Correlation Estimation in SAR Interferograms

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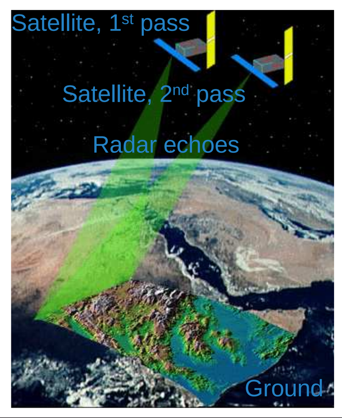
Background

Knowing the relative accumulation and loss rates of ice sheets in the Earth's polar regions is critical for understanding global climate change. **Satellite radar interferometry (SAR)** is a remote sensing technique that generates maps of ice accumulation by measuring the **correlation** of the radar echoes. Here, we present an algorithm that obtains more accurate estimates of radar correlation compared to previous remote sensing measurements. It then follows that we can compute more accurate estimates of snow accumulation.



What is correlation?

Correlation is a statistical measure indicating how related two radar signals are. Values run between zero and one where zero indicates no relation and one indicates an exact match-up in the two signals. The equation on the right is the mathematical definition of correlation where c_1 and c_2 are two signals. In practice, we estimate local correlation by averaging over a small box of pixels in the interferogram.

Correlation equation

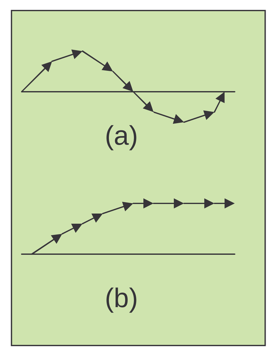
$$\rho = \frac{\left|\left\langle c_1 \cdot c_2^* \right\rangle\right|}{\sqrt{\left\langle c_1 \cdot c_1^* \right\rangle \left\langle c_2 \cdot c_2^* \right\rangle}}$$

What is an interferogram?

Interferograms, like the two depicted at the right, are formed by cross-correlating the complex-numbered radar images of a region on the ground for two separate satellite passes. Each complex element of an image matrix is represented as a pixel where the magnitude is mapped to a grayscale shade and the phase is mapped to a given colormap color. Because phase depends on topography and ice motion, as well as accumulation rate, phase images tend to have colored bands called **fringes**.

The Problem

The presence of interferometric fringes lowers the estimated correlation below true values. The diagram on the right depicts eight complex entries in vector form. As depicted by cartoon (a), an area with quickly varying phases will have an average signal of zero or nearly zero, implying a low calculated correlation. Our goal is to remove the quickly varying fringe patterns caused by ice motion and topography to obtain a slowly varying fringe pattern caused by snow accumulation. In doing so, we will obtain higher and more accurate correlation estimations as depicted by cartoon (b) of the same diagram.



Our Solution

To remove the decorrelating effects of fringes, we use a local Fourier filter to estimate fringe rates and then we subtract the fringe component of the signal before calculating correlation. We refer to this technique as **defringing** or **phase-flattening**. In addition, this algorithm creates its own bias which we remove in a second step.

The Methods

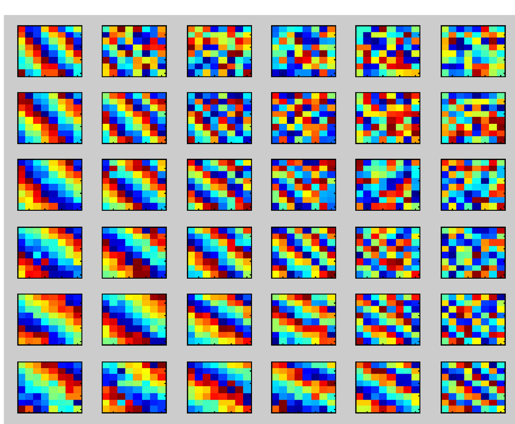
Removing phase fringes

High-fringe rates caused by topography and motion must be removed to estimate correlation accurately. The following depicts phase-flattening over a 48x48 pixel sample of the larger interferogram.

Original phase matrix

$$A = e^{j2\pi(fx)t} e^{j2\pi(fy)t} e^{jp}$$

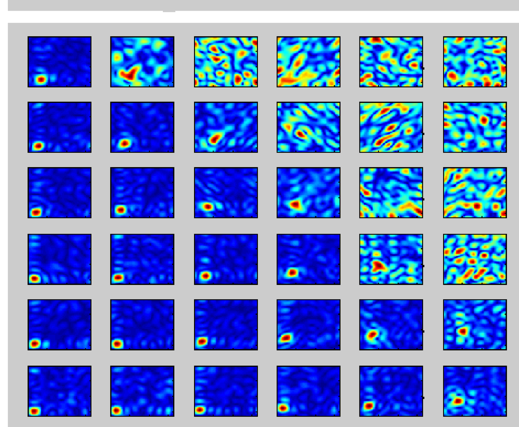
We subdivide the full image into 8x8 pixel boxes to be evaluated individually. In the equation above, A represents one 8x8 block of the original phase matrix where fx and fy are the fringe rates in the x- and y- directions and ρ is the residual phase.



Interpolated FFT

$$F = fft(A)$$

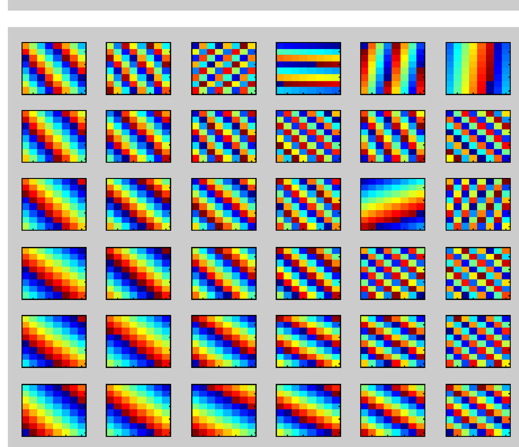
We estimate the average fringe rate in each box using Fourier techniques. We interpolate the FFT of each box by a factor of 8 to better identify the exact fringe rate. The peak location in the transform gives the estimated fringe rates in the x and y directions. We also determine the phase of the fringe signal at the maximum point.



Estimated Phase Rate

$$E = e^{-j2\pi(efx)t} e^{-j2\pi(efy)t} e^{-j\theta}$$

In the above equation, efx and efy are the fringe-rates across each 8x8 box estimated by the position of the FFT maximum and θ is the phase at the FFT peak. Note that the colors run in opposite directions from the original matrix because the exponent is negative. There are noticeable discrepancies in the upper-right corner where the original matrix was noisy.

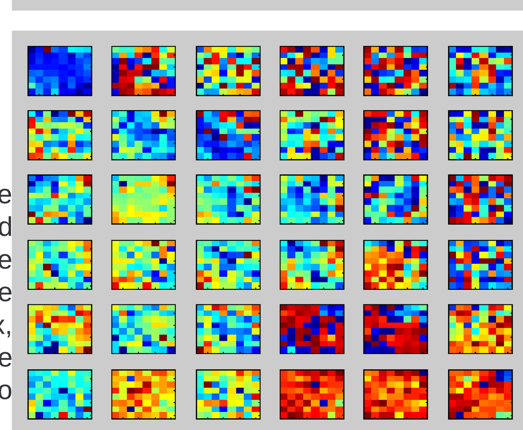


Phase-flattened matrix

If $fy \approx efy$ $fx \approx efx$ and $\rho \approx \theta$

Then $A \cdot B \approx e^0 = 1$

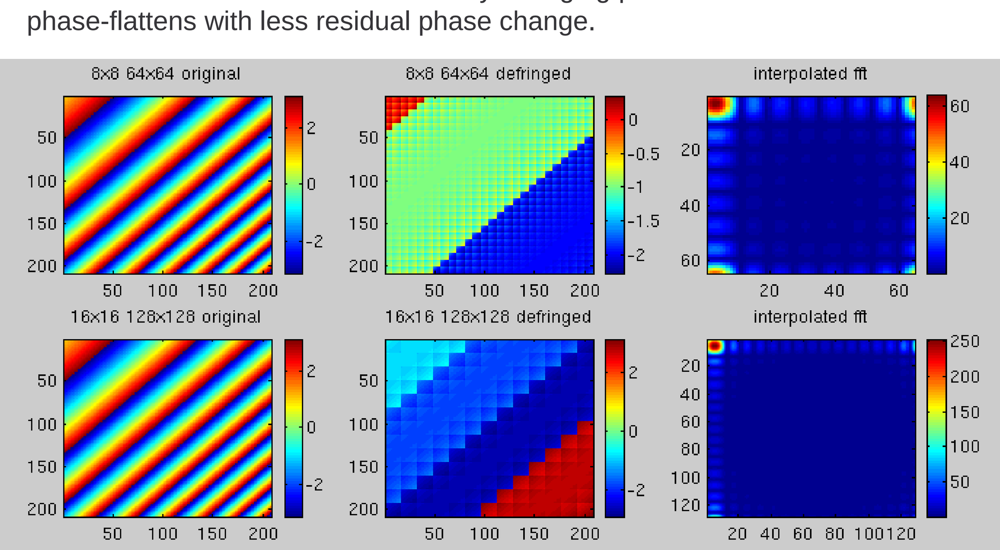
The corrected data results from the cross multiplication of the original and the estimated matrices. If the estimated fringe rate exactly equals the real fringe rates of the original matrix, the result is an 8x8 box with no phase variation. We reassemble 8x8 boxes to form complete phase-flattened image.



Problems with defringing

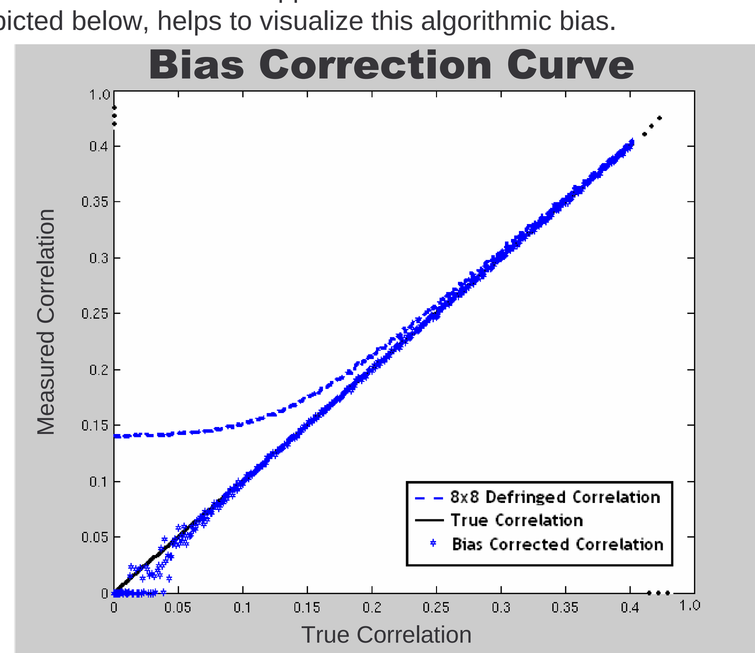
▪Removing phase fringe gradients is all about **FFT box size**. The box must be small enough to detect very small phase fringes and yet large enough so that the algorithm will not confuse noise for fringes.

▪The algorithm has difficulty phase-flattening areas where fringes gradually change rate. The result will not be a uniformly phase-flattened area as a residual fringe pattern will remain. The figure below depicts how an 8x8 and a 16x16 FFT box size deals with the same uniformly changing phase rate. The smaller box size phase-flattens with less residual phase change.



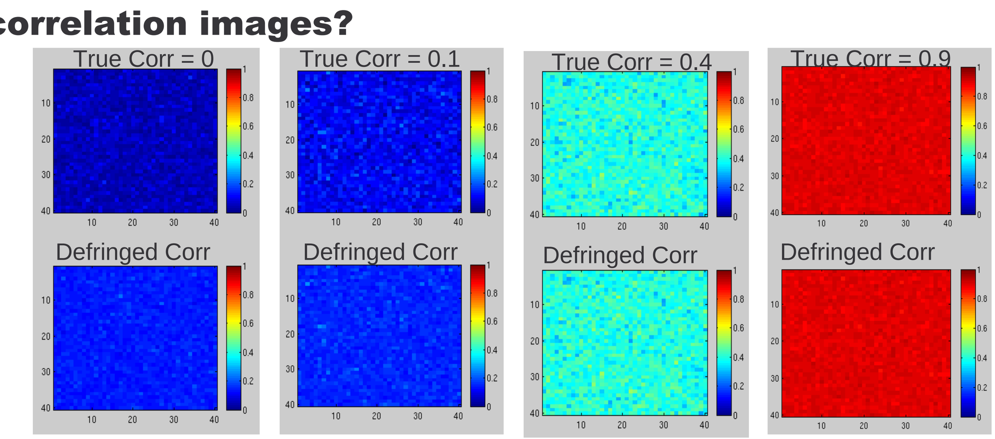
Algorithmic Bias Correction

Applying the phase-flattening algorithm with small FFT box sizes causes the program to defringe noisy areas that do not have actual fringes. Because of this, it over-estimates correlations in areas that are supposed to have low correlations. The Bias Correction Curve, depicted below, helps to visualize this algorithmic bias.



Ideally, we would like measured correlations to exactly equal true correlations as represented by the solid black line in the plot above. The dotted blue line represents the calculated correlations after applying the defringing algorithm with an 8x8 pixel FFT box. For true correlations above 0.4, we are able to approximate correlations very well. However, for true correlations below 0.4, the calculated correlations are higher than actual values. This is what is meant by algorithmic bias. If the shape of the bias curve is known, the bias can be subtracted in the final estimation step.

What does this bias look like in the actual correlation images?



The top row figures are correlation images of simulated noisy signals with set true correlation values 0, 0.1, 0.4, and 0.9. The bottom row figures are correlation images of 8x8 FFT box defringed versions of their corresponding top row figures. At high correlations, the difference is unnoticeable. However, as true correlations drop, the defringing algorithm causes greater correlation misestimation.

How do we obtain the Bias Correction Curve?

We use a simulation method to obtain the dashed blue bias curve seen in the plot above entitled "Bias Correction Curve". Then we subtract the difference between a polynomial fit of the bias curve and the solid black true correlation curve to remove bias from the correlation images shown at the right.

Steps in the simulation algorithm

1. Generate noisy matrix with a set true correlation value ρ
2. Apply phase fringe gradient removal algorithm for FFT box size 8x8.
3. Calculate mean correlations for each of the resulting matrix.
4. Repeat process for true correlation values between 0 and 0.4 in steps of 0.01.
5. Plot mean calculated correlation vs. true correlation to form bias curve.
6. Fit curve to a high-order polynomial. An 8^{th} order polynomial was used in the plot above.
7. Calculate the difference between the polynomial fitted bias curve and the true correlation.
8. Subtract the difference from correlation images to correct for biases due to the phase-flattening algorithm.

Problems with bias correction

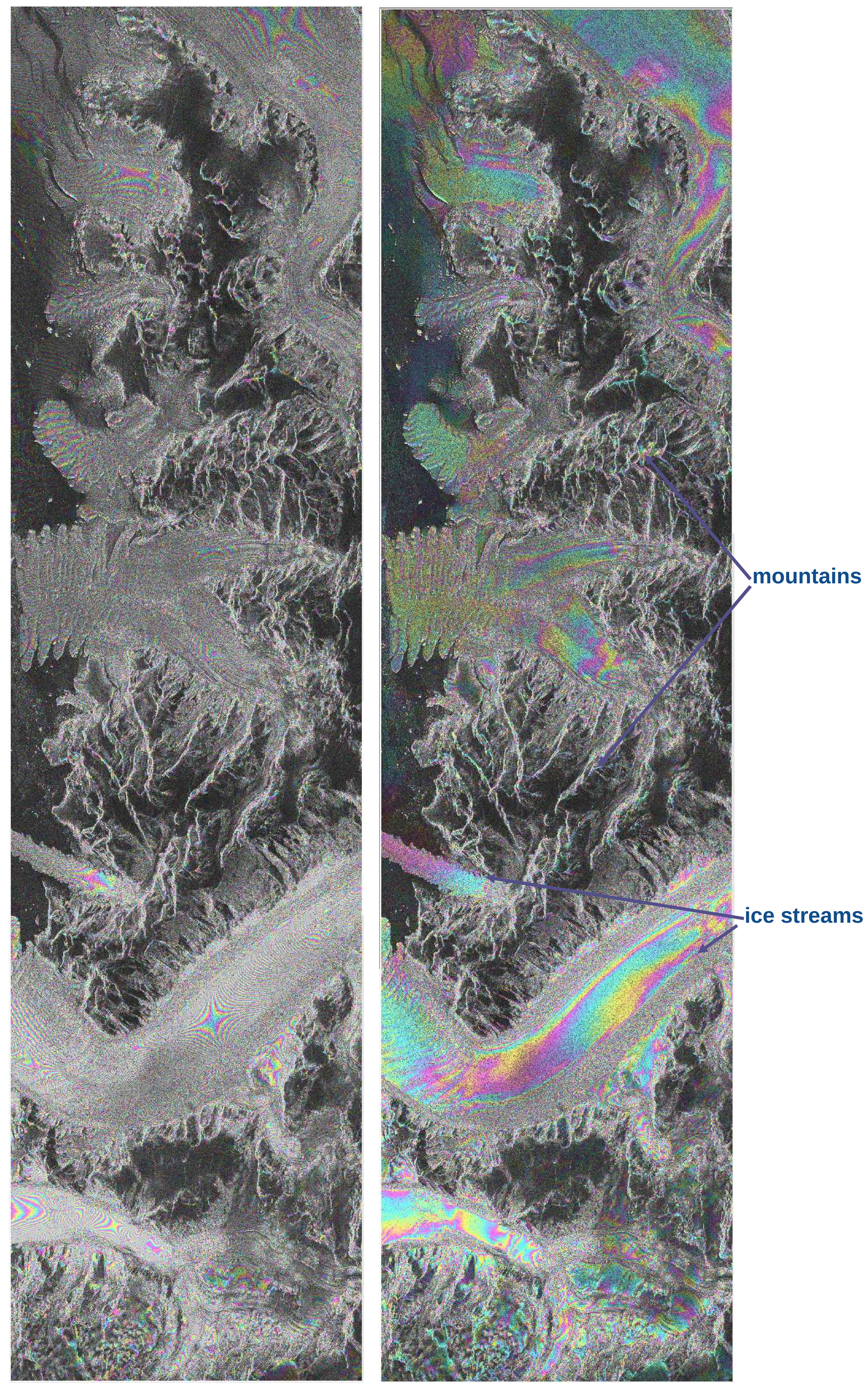
From the bias correction curve depicted above, the bias is well corrected for true correlation values of 0.1 and above. However, as the measured correlation flattens out for true correlation values below 0.1, the algorithmic bias is harder to correct for because a small variation in measured bias becomes a large variation in estimated true correlation Thus, the blue stars representing the Bias Corrected Correlation in the above plot appear to be scattered for true correlation values below 0.1 instead of perfectly fitting the solid black true correlation curve as they do for true correlation values above 0.1.

Results

We apply the defringing techniques described under the Methods section, to a 6052x25253 pixel interferogram from Antarctica as shown in the figures below. The area covers approximately 45x135km from the location indicated by the red box on the globe to the right. Note that the following images are reduced by a factor of twelve and resized to better accommodate the size of the poster.

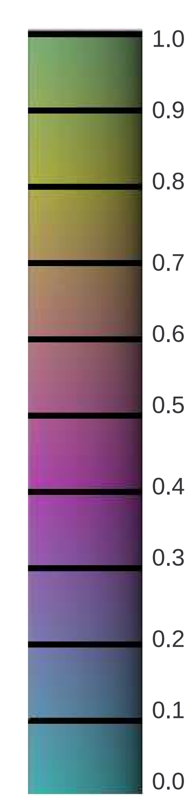
Interferograms

Original **Defringed**



Correlation Images

(a) Original **(b) Defringed** **(c) Bias-corrected**



Correlation values increase going up the colorbar.

Magnitude values increase going left across the colorbar.

As can be seen by comparing correlation images (a) and (b), the presence of interferometric fringes lowers measured correlations below actual values. To increase measured correlations, we employ an algorithm to remove the fringes with a local Fourier filter. The defringed image (b) is noticeably more correlated, especially in ice streams.

The algorithm creates its own bias where it overestimated correlations in areas with low actual values. We removed this bias with a method also described on the left, resulting in the defringed and bias-corrected correlation image, (c). The mountainous areas of the bias-corrected image now have places with zero correlation while still maintaining high correlations in the ice streams.