Solute transport in heterogeneous porous formations,*J. of Fluid Mechanics*, 145:151-177, 1984.

Dagan, G., Stochastic modeling of groundwater flow by unconditional and conditional probabilities: The inverse problem, *Water Resour. Res.*, 21(1):65-72, 1985a.

Dagan, G., A note on the high-order corrections of the head covariances in steady aquifer flow, *Water Resour. Res.*, 21(4):573-578, 1985b.

Dagan,G., Theory of solute transport by groundwater, *Annual Review of Fluid Mechanics*, 19:183-215, 1987.

Dagan, G., Time dependent macrodispersion for solute transport in anisotropic heterogeneous aquifers, *Water Resour. Res.*, 24(9):1491-1500,1988.

Delfiner, P., Linear estimation of nonstationary spatial phenomena, in *Advanced Geostatistics in the Mining Industry*, ed. by M. Guarascio, M. David, and C. Huijbregts, pp. 49-68, D. Reidel, Bingham, Massachusetts, 1976.

Desbarats, A. J., and R. M. Srivastava, Geostatistical simulation of groundwater flow parameters in a simulated aquifer, *Water Resour. Res.*, 27(5):687-698, 1991.

Desbarats, A. J., and S. Bachu, Geostatistical analysis of aquifer heterogeneity from the core scale to the basin scale: A case study, *Water Resour. Res.*, 30(3), 673-684, 1994.

Dietrich, C. R., and M. R. Osborn, Estimation of covariance parameters in kriging via restricted maximum likelihood,*Mathematical Geology*, 23(1), 119-135, 1991.

Farrel, D. A., A. D. Woodbury, E. A. Sudicky, and M. O. Rivett, Stochastic and deterministic analysis of dispersion in unsteady flow at the borden Tracer-Test site, Ontario, Canada, *J. of Contaminant Hydrology*, 15:159-185, 1994.

Gelhar, L. W., and C. L. Axness, Three dimensional stochastic analysis of of macrodispersion in aquifers, *Water Resour. Res.*, 19(1):161-180, 1983.

Gelhar, L. W., *Stochastic Subsurface Hydrology*, Prentice Hall, 1993.

Harvey, C. F., and S. M. Gorelick, Mapping hydraulic conductivity: Sequential conditioning with measurements of solute arrival time, hydraulic head, and local conductivity, to appear in *Water Resour. Res.*, 1994.

Hoeksema, R. J., and P. K. Kitanidis, An application of the geostatistical approach to the inverse problem in two-dimensional groundwater modeling, *Water Resour. Res.*, 20:1003:1020,1984.

Hoeksema, R. J., and Clapp, R. B., Calibration of groundwater flow models using Monte Carlo simulations and geostatistics, in ModelCARE 90: Calibration and Reliability in Groundwater Modelling (pp. 33-42). IAHS Publ. No 195, 1990.

Journel, A. G., and C. V. Deutsch, Entropy and spatial disorder, *Mathematical Geology*, 25(3), 329-355, 1993.

Keidser, A., D. Rosbjerg, K. H. Jensen, and K. Bitsch, A Joint Kriging and Zonation Approach To Inverse Groundwater Modeling, In Proceedings *ModelCARE 90:Calibration and Reliability in Groundwater Modeling*, The Hague, 1990.

Kitanidis, P. K., Minimum variance unbiased quadratic estimation of polynomial generalized covariances of regionalized variables, *Mathematical Geology*, 17(2):195-208, 1985a.

Kitanidis, P. K., Use of Prior Information in the Geostatistical Approach to the Inverse Problem, Proceedings of the ASCE "Computer Applications in Water Resources" Conference, Buffalo, New York, pp. 897-906, June 10-12, 1985b.

Kitanidis, P. K., Orthonormal residuals in Geostatistics : Model Criticism and Parameter Estimation, *Mathematical Geology*, 23(5):741-758, 1991.

Kitanidis, P. K, Generalized covariance functions in estimation, *Mathematical Geology*, 25(5):525-540, 1993.

Kitanidis, P. K., Geostatistics, Chapter 20 in *Handbook of Hydrology*, Edited by D. R. Maidment, pp. 20.1-20.39, MacGraw-Hill, New York, 1993.

Keidser, A., and D. Rosbjerg, Comparison of Four Inverse Approaches to Groundwater Flow and Transport Parameter Identification, *Water Resour. Res.*, 27(9):2219-2232, 1991.

Lall, U., Recent advances associated with nonparametric methods in Hydrology, this volume.

LaVenue, A. M., and J. F. Pickens, Application of a coupled adjoint sensitivity and Kriging approach to calibrate a Groundwater Flow Model, *Water Resour. Res.*, 28(6):1543-1569, 1992.

Lee, S. -I., and Kitanidis, P. K., Optimal Estimation and Scheduling in Aquifer Remediation with Incomplete Information, *Water Resour. Res.*, 27(9), 2203-2217, 1991.

Loaiciga, H. A., R. J. Charbeneau, L. G. Everett, G. E. Fogg, B. F. Hobbs, and S. Rouhani, Review of groundwater quality monitoring network design, *ASCE J. of Hydraulic Engineering*, 118(1), 11-37, 1992.

Matheron, G., *The Theory of Regionalized Variables and Its Applications*, Ecole de Mines, Fontainbleau, France, 1971.

McKinney, D. C., and D. P. Loucks, Network Design for Predicting Groundwate Contamination, *Water Resour. Res.*, (28) 1, 133-147, 1992.

Neuman, S. P., Adjoint-state finite element equations for parameter estimation. In S. W. Wang, C. V. Alonso, C. A. Brebbia, W. G. Gray, and G. F. Pinder (Eds.), Finite Elements in Water Resources: Proceedings of Third International Conference New York: Springer-Verlag, 1980.

Neuman, S. P., C. L. Winter, and C. M. Newman, Stochastic theory of field-scale Fickian dispersion in anisotropic porous media, *Water Resour. Res.*, 23(3):453-466, 1987.

Rajaram, H., and D. McLaughlin, Identification of large scale spatial trends in hydrologic data, *Water Resour. Res.*, 26(10),2411-2424, 1990.

Rubin, Y., and G. Dagan, Stochastic analysis of boundaries effects on head spatial variability in heterogeneous aquifers, 1, Constant Head Boundary, *Water Resour. Res.*, 24(10):1689-1697, 1988.

Rubin, Y., Prediction of tracer plume migration in disordered porous media by the method of conditional probabilities, *Water Resour. Res.*, 27(6), 1291-1308, 1991a.

Rubin, Y., The spatial and temporal moments of tracer concentration in disordered porous media, *Water Resour. Res.*, 27(11), 2845-2854, 1991b.

Rubin, Y., Transport in heterogeneous porous media: Prediction and uncertainty, *Water Resour. Res.*, 27(7), 1723-1738, 1991c.

Rizzo, D. M., and D. E. Dougherty, Characterization of aquifer properties using artificial neural networks: Neural Kriging, *Water Resour. Res.*, 30(2):483-497, 1994.

Rouhani, S., and H. Wackernagel, Multivariate geostatistical approach to Space-Time Data Analysis, *Water Resour. Res.*, 26(4):585-591, 1990.

Rouhani, S., and D. E. Myers, Problems in space-time kriging of geohydrological data, *Mathematical Geology*, 22(5), 611-623, 1990.

Rouhani, S., Georgakakos, A. P., Kitanidis, P. K., Loaiciga, H. A., Olea, R. A. and Yates, S. R., Geostatistics in Geohydrology: Part I. Basic Concepts, *ASCE J. of Hydraulic Engineering*, 116(5), 612-632, 1990a.

Rouhani, S., Georgakakos, A. P., Kitanidis, P. K., Loaiciga, H. A., Olea, R. A. and Yates, S. R., Geostatistics in Geohydrology: Part II. Applications, *ASCE J. of Hydraulic Engineering*, 116(5), 633-658, 1990b.

Rubin, Y., and A. G. Journel, Simulation of non-Gaussian space random functions for modeling transport in groundwater, *Water Resour. Res.*, 27(7):1711-1721, 1991.

Rubin, Y., G. Mavko, and J. Harris, Mapping Permeability in Heterogeneous Aquifers Using Hydrologic and Seismic Data, *Water Resour. Res.*, 28(7):1809-1816, 1992.

Russo, D., and M. Bouton, Statistical Analysis of Spatial Variability in Unsaturated Flow Parameters, *Water Resour. Res.*, 28(7):1911-1925, 1992.

Savage, L. J. (1962). The Foundations of Statistical Inference. New York: Wiley and Sons.

Schweppe, F. C., *Uncertain Dynamic Systems*, Prentice-Hall, Englewood-Cliffs, NJ, 1973.

Shafer, J. M., and M. D. Varljen, Approximation of Confidence Limits on Sample Semivariograms From Single Realizations of Spatially Correlated Random Fields, *Water Resour. Res.*, 26(8):1787-1802, 1990.

Sun, N-Z., and W. W-G. Yeh, A stochastic Inverse Solution for Transient Groundwater Flow: Parameter Identification and Reliability Analysis, *Water Resour. Res.*, 28(12):3269-3280, 1992.

Suro-Perez, V., and A. G. Journel, Indicator Principal Component Kriging,*Mathematical Geology*, 23(5):759-788, 1991.

Sudicky, E. A., A natural gradient experiment on solute transport in a sand aquifer: Spatial variability of hydraulic conductivity and its role in the dispersion process, *Water Resour. Res.*, 22(13):2069-2082, 1986.

Tucciarelli, T., and G. Pinder, Optimal data acquisition strategy for the development of a transport model for groundwater remediation, *Water Resour. Res.*, 27(4):577-588, 1991

Tuckey, J. W., Discussion emphasizing the connection between analysis of variance and spectrum analysis. *Technometrics*, 3, p. 191, 1961.

Van der Linde, A., On least squares estimation of generalized covariance functions, *Mathematical Geology*, 25(1), 1993.

Van Geer, F. C., C. B. M. Te Stroet, and Y. Zhou, Using Kalman filtering to improve and quantify the uncertainity of numerical groundwater simulations,1, The role of sys-

tem noise and its calibration, *Water Resour. Res.*, 27(8), 1987-1994, 1991.

Van Tonder, G. J., J. F. Botha, and D. J. De Waal, Bayesian Estimation of Water Levels, In Proceedings, *ModelCARE 90:Calibration and Reliability in Groundwater Modeling*, The Hague, 1990.

Woodbury, A. D., and E. A. Sudicky, The Geostatistical characteristics of the Borden aquifer, *Water Resour. Res.*, 27(4):533-546, 1991.

Woodbury, A. D., and E. A. Sudicky, Inversion of the Borden Tracer Experiment Data: Investigation of Stochastic Moment Models, *Water Resour. Res.*, 28(9):2387-2398, 1992.

Woodbury, A. D., and T. J. Ulrych, Minimum Relative Entropy : Forward Probabilistic Modeling, *Water Resour. Res.*, 29(8):2847-2860, 1993.

Zhou, Y., Z., C. B. M. Te Stroet, and F. C. Van Geer, Using Kalman filtering to improve and quantify the uncertainity of numerical groundwater simulations,1, The role of system noise and its calibration, *Water Resour. Res.*, 27(8), 1995-2006, 1991.

P. K. Kitanidis, Department of Civil Engineering, Stanford University, Stanford, CA 94305-4020. (e-mail: pkk@hyetose.stanford.edu)

(Received June 13, 1994; accepted December 9, 1994.)