A Formula for Solution Separation without Subset Solutions for Advanced RAIM

Juan Blanch
Dept. of Aero&Astro
Stanford University
Stanford, California
blanch@stanford.edu

Todd Walter
Dept. of Aero&Astro
Stanford University
Stanford, California
twalter@stanford.edu

Per Enge
Dept. of Aero&Astro
Stanford University
Stanford, California
penge@stanford.edu

Abstract— **Advanced RAIM (ARAIM) algorithms developed for aviation can be applied in situations where the probabilities of fault are arbitrarily high. However, with large probabilities, the user receiver needs to protect against a large number of fault modes (which results from the combination of simultaneous independent faults). In baseline ARAIM algorithms, the user receiver must compute a fault tolerant position solution for each fault mode (computed using the subset of the available measurements that would not be affected by the fault mode), or at least its error covariance. After showing how to use a solution separation algorithm without computing the fault tolerant position solutions, we present a technique to obtain upper bounds on the subset error covariance for a given subset size. This upper bound does not require the computation of every subset error covariance, and can be computed without additional matrix inversions. We evaluate the potential of this technique by examining a specific satellite geometry.**

Keywords— *Advanced RAIM, fault tolerant filters, integrity, radio navigation*

I. INTRODUCTION

The safe operation of UAVs and autonomous vehicles in general is likely to be dependent on an assured navigation solution. That is, the position error in the solution computed onboard should be within a known bound (the protection level) with very high probability. In radio-navigation, such position error bounds are easy to compute when the error distribution of the pseudorange errors is known. When at most one measurement is faulty, Receiver Autonomous Integrity Monitoring (RAIM) can be used to compute a guaranteed error bound. RAIM was developed for aviation applications, where the probability of fault of more than one measurement can be considered negligible. However, autonomous vehicles will be expected to operate in environments where this is no longer true. Multipath, erroneous clocks (in the case of terrestrial signals of opportunity), or even spoofing, could cause very large delays, and with high probability.

Advanced RAIM (ARAIM) algorithms developed for aviation [1] can, in theory, be applied in situations where the probabilities of fault are arbitrarily high. This class of algorithms typically forms a list of subsets corresponding to a fault mode (a fault mode being a combination of m simultaneous measurement faults), and assigns a probability to it. As the probabilities of fault of each measurement increases, more

subsets need to be characterized. In order to compute a protection level, the receiver must compute the covariance of the position error corresponding to each subset. This is practical when no more than two simultaneous faults need to be considered, but it becomes impractical when the receiver must list all subsets with N-m measurements when m is three or more (N being the number of measurements). For example, for 30 range measurements, a target integrity of 10^{-7}, and a probability of fault of 1%, the receiver would need to form a list of 2 million subsets [5].

Many techniques to reduce the number of subsets have been described [2],[3],[4]. In this paper we develop a new technique to monitor and account for the subset geometries without having to compute, neither the corresponding fault tolerant solution nor their covariance explicitly. The key to this technique is a formula that can provide an upper bound on the standard deviation of all subsets of a given size.

We will start by showing how to implement a relaxation of solution separation that does not require the explicit computation of all the subset position solutions that are monitored, but that still requires the computation of their standard deviation. Then we will derive an upper bound on the standard deviations that does not require making an exhaustive list of all standard deviations. Finally we show a very preliminary evaluation the performance this upper bound by comparing it to the actual maximum in one example.

II. SOLUTION SEPARATION WITHOUT COMPUTING SUBSET SOLUTIONS

In this section we show how we can obtain an upper bound on the protection level without computing explicitly the solution separation subset solutions. The notations and results used in this section are mostly drawn from [1] and [5]. In order to focus on the contribution of this paper, we will assume that the nominal biases are zero and that the accuracy error model and the integrity error model are the same.

A. Solution separation test statistic

The idea of solution separation, as described for example for ARAIM in [1], is to compute a fault tolerant position solution $x^{(k)}$ for each fault mode k and compare it to the all-in view solution. For each fault mode k and position coordinate position q, the consistency test passes if:

$$\left|\hat{x}_q^{(k)} - \hat{x}_q^{(0)}\right| \le T_{k,q} \tag{1}$$

With:

$$T_{k,q} = K_{fa,q}^{(k)} \sigma_{ss,q}^{(k)} \tag{2}$$

Where $\sigma_{ss,q}^{(k)}$ is the standard deviation of the solution separation $\hat{x}_q^{(k)} - \hat{x}_q^{(0)}$ under nominal conditions and $T_{k,q}$ is a scalar set to meet a pre-determined false alert rate under fault free conditions [1]. A valid protection level can be output if all consistency tests pass.

B. Protection Level Equation

The protection level in the coordinate q, PL_q, can be computed by solving the following equation:

$$2Q\left(\frac{PL_q}{\sigma_q^{(0)}}\right) + \sum_{k=1}^{N_{\text{fault modes}}} p_{fault,k} Q\left(\frac{PL_q - T_{k,q}}{\sigma_q^{(k)}}\right) = PHMI_q \tag{3}$$

where:

Q is the right hand side normal cdf

$P = W - WG\left(G^T W G\right)^{-1} G^T W$ is the standard deviation of the error of the kth position solution

$p_{fault,k}$ is the prior probability of fault mode k

$PHMI_q$ is the integrity allocation to the qth coordinate

As shown in [1], this equation can be solved efficiently using a half interval search. Since the right hand term is the integrity risk, it is sufficient to have:

$$2Q\left(\frac{PL_q}{\sigma_q^{(0)}}\right) + \sum_{k=1}^{N_{\text{fault modes}}} p_{fault,k} Q\left(\frac{PL_q - T_{k,q}}{\sigma_q^{(k)}}\right) \le PHMI_q \tag{4}$$

C. Solution separation test without evaluating the subset solutions

Now let us assume that for a subset Ω of the fault modes we have not computed the subset solutions. In [5], we showed that the chi-square statistic is a bound on the normalized solution separation, that is:

$$\forall k \quad \frac{\left|\hat{x}_q^{(k)} - \hat{x}_q^{(0)}\right|}{\sigma_{ss,q}^{(k)}} \le \sqrt{y^T \left(W - WG\left(G^T W G\right)^{-1} G^T W\right) y} \tag{5}$$

where y are the linearized measurements, W is the inverse of the pseudorange error covariance, and G is the geometry matrix. Therefore, if:

$$\sqrt{y^T \left(W - WG\left(G^T W G\right)^{-1} G^T W\right) y} \le K_{fa,q}^{\Omega} \tag{6}$$

we have:

$$\forall k \quad \frac{\left|\hat{x}_q^{(k)} - \hat{x}_q^{(0)}\right|}{\sigma_{ss,q}^{(k)}} \le K_{fa,q}^{\Omega} \tag{7}$$

We can therefore use Equation (3) to compute the protection level with:

$$\forall k \in \Omega \quad T_{k,q} = K_{fa,q}^{\Omega} \sigma_{ss,q}^{(k)} \tag{8}$$

This means that we do not need to evaluate the subset position solutions to check the consistency of the measurements. However, we still need the standard deviations $\sigma_q^{(k)}$ and $\sigma_{ss,q}^{(k)}$ to evaluate the Protection Level.

D. Computing a Protection Level without evaluating all subset solutions

In [5] we showed that when using the all-in-view least squares position solution, we have the identity:

$$\sigma_{ss,q}^{(k)2} = \sigma_q^{(k)2} - \sigma_q^{(0)2} \tag{9}$$

Now let us assume that that we have an upper bound σ_q^{Ω} of all the standard deviations $\sigma_q^{(k)}$ for all k in Ω:

$$\forall k \in \Omega \quad \sigma_q^{(k)} \le \sigma_q^{J} \tag{10}$$

For any value of L and any k in Ω we have:

$$Q\left(\frac{L - T_{k,q}}{\sigma_q^{(k)}}\right) \le Q\left(\frac{L - K_{fa,q}^{\Omega} \sqrt{\sigma_q^{\Omega 2} - \sigma_q^{(0)2}}}{\sigma_q^{\Omega}}\right) \tag{11}$$

The important feature of this upper bound is that it does not depend on the index k. We can further write:

$$\sum_{k \in \Omega} p_{fault,k} Q\left(\frac{L - T_{k,q}}{\sigma_q^{(k)}}\right) \le$$
$$\left(\sum_{k \in \Omega} p_{fault,k}\right) Q\left(\frac{L - K_{fa,q}^{\Omega} \sqrt{\sigma_q^{\Omega 2} - \sigma_q^{(0)2}}}{\sigma_q^{\Omega}}\right) \tag{12}$$

Note that factor is now the same for all fault modes in Ω. We can compute a valid protection level with the following equation:

$$2Q\left(\frac{PL_q}{\sigma_q^{(0)}}\right) + \sum_{k \notin \Omega} p_{fault,k} Q\left(\frac{PL_q - T_{k,q}}{\sigma_q^{(k)}}\right) +$$
$$\left(\sum_{k \in \Omega} p_{fault,k}\right) Q\left(\frac{L - K_{fa,q}^{\Omega} \sqrt{\sigma_q^{\Omega 2} - \sigma_q^{(0)2}}}{\sigma_q^{\Omega}}\right) = PHMI_q \tag{13}$$

However, this equation assumes that we have the upper bound σ_q^{Ω}. The purpose of the next section is to obtain such a bound.

III. UPPER BOUND ON THE ERROR COVARIANCES OF SUBSETS OF A GIVEN SIZE

The formula for the standard deviation of the all-in-view position solution position error with nominal error is given by:

$$\sigma_q^{(0)2} = e^T \left(G^T W G \right)^{-1} e \tag{14}$$

where the vector e projects the position onto the coordinate of interest. For a subset position solution where we have removed all the indices j in a set J the standard deviation is given by:

$$\sigma_q^{(J)2} = e^T \left(G^T W G - G_J^T W_J G_J \right)^{-1} e \tag{15}$$

where G_J and W_J are respectively the geometry matrix and the weighting matrix corresponding to the measurements in the set J. (We have changed the notation because we do not need to refer to the fault index k).

The goal of the rest of this section is to obtain an upper bound on $\sigma_q^{(J)2}$ for a fixed size of J, that does not require listing all the possible sets and performing the matrix inversion in Equation (15).

A. Matrix inversion lemma

We start by applying the matrix inversion lemma (also called Woodbury matrix identity) [6] to Equation (15). This step allows us to express $\sigma_q^{(J)2}$ as a function of $\sigma_q^{(0)2}$. After applying the identity, we get:

$$\sigma_q^{(J)2} = e^T \left(G^T W G \right)^{-1} e +$$
$$e^T \left(G^T W G \right)^{-1} G_J^T W_J \left(W_J - W_J G_J \left(G^T W G \right)^{-1} G_J^T W_J \right)^{-1} W_J G_J \left(G^T W G \right)^{-1} e$$
$$= \sigma_q^{(0)2} + e^T S_J \left(P_{JJ} \right)^{-1} S_J^T e$$

$$\tag{16}$$

where:

P_{JJ} is obtained by selecting indices J in the rows and columns of the matrix $P = W - W G \left(G^T W G \right)^{-1} G^T W$

$e^T S_J$ composed of the indices J of $e^T S$ where $S = \left(G^T W G \right)^{-1} G^T W$

So even if at first sight Equation (16) seems very convoluted, many of the terms are byproducts of the all-in-view position solution. We note that this equation is a generalization of the rank one update formula (where the matric P_{JJ} is a scalar).

However, this form should not be applied directly when $|J|$ is larger than four, because it would be more computationally intensive than just computing the subset covariance directly. It can however be exploited to find upper bounds on the worst covariance.

We further modify Equation (16) by defining D_J, the matrix formed by the diagonal terms of P_{JJ}. We have:

$$\sigma_q^{(J)2} = \sigma_q^{(0)2} + s_{norm,J} \left(P_{norm,JJ} \right)^{-1} s_{norm,J}^T \tag{17}$$

where:

$$P_{norm} = D^{-\frac{1}{2}} P D^{-\frac{1}{2}}$$

$s_{norm} = D^{-\frac{1}{2}} S^T e$ and $s_{norm,J}$ designates the vector obtained extracting the indices J in s_{norm}.

The matrix $P_{norm,JJ}$ has only ones in the diagonal. In the case where $|J|=1$, P_{JJ} is equal to the scalar 1, and the equation above is simply a rank one update.

To go further, we use the following inequality [7]:

$$s_{norm,J} \left(P_{norm,JJ} \right)^{-1} s_{norm,J}^T \leq \left| s_{norm,J} \right|^2 \max \left(\lambda \left(P_{norm,JJ} \right) \right) \tag{18}$$

where $\lambda(A)$ designates the set of eigenvalues of the matrix A. We also have:

$$\max \left(\lambda \left(P_{norm,JJ} \right) \right) = \frac{1}{\min \left(\lambda \left(P_{norm,JJ} \right) \right)} \tag{19}$$

Therefore:

$$\sigma_q^{(J)2} \leq \sigma_q^{(0)2} + \frac{\left| s_{norm,J} \right|^2}{\min \left(\lambda \left(P_{norm,JJ} \right) \right)} \tag{20}$$

As mentioned above, we want an upper bound valid for any set J of a fixed size, so we take the maximum across all subsets of size m:

$$\max_{J \text{ s.t. } |J|=m} \sigma_q^{(J)2} \leq \sigma_q^{(0)2} + \frac{\max\limits_{J \text{ s.t. } |J|=m} \left| s_{norm,J} \right|^2}{\min\limits_{J \text{ s.t. } |J|=m} \min \left(\lambda \left(P_{norm,JJ} \right) \right)} \tag{21}$$

An upper bound of the first part of the product can be obtained by: first, sorting the squares of the vector s in decreasing order, and second, taking the sum of the first m terms. To compute the second part, we need a lower bound on $\min \left(\lambda \left(P_{norm,JJ} \right) \right)$.

B. Lower bound the smallest eigenvalue of $P_{norm,JJ}$ for any set J with m elements

This lower bound can be computed using the Gershgorin circle theorem [8]. This theorem states that each eigenvalue of a matrix A lies at least within one of the discs $D(a_{ii}, R_i)$ (called Gershgorin circles) where:

$$R_i = \sum_{j \neq i} \left| a_{ij} \right| \tag{22}$$

For symmetric matrices, where the eigenvalues are real numbers, this result means that for any eigenvalue λ, there exists an index i such that:

$$\lambda \in \left[a_{ii} - \sum_{j \neq i} \left| a_{ij} \right|, a_{ii} + \sum_{j \neq i} \left| a_{ij} \right| \right] \tag{23}$$

As a consequence, for any eigenvalue λ we have:

$$\min_i \left(a_{ii} - \sum_{j \neq i} |a_{ij}| \right) \leq \lambda \qquad (24)$$

Let us now apply this lower bound to the matrix $P_{norm,JJ}$. We have:

$$\min_{i \in J} \left(1 - \sum_{j \neq i} |P_{norm,ij}| \right) \leq \lambda \left(P_{norm,JJ} \right) \qquad (25)$$

We now take the minimum across all sets J of size m:

$$\min_{J \text{ s.t. } |J|=m} \min_{i \in J} \left(1 - \sum_{j \neq i} |P_{norm,ij}| \right) \leq \min_{J \text{ s.t. } |J|=m} \lambda \left(P_{norm,JJ} \right) \qquad (26)$$

Now let us consider the left hand side term. We write:

$$\min_{J \text{ s.t. } |J|=m} \left\{ \min_{i \in J} \left(1 - \sum_{\substack{j \neq i \\ j \in J}} |P_{norm,ij}| \right) \right\} =$$
$$\min_{\substack{J \text{ s.t. } |J|=m \\ i \in J}} \left(1 - \sum_{\substack{j \neq i \\ j \in J}} |P_{norm,ij}| \right) \qquad (27)$$

Next, we rewrite the minimum by considering all the subsets K of size m-1:

$$\min_{\substack{J \text{ s.t. } |J|=m \\ i \in J}} \left(1 - \sum_{\substack{j \neq i \\ j \in J}} |P_{norm,ij}| \right) = \min_{\substack{i \in [1,\cdots,n] \\ K \text{ s.t. } |K|=m-1 \\ i \notin K}} \left(1 - \sum_{j \in K} |P_{norm,ij}| \right) \qquad (28)$$

We can further write it as:

$$\min_{\substack{J \text{ s.t. } |J|=m \\ i \in J}} \left(1 - \sum_{\substack{j \neq i \\ j \in J}} |P_{norm,ij}| \right) =$$
$$\min_{i \in [1,\cdots,n]} \min_{\substack{K \text{ s.t. } |K|=m-1 \\ i \notin K}} \left(1 - \sum_{j \in K} |P_{norm,ij}| \right) \qquad (29)$$

This step is very important, because for each index i we can compute the inner minimum without listing all the subsets K. We can see this by writing:

$$\min_{\substack{K \text{ s.t. } |K|=m-1 \\ i \notin K}} \left(1 - \sum_{j \in K} |P_{norm,ij}| \right) =$$
$$1 - \max_{\substack{K \text{ s.t. } |K|=m-1 \\ i \notin K}} \sum_{j \in K} |P_{norm,ij}| \qquad (30)$$

The subset that realizes this maximum is composed of the largest m-1 values of the ith row of $|P_{norm,ij}|$, which can be obtained by sorting the coefficients in decreasing order and taking the first m-1.

If this lower bound is negative, it cannot be exploited, but if it is positive, we have the upper bound:

$$\max_{J \text{ s.t. } |J|=m} \sigma_q^{J\,2} \leq \sigma_q^{(0)2} +$$
$$\frac{\displaystyle \max_{J \text{ s.t. } |J|=m} \left| s_{norm,J} \right|^2}{1 - \displaystyle \max_{i \in [1,\cdots,n]} \max_{\substack{K \text{ s.t. } |K|=m-1 \\ i \notin K}} \sum_{j \in K} |P_{norm,ij}|} \qquad (31)$$

Although this equation appears to be very complicated, it can be evaluated by finding the m larger coefficients (in absolute value) in s_{norm}, and the m-1 larger entries (excluding 1) in each row of the matrix P_{norm}. This is therefore orders of magnitude faster than performing an exhaustive search on all subsets of size m and computing the corresponding covariance (even if we use rank one updates). We do note that this upper bound can only be computed when the denominator in the second term is strictly positive.

IV. EVALUATION OF THE UPPER BOUND

In this section we evaluate the upper bound developed in the previous section by comparing it to the actual maximum for a given satellite geometry for a range of subset sizes. We will assume that the standard deviation of the measurement noise is one (meter, for example) for all measurements, so that the weighting matrix W is the identity matrix.

The geometry used for this example corresponds to a triple constellation configuration GPS-Galileo-GLONASS, where the clock offsets per constellation are estimated by the receiver. We include the G matrix in the Appendix. Fig. 1 through 4 show the histograms of the standard deviations of the subset position solutions $\sigma_q^{(k)}$ normalized by the all-in-view $\sigma_q^{(0)}$ for subset sizes from n-2 to n-5.

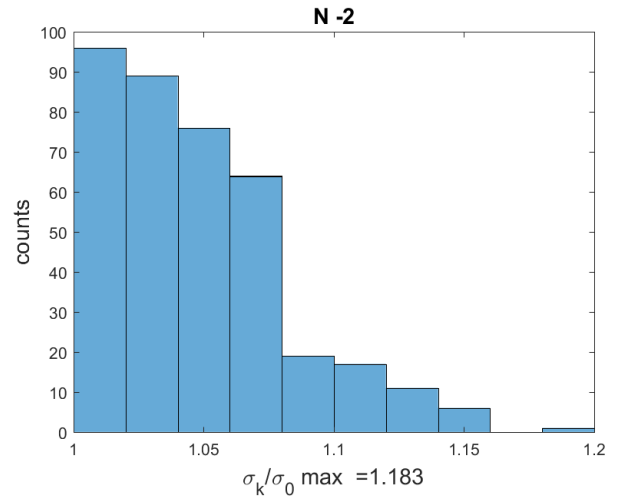

Fig. 1. Histogram of the subset position sigma normalized by the all-in-view for m=2 for the example geometry (379 subsets).
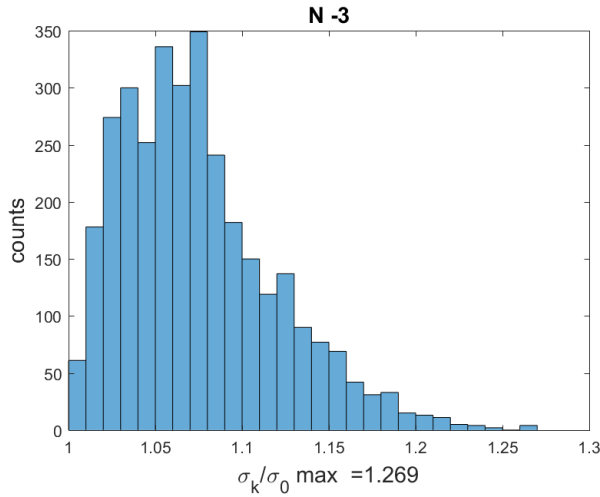
Fig. 2. Histogram of the subset position sigma normalized by the all-in-view for m=3 for the example geometry (3277 subsets).
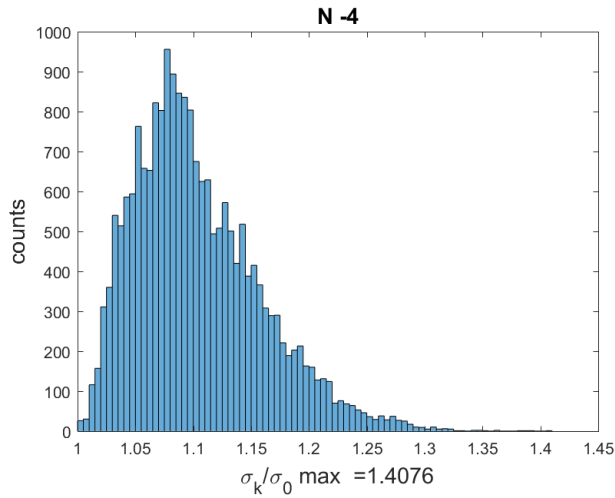

Fig. 3. Histogram of the subset position sigma normalized by the all-in-view for m=4 for the example geometry (20476 subsets).
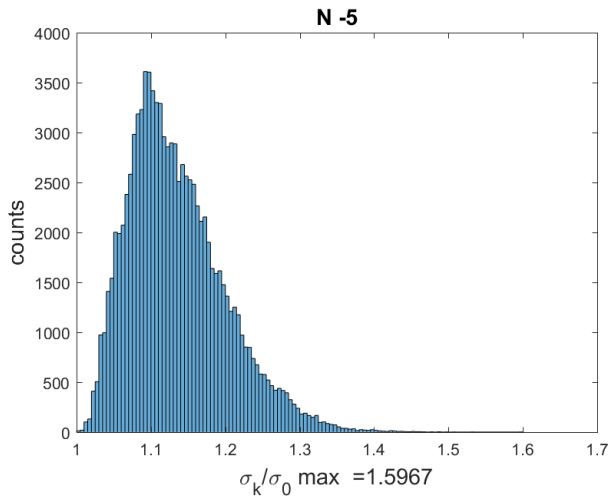

Fig. 4. Histogram of the subset position sigma normalized by the all-in-view for m=5 for the example geometry (98281 subsets).

As expected all values are above one. We also see that although there are many subset solutions (from N=379 for m=2 to N=98281 for m=5) the subset geometries remain very strong. It therefore seems a waste of computational power to list and compute the error covariance for every single one of them. It is in this type of geometry that the formula developed above (Equation (31)) can dramatically lower the amount of computations.

We now compute the upper bound provided by the formula given in Equation (31). Table 1 shows the results compared to the actual maximum:

TABLE I.   TABLE TYPE STYLES

	Bounds on the worst subset standard deviation	
	Worst case σ_k / σ_0 column subhead	*Upper bound computed using Equation (31)*
n-2	1.1830	1.2159
n-3	1.2690	1.3755
n-4	1.4076	1.6853
n-5	1.5967	2.7145

The results in Table 1 are very encouraging, because the bound is very tight up to m=4, and it could still be useful for m=5. Considering the savings in computational load, these preliminary results are very encouraging. The utility of the upper bound in this paper will be very dependent on the application. In general, it will be valuable in computation constrained applications with a large number of measurements and high fault rates.

APPENDIX

$$
\begin{aligned}
G = [\ &-0.0939 \quad -0.6984 \quad -0.7095 \quad 1.0000 \quad 0 \quad 0; \\
&-0.7752 \quad 0.4972 \quad -0.3897 \quad 1.0000 \quad 0 \quad 0; \\
&0.1627 \quad -0.9720 \quad -0.1697 \quad 1.0000 \quad 0 \quad 0; \\
&-0.7894 \quad -0.0651 \quad -0.6104 \quad 1.0000 \quad 0 \quad 0; \\
&0.0368 \quad 0.7023 \quad -0.7109 \quad 1.0000 \quad 0 \quad 0; \\
&0.7906 \quad -0.3823 \quad -0.4783 \quad 1.0000 \quad 0 \quad 0; \\
&0.8010 \quad 0.1143 \quad -0.5877 \quad 1.0000 \quad 0 \quad 0; \\
&-0.2378 \quad -0.9627 \quad -0.1294 \quad 0 \quad 1.0000 \quad 0; \\
&-0.5876 \quad -0.5164 \quad -0.6230 \quad 0 \quad 1.0000 \quad 0; \\
&-0.6473 \quad 0.2850 \quad -0.7070 \quad 0 \quad 1.0000 \quad 0; \\
&-0.3606 \quad 0.8821 \quad -0.3030 \quad 0 \quad 1.0000 \quad 0; \\
&-0.5713 \quad 0.7573 \quad -0.3165 \quad 0 \quad 1.0000 \quad 0; \\
&0.0956 \quad 0.6960 \quad -0.7116 \quad 0 \quad 1.0000 \quad 0;
\end{aligned}
$$

```
0.7527    0.2368   -0.6143        0   1.0000        0;
0.9512   -0.2862   -0.1153        0   1.0000        0;
0.8716   -0.0631   -0.4862        0   1.0000        0;
0.3268   -0.5866   -0.7410        0   1.0000        0;
-0.3965  -0.7858   -0.4746        0   1.0000        0;
-0.5517  -0.6429   -0.5314        0        0   1.0000;
-0.4731  -0.7189   -0.5093        0        0   1.0000;
-0.5218   0.1712   -0.8357        0        0   1.0000;
0.3261   -0.4333   -0.8402        0        0   1.0000;
-0.1731   0.9293   -0.3263        0        0   1.0000;
0.6479    0.1610   -0.7445        0        0   1.0000;
0.8936   -0.4346   -0.1123        0        0   1.0000;
0.9331    0.0734   -0.3521        0        0   1.0000;
-0.6882   0.7189   -0.0974        0        0   1.0000;
-0.0920   0.6799   -0.7275        0        0   1.0000];
```

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