Structural Health and Strain Monitoring Through Fourier-based Transducers

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Background

• Structural Health and Usage Monitoring:
  – Loads monitoring: usage and fatigue
  – Health monitoring: damage

• Investigation of novel sensors for:
  – Strain measurements
  – Guided wave inspection
  – Integration (?)
• Roadmap to structural health monitoring (SHM) and usage monitoring:
  “…in the not too distant future, to transition from usage monitoring to individual component damage tracking, to enhance our health assessment capability with better prediction of fatigue for each dynamic component and fatigue critical area…”(*)

Locations of defects in UH60 airframe (**)
Objectives

• Legacy sensors have inherent limitations related to:
  – Wiring
  – Hardware requirements
  – Power needs
  – Reliability

• Basic research is needed to:
  – Simplify health & usage monitoring tasks
  – Enable new functionalities
    (remote, wireless, distributed, embedded sensing....)
Objectives

- Fundamentally new transducer concepts must support an integrated Health & Usage Monitoring process

- Opportunities offered by:
  - Additive manufacturing
  - Advanced micro/nano fabrication technologies
  - Direct write technology for in-situ deposition of sensing material

- Patterning of transducer surface/electrodes leads to novel and tailored functionalities
Outline

- Frequency Steerable Acoustic Transducers (FSATs) for directional sensing and actuation of guided waves
- Acoustic Wave Rosettes for multi-component strain sensing
- Conclusions & Outlook
Wave-based techniques
Directional wave generation

Omnidirectional transducers

Directional transducers

Damage
Frequency Steerable Transducers

- Phased-array technology
  - Phasing/control of each component of an array
  - Hardware complexity

- Frequency Steerable Acoustic Transducer (FSAT):
  - Frequency-dependent directionality provided by spatial arrangement of active material
  - Patterning of sensing geometry leads to focused sensing/actuation with reduced hardware

Array systems and related methods for structural health monitoring, *US Patent* 8,286,490
- Frequency-steered acoustic transducer using a spiral array *US Patent* 20140157898 A1
Concept: 1D Combs

Concept

- A 1D periodic sensing surface behaves as a narrow band filter

- A 2D periodic sensing surface behaves as a narrow band filter with directional sensitivity

- Same considerations apply for ACTUATION, i.e. wave generation
Configuration & Basic Relations: Sensing Mode

Constitutive relations (Piezo):
\[
\sigma = C^E \varepsilon - e^T E \\
D = e \varepsilon + \varepsilon^E E
\]

Matrix formulation

\[
\begin{bmatrix}
\sigma \\
D
\end{bmatrix} = \varphi(\mathbf{x}) \begin{bmatrix}
C^E & -f(\mathbf{x})e^T \\
f(\mathbf{x})e & \varepsilon^E
\end{bmatrix} \begin{bmatrix}
\varepsilon \\
E
\end{bmatrix}, \quad \mathbf{x} \in \Omega
\]

Measured Voltage

\[
V = \frac{t_P}{A_P} \frac{b^T (d^\sigma C^E d^\sigma^T - \varepsilon^\sigma) b}{b^T d^\sigma C^E} \int_{\Omega} \varepsilon f(\mathbf{x}) d\mathbf{x}
\]
Sensor’s directionality

- Measured voltage can be expressed as:

\[ V(\omega) = jU_{10}(\omega) k_0(\omega) H(\theta) D(\omega, \theta) \]

- Where

\[ H(\theta) = \frac{t_P}{A_P} \frac{b^T d^\sigma C^E r(\theta)}{[b^T (d^\sigma C^E d^{\sigma T} - \epsilon^\sigma)b]} \]

\[ D(\omega, \theta) = \int_{\Omega} e^{jk_0(\omega)(x_1 \cos \theta + x_2 \sin \theta)} f(\mathbf{x}) d\mathbf{x} \]

- Geometric directionality can be rewritten as

\[ D(\omega, \theta) = \int_{-\infty}^{+\infty} e^{-jk_0(\omega) \cdot \mathbf{x}} f(\mathbf{x}) d\mathbf{x} \]

\[ D(k_0(\omega), \theta) = \mathcal{F}[f(\mathbf{x})] \]

**Material directionality**

**Geometric directionality**

**General framework for design**
Example:
Rectangular array of point sources

\[ f(x_1, x_2) = f_0 \sum_{n=1}^{N} \sum_{m=1}^{M} \delta(x_1 - nd_1)\delta(x_2 - md_2) \]

\[ f(k_1, k_2) = f_0 \frac{\sin(\frac{Nk_1d_1}{2})}{\sin(\frac{k_1d_1}{2})} \frac{\sin(\frac{Mk_2d_2}{2})}{\sin(\frac{k_2d_2}{2})} \]

\[ k_1 = p \frac{2\pi}{d_1}, \quad k_2 = q \frac{2\pi}{d_2}, \quad p, q \in \mathbb{Z} \]
Frequency-dependent radiation
Experimental validation: Plate and array configuration

Thin aluminum plate

Array (7*7)

\[ d_1 = 4.3 \text{ cm} \]
\[ d_2 = 3.5 \text{ cm}, \]
\[ \alpha = 55^\circ \]
\[ \beta = 165^\circ \]

Experimental validation: Directional Radiation
FSAT Spiral Array

\[ \mathcal{D}(k_0(\omega), \theta) = \mathcal{F}[f(x)] \]

\[ f(x) = \frac{1}{N} \text{rect} \left( \frac{|x|}{a} \right) \sum_{n=1}^{N} \sin(k_n \cdot x) \]

**Each frequency corresponds to a given direction of sensing and radiation**
Further development of Eq. 17, which can be rewritten as follows:

\[ D(r_s) = \sum_{n=1}^{N} \sin(k_n \cdot x) \]

where Eq. 18 can be easily recognized as the spatial Fourier Transform (rFT) of the function \( f(x) \) so that the integration limits can be extended to infinity without affecting the value of the integral. This simple observation leads to the convenient estimation of the directivity for various sensor shapes and polarizations through the identification of the proper FT pairs:

\[ D(k_0(\omega), \theta) = \mathcal{F}[f(x)] \]

2.3 Examples of directivities for simple geometries

2.3.1 Circular sensor

The case of a circular piezo sensor can be modeled through the following expression for the function \( f(x) \):

\[ f(x) = \frac{1}{N} \text{rect}\left(\frac{|x|}{a}\right) \sum_{n=1}^{N} \sin(k_n \cdot x) \]

where the function \( \text{rect} \) is defined as follows:

\[ \text{rect}(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases} \]

and where \( a \) defines the radius of the disc. Figure 3 shows a configuration of the disc where the black domain identifies the location of the piezoelectric material. The corresponding directivity is given by:

\[ D(k_0(\omega), \theta) = \mathcal{F}[f(x)] \]

which confirms results previously presented for the case of a circular disc. The directivity function clearly shows the absence of any sensing directivity and indicates that preferential tuning occurs for wave modes corresponding to local maxima of the sinc function. Such maxima can be found at:

\[ k_0 a = 2 \pi n \]

where \( n = 1, 2, \ldots \) is an integer. The directivity function in the wavenumber domain as well as the directivity curve for \( k_0 = \pi / a \) is presented in Fig. 4 to confirm the absence of any preferential direction of sensitivity.
Iso-frequency circle (radius proportional to excitation frequency)
Printed bitmap image

- Diameter = 50 mm
- Bandwidth = [50, 350] kHz
- Angular range = [0, 180] deg.
- Double polarity
- Image resolution = 625 x 625 px
- Resulting pixel spacing = 80 µm

Experimental Results

Electrode 1

Electrode 2
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
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Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Plate response to chirp excitation
Experimental Results:
Source localization

Electrode 1

Electrode 2
Experimental Results:
Source localization
Experimental Results: Two-source localization
FSAT Fabrication on varied substrates

- PVDF
- PZT
- MFC
In collaboration with Prof. Cesnik @ UMich
Compensation Function

\[ V(\omega) = jU_{10}(\omega)k_0(\omega)\mathcal{H}(\theta)\mathcal{D}(\omega, \theta) \]

\[ \mathcal{H}(\vartheta) = \frac{t_P}{A_P} \left( d_{31}c_{11} + d_{32}\nu_{12}c_{22} \right) \cos^2(\vartheta) + \left( d_{31}\nu_{12}c_{22} + d_{32}c_{22} \right) \sin^2(\vartheta) \]

\[ b^T (d^\sigma C^E d^\sigma^T - \epsilon^\sigma) b \]

\[ \mathcal{C}(\vartheta) = \frac{1}{\mathcal{H}(\vartheta)} \]

\[ a_n = a_0 \bar{C}_n \quad \text{with} \quad \bar{C}_n = \frac{C_n}{\max(C_n)} \]
Directionality Plots

- Analytical
- Experimental

80 kHz

90 kHz

100 kHz
Acoustic Wave Rosettes for multi-component strain sensing
Conceptual Idea

• Think of patterning as a **QR code** where stretching of patterning carries information about strain components

Scan info → No strain

Scan info → $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$

Scan device → Surface Acoustic Wave (SAW)
Surface Acoustic Wave (SAW) Strain Sensors

- Sensing material distribution

\[ f(x) = \sum_{i=1}^{N} \delta(x-id(1+\varepsilon)) \]

- Strain-Frequency relation

\[ c_R = \frac{0.87 + 1.12\nu}{1 + \nu} \sqrt{\frac{G}{\rho}} \]

\[ f = \frac{c_R}{d(1+\varepsilon)} \]
\[ f(\varepsilon) = \frac{c}{d(1 + \varepsilon)} \]

\[ d = \frac{c}{f_0} \Rightarrow f_0 = 4\text{MHz} \rightarrow d = 1.3\text{mm} \]
Rosette Configuration

$f_0 = 50 \text{ MHz}$

$N = 32$

$-30^\circ, 90^\circ, 210^\circ$
Pattern is defined in the wavenumber domain

\[ f(\mathbf{x}) = \frac{1}{N} \text{rect}\left(\frac{|\mathbf{x}|}{a}\right) \sum_{n=1}^{N} \sin(k_n \cdot \mathbf{x}) \]
Determination of surface strain components

- Strain rosette equation

\[
\varepsilon_x \cos^2(\vartheta_0) + \varepsilon_{xy} \sin(2\vartheta_0) + \varepsilon_y \sin^2(\vartheta_0) = \frac{1}{2} \left[ 1 - \left( \frac{f_d}{f_0} \right)^2 \right]
\]

- \(f_0\) and \(\vartheta_0\) assigned at design stage. They correspond to the radiation peak in wavenumber domain
- \(f_d\) is a sensor output
- \(\varepsilon_x, \varepsilon_y, \varepsilon_{xy}\) are the unknowns

\[A\epsilon = b\]
Numerical test

Displacement field

\[ u = \frac{q}{E} x \]
\[ v = -\nu \frac{q}{E} y \]

Strain Field

\[ \varepsilon_x = \frac{q}{E} \]
\[ \varepsilon_y = -\nu \frac{q}{E} \]
\[ \varepsilon_{xy} = 0 \]
Example Solution: $q = 1 \text{ GPa}$

- Excitation Input
- Output, Reference ($q = 0 \text{ GPa}$)
- Output, Stretched ($q = 1 \text{ GPa}$)

Peaks Shift leads to strain mapping

<table>
<thead>
<tr>
<th></th>
<th>Analytical ($\mu \varepsilon$)</th>
<th>Measured ($\mu \varepsilon$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>1414</td>
<td>1434</td>
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<tr>
<td>$\varepsilon_y$</td>
<td>-467</td>
<td>-485</td>
</tr>
<tr>
<td>$\varepsilon_{xy}$</td>
<td>0</td>
<td>-31</td>
</tr>
</tbody>
</table>
Steering of Bulk Waves

Transducer Design and Numerical Simulations
Steering properties in the bulk

\[ \sin \theta = \frac{dc}{2f} \]

- Electrode 1
- Electrode 2
- \( d = 2 \text{ mm} \)
- \( f_0 = 3.1 \text{ MHz} \)

Usable steering range: 3.3 MHz to 12.0 MHz
$f = 4 \text{ MHz}, \ Angle = 39.2^\circ$

**Time snapshot**

- S-wave
- P-wave

**FFT 3D**

- Expected directionality
- Expected wavenumber

IWSHM - Stanford University
\[ f = 5 \text{ MHz}, \ Angle = 51.7^\circ \]
$f = 7 \text{ MHz, Angle} = 63.7^\circ$
f = 9 MHz, Angle = 69.9°

Time snapshot

FFT 3D

S-wave

P-wave

Expected directionality

Expected wavenumber
Inspection scenario

Specimen detail

<table>
<thead>
<tr>
<th>Hole</th>
<th>Angle [deg]</th>
<th>Frequency [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.6</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>32.4</td>
<td>3.7</td>
</tr>
<tr>
<td>3</td>
<td>43.6</td>
<td>4.3</td>
</tr>
<tr>
<td>4</td>
<td>51.8</td>
<td>5.0</td>
</tr>
<tr>
<td>5</td>
<td>62.3</td>
<td>6.7</td>
</tr>
</tbody>
</table>
Results: $f = 3.25$ MHz

P-wave reflections from top holes
Results: $f = 6.67$ MHz
Summary and Outlook

• Class of transducers designed through a Fourier-based approach
  – Sensing material distribution is designed and analyzed in spatial Fourier domain
  – Exploitation of inherent frequency-dependent directional properties (SHM application)
  – Frequency shifts of radiation associated with local straining (strain sensing application)

• Currently working on Bulk wave FSAT for thickness steering/interrogation

• Potential for integration for a health&usage monitoring device
Thank you

Collaborators
M. Carrara, B. Xu,
E. Baravelli, M. Senesi, S. Gonella

Task 11 Multifunctional Sensors for Loads Monitoring and Structural Diagnostic
Georgia Tech VLCROE (2011-2016).
• Monitoring issues:
  – Airframe:
    • Fatigue cracking and corrosion drivers of inspections and maintenance
    • Damage from impact/ballistic events
  – Rotor hub:
    • Dynamic components subject to high-cycle fatigue
    • Impact damage (ballistic damage, delaminations, voids)

• Structural Health and Usage Monitoring strategies:
  – Loads monitoring through strain measurements for fatigue estimation
  – Active monitoring of large areas or inaccessible hotspots
  – Passive sensing and localization of impacts
SHM Distributed Transducer Arrays

- CFRP composite panel
- Quasi-isotropic layup of $[0/90/-45/45]_2$

Courtesy of J. & T. Michaels – Georgia Tech
Quadrilateral array

\[ f(x_1, x_2) = f_0 \sum_{n=1}^{N} \sum_{m=1}^{M} \delta(x - x_{n_1,n_2}) \quad x_{n_1,n_2} = n_1 e_1 + n_2 e_2 \]

\[ e_1 = d_1 \cos \alpha i_1 + d_1 \sin \alpha i_2 \]

\[ e_2 = d_2 \cos \beta i_1 + d_2 \sin \beta i_2 \]

Frequency-dependent radiation
FSAT Spiral Array: thresholding

\[ \bar{f}(\mathbf{x}) = \begin{cases} 
  1, & f(\mathbf{x}) \geq \varepsilon \\
  0, & f(\mathbf{x}) < |\varepsilon| \\
  -1, & f(\mathbf{x}) \leq -\varepsilon 
\end{cases} \]
Numerical Results:
Chirp Excitation
Determination of surface strain components

- Three equations needed to solve for strain components (linear system)

\[
A \varepsilon = b \quad \iff \quad \varepsilon = A^{-1}b
\]

- where:

\[
\varepsilon = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}]^T
\]

\[
b_i = \frac{1}{2} \left[ \left( \frac{f_{d_i}}{f_{0_i}} \right)^2 - 1 \right]
\]

\[
A(i,:) = [\cos^2(\vartheta_{0_i}), \sin^2(\vartheta_{0_i}), \sin(2\vartheta_{0_i})]
\]
Numerical Test

Displacement field

\[ u = 0 \]

\[ v = \frac{q}{2G}x \]

Strain Field

\[ \varepsilon_x = 0 \]

\[ \varepsilon_y = 0 \]

\[ \varepsilon_{xy} = \frac{q}{2G} \]
Example Solution: $q = 1$ GPa

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<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>0</td>
<td>49.5</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
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<td>-52</td>
</tr>
<tr>
<td>$\varepsilon_{xy}$</td>
<td>1881</td>
<td>1824</td>
</tr>
</tbody>
</table>
Outlook: higher order symmetries