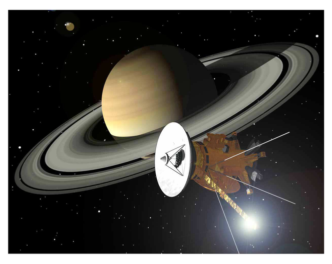
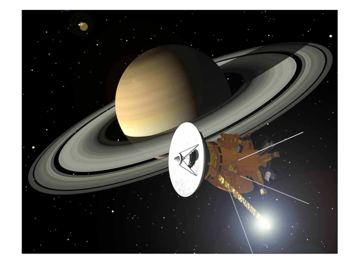


Analysis of Phoebe Rotation Rate and Spectral Bandwidth based on Cassini Satellite Data



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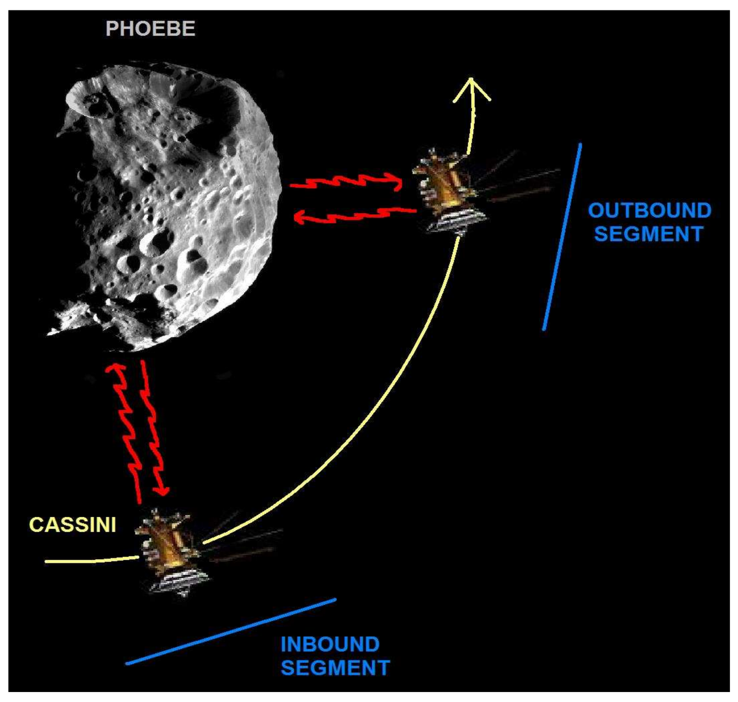
Introduction



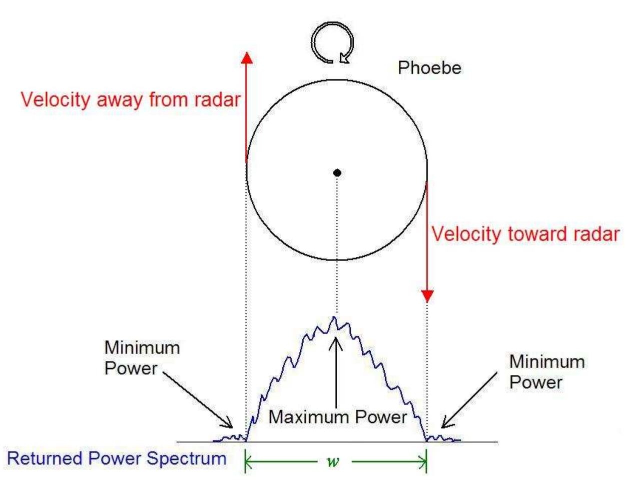
The Cassini- Huygens spacecraft mission is composed of two elements: the Cassini orbiter that will orbit Saturn and its moons for four years, and the Huygens probe that will enter Titan's atmosphere and land on its surface. In this project we analyzed data acquired by the radar instrument on board the Cassini orbiter. The Cassini spacecraft imaged Saturn's most distant moon, Phoebe, on June 11, 2004. Phoebe is of great interest because it is likely a captured Kuiper belt object dating back to the early origins of the solar system. While most of Saturn's other moons orbit directly in Saturn's ring plane, Phoebe orbits in an inclined plane. Also, its orbit is retrograde, which is the opposite direction with respect to most other objects in the solar system. Thus, Phoebe just might hold clues about the formation of the solar system. Data collected during Cassini's close flyby of Phoebe will help determine where the moon came from and allow further analysis of its original chemical composition.

Two sets of data of power, time, and relative coordinates were collected during the flyby: one as Cassini approached Phoebe and the other as it left. We refer to these as the inbound and outbound segments of data.

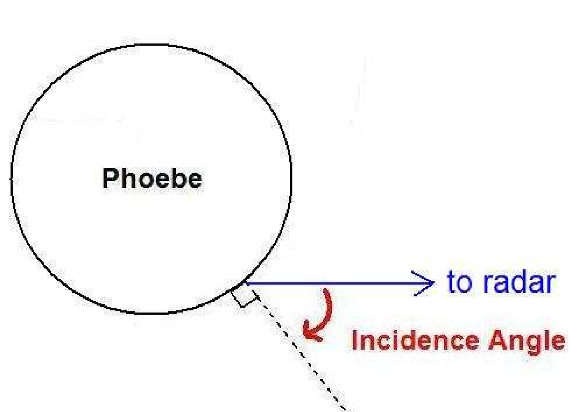
Objective

The data were acquired by the Cassini radar in a scatterometer mode. The scatterometer data are analyzed through their Doppler spread and spectral weighting function, which depends on the backscatter law. Various properties of Phoebe, including rotation rate, bandwidth, reflectivity, and chemical composition, can be investigated using these observations. Our analysis confirms previous estimates of spin rate made before the flyby and measures the surface backscatter function for the first time.

How Physical Properties Affect Observations



Doppler Spread -
Planet motion causes the radar echoes to be shifted in frequency. For example, signals from the right hand portion of the spinning planet at left are shifted higher in frequency due to the Doppler Effect. Signals from the left hand side have lower frequency. The amount of shift depends on the spin rate. Hence, the shape of the observed signal spectrum is related to both the planet motion and the reflectivity of Phoebe at each location.



Spectral Shape -
The total power observed at each frequency depends on the reflectivity of the surface and the angle of incidence as shown. The edges of the spectrum have minimum power, and the middle has maximum power returned. For icy bodies such as Phoebe, reflectivity falls off with incidence angle. When at the maximum incidence angle (90°), minimal power is reflected back; at minimum incidence angle (0°), the greatest amount of power is returned.

Method

We analyzed the signals by relating the shape of the observed spectrum to a modeled backscatter function of the form *cos*^{n}(incidence angle). The parameter n is used to distinguish icy from rocky surfaces while width of spectrum is proportional to the rotation rate. Our procedure is:

1. Produce averaged spectrum

Spacing in Samples = (FFT size * Pulse Repetition Rate)/Sampling Rate

The radar broadcasts signals at the pulse repetition rate. These are reflected back to the radar and are digitized on the spacecraft at the sampling rate. Echoes from many pulses are transformed and averaged.

2. Evaluate model functions

The best fit backscatter law is assumed to be the *cos*^{n} curve so the following integral is used to model the data spectrum:

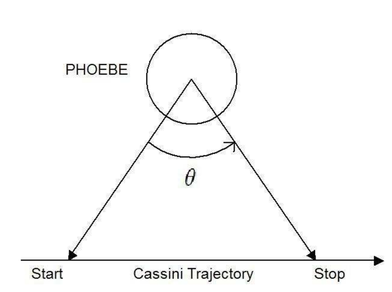
$$\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \cos^n(\sin^{-1}\sqrt{x^2+y^2})\,dy \quad \text{for } -1 < x < 1$$

We assume nadir latitude (λ_0 as defined below) is zero to simplify the model function, but should still get close results.

3. Find parameters, exponent n and width w, to minimize squared error

4. Analyze both inbound and outbound spectra

5. Calculate rotation rate



Because spacecraft motion adds an apparent rotation to the moon, we calculate and subtract this from the observed spectrum.

Net rotation rate = Actual rate - Apparent rate ($\frac{\theta}{\Delta t}$)

Bandwidth (Doppler Spread) = $\Delta f = \frac{4\omega r}{\lambda}\cos\lambda_0 = \frac{8\pi r}{\lambda T}\cos\lambda_0$

Real rotation rate = ω- Net rotation rate (subtract because Phoebe rotates clockwise)

Results

Figures I and III show the measured spectra for the inbound and outbound cases.

Figure I

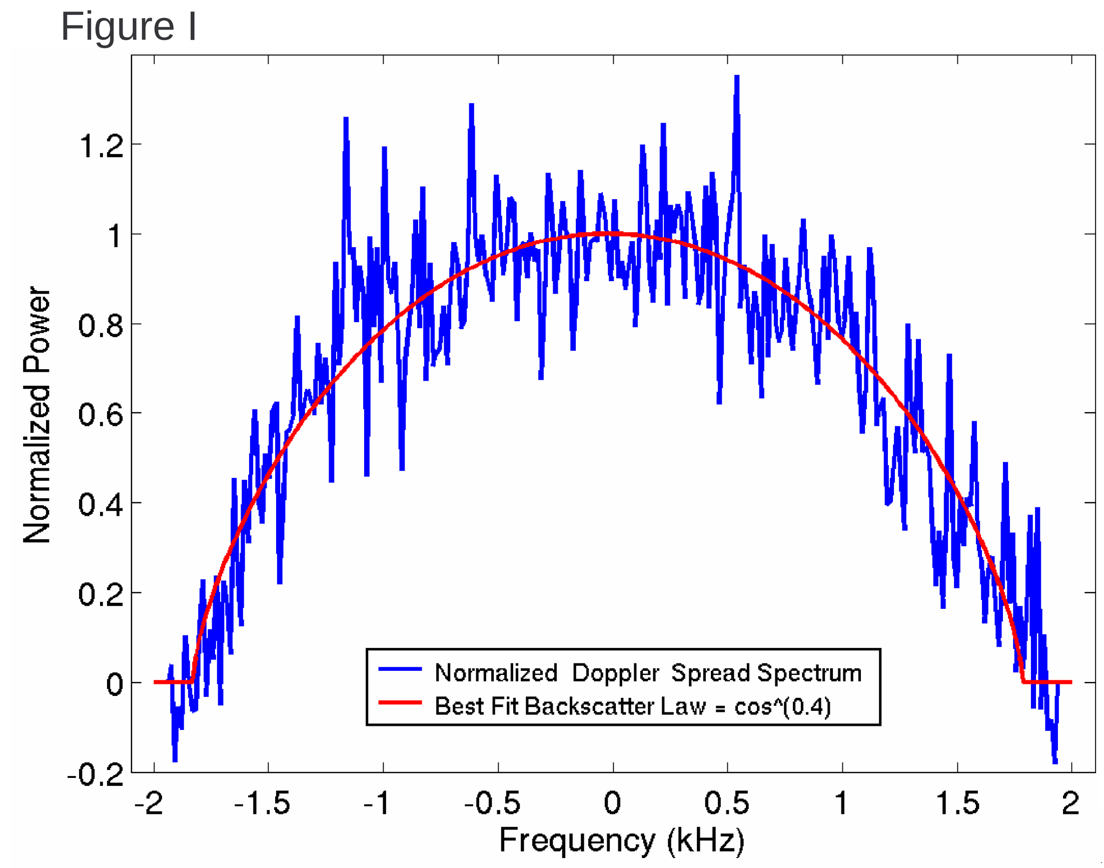
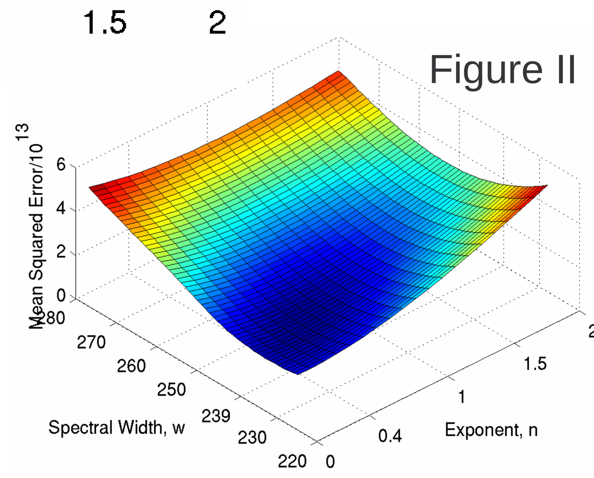


Figure II



The inbound data are acquired from a farther distance so the received signal power is much lower and noise is more evident. The best fit model curve has n = 0.4 and w = 239 FFT bins, which is the minimum mean squared error point over reasonable values of n and w, as seen in Figure II.

Figure III

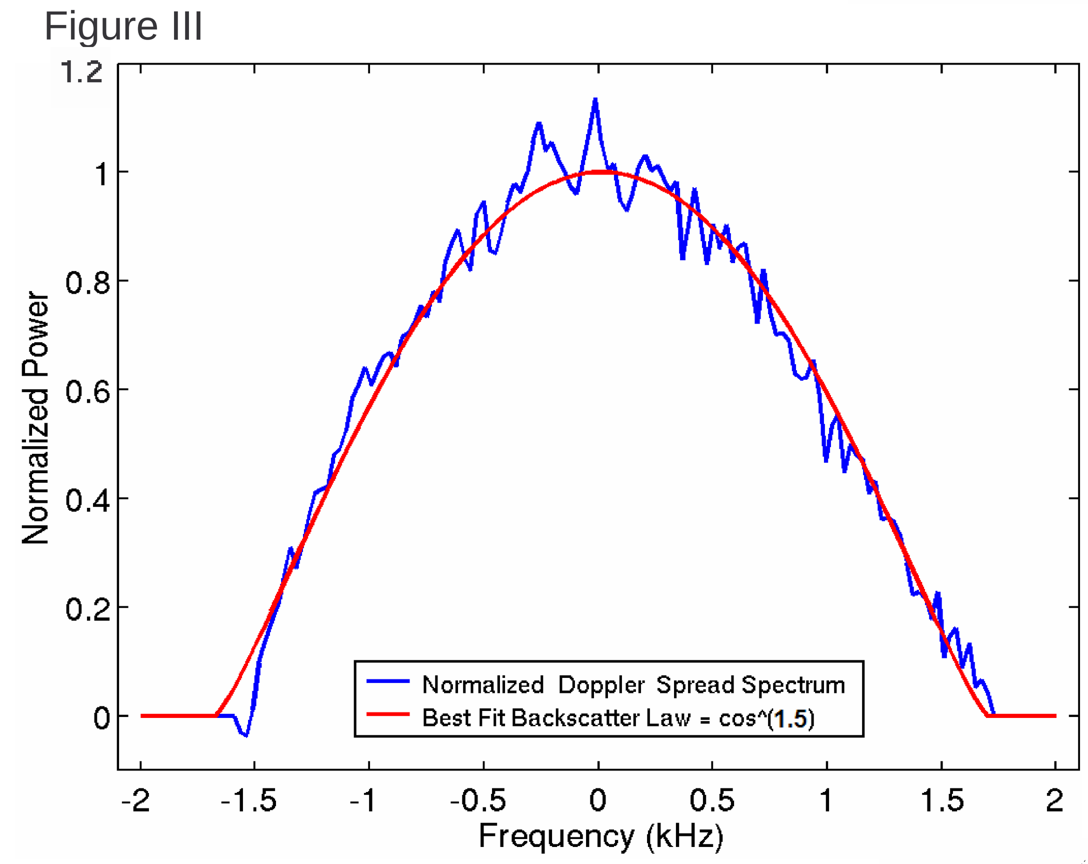
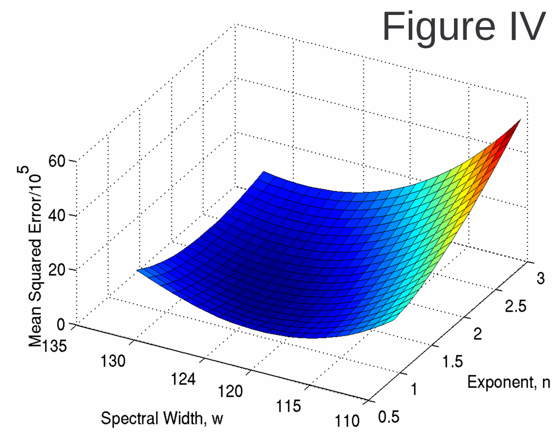


Figure IV



The outbound data are acquired at a closer distance so the signal is stronger and less noisy. The different inbound and outbound spectral shapes may result from observing different parts of Phoebe's surface and differing scattering properties over the entire surface. From Figure IV, it can be seen that the optimal model curve that minimizes the mean squared error has n = 1.5 and w = 124.

Table I

Parameter	Inbound Results	Outbound Results
Measured exponent in *cos*^{n} model (n)	0.4	1.5
Measured Spectral Width (w)	239 FFT bins	124 FFT bins
Measured Doppler Spread	3646.8515 Hz	3784.1806 Hz
Measured Rotation Rate	179.375 μrad/s	184.352 μrad/s
Actual Rotation Rate	187.973 μrad/s	187.973 μrad/s
Measured Rotation Period	9.730 hrs	9.467 hrs
Actual Rotation Period	9.285 hrs	9.285 hrs

Conclusion

The results of fitting a *cos*^{n} model curve to the data spectra are tabulated in Table I and plotted in Figures I and III. The inbound average spectra is less peaked than a straight cosine curve, thus the smaller exponent n, while the outbound spectra is more pointed with a higher n. The measured spectral width, 3646.8515 Hz, of the inbound results is fairly close to the outbound spectral width, 3784.1806 Hz.

Other than a slight difference between the measured Doppler spread of the inbound and outbound spectra, there is also some deviation between the measured and estimated rotation rate. There are several explanations for these inconsistencies. One explanation is that the nadir latitude is not included in the model curve, but is used in calculating the rotation rate. A more accurate model curve would involve varying both x and y according to Phoebe surface properties. Another source of error would be assuming a spherical model for Phoebe. All of the equations used assume Phoebe to be a perfect sphere, which it is not. Utilizing the correct target body model in the formula would give a more accurate estimation for the Doppler spread and would likely correct most of the differences between the measured rotation rate and the rate estimated.