Ionospheric Estimation using Extended Kriging for a low latitude SBAS

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ABSTRACT

The ionosphere causes the most difficult error to mitigate in Satellite Based Augmentation Systems (SBAS). The problem has been solved for the mid latitude regions using the thin shell approximation. There, it is very accurate on quiet days and allows the augmentation system so send the information in a two dimensional grid with a five by five degree resolution. However, even during quiet days, this approximation does not model correctly the ionosphere in the low latitudes: the decorrelation of the projected ionospheric vertical delays over the thin shell is very large. Several ionospheric estimation methods have been proposed to decrease the User Ionospheric Vertical Error (UIVE), among which are the 'conical domain' approach, tomography and extended kriging. The conical domain approach requires several measurements from the same satellite to work properly and in tomography the equation to solve is underdetermined, leading to artificial constraints and very large estimation errors at the edge of coverage. Extended kriging was developed to avoid these problems. The idea is to use kriging with several layers and an average vertical density profile to define the covariance between measurements (unlike in previous applications of kriging, where only one layer, the thin shell at 350 km, is used). Early results show that extended kriging gives estimation errors 30% to 50 % lower than the planar fit using the thin shell model. As a consequence this method has the potential to reduce the UIVEs by the same amount, thus increasing the availability of the augmentation system.

In this paper we will first recall the basics of extended kriging and the assumptions needed. Then we will present a new error analysis more adapted to disturbed ionospheric conditions and apply it to real ionospheric delay measurements taken at reference stations over Brazil. Based on this error analysis, we will propose a new Vertical Position Level equation and evaluate it using an SBAS simulation tool. The results show that, even under severe ionospheric disturbances, the 95th percentile of the Vertical Protection Level is to not too far from 50 meters.

INTRODUCTION

Up to now, no ionospheric estimation method fitting in the current standards has proven to be good enough to provide an acceptable level of service in the Equatorial regions for single frequency SBAS users. In the best cases, the residual errors appear to be almost five times larger than in the mid latitudes [1], [2], [3]. Several factors are behind this. Above all of them is the ionospheric behavior, which is characterized by sharp Total Electron Content gradients both spatial and temporal and large TEC values which are difficult to predict and describe [4]. But we can also blame: the current ionospheric algorithms, the message standards, and an error analysis based on Gaussian statistics - that is well suited for quiet ionospheric conditions but that predicts very large errors in disturbed conditions.

With the coming new signals (L5, L2C and Galileo signals), ionosphere induced delay on the pseudoranges will no longer be an issue for Satellite Based Augmentation Systems (SBAS), as dual frequency will enable users to remove it. Therefore, it might seem that the best option for providing SBAS in the Equatorial regions – where the ionosphere is not well modeled by the thin shell model and the planar approximations used in the mid latitudes – is to wait until dual frequency is available.

However, civil dual frequency will not be operational before 2015 and even this date is uncertain; so there is a risk involved in relying only on the new signals. Moreover, single frequency will still be a fall back mode for dual frequency users. As such, and taking into account that the new signals will require new standards, it is worthwhile finding: an ionospheric estimation algorithm adapted to disturbed conditions, a way to analyze the errors during these conditions that is not overly pessimistic, and the ideal way to send the ionospheric corrections to the user.

In this paper, we explore the benefits of combining Extended Kriging [3] and a new error analysis which does not rely on Gaussian statistics. First the main ideas behind Extended Kriging will be presented; second, an error analysis departing from the usual vertical error residual [3] will be described; finally, based on this error analysis, the expected performance in the position domain of an SBAS in Brazil will be evaluated. (In this paper, we will not investigate the message structure allowing the application of Extended Kriging. Instead we will assume that all measurements are known by the user.)

EXTENDED KRIGING

Extended Kriging [3] is an extension of the two dimensional estimation technique known as kriging and that has already been successfully applied to ionospheric estimation [5]. Kriging takes advantage of the random structure of the ionospheric delay as projected onto the thin shell: the measurements taken at the reference stations are projected on the thin shell and transformed in equivalent vertical delay. This random structure allows us to define a distance dependent covariance among the available measurements and between the measurements and the location to be estimated. Assuming this covariance structure, one can find the optimal estimator in a least squares sense. For more details about kriging see [5].

Because the thin shell model fails to capture the characteristics of the low latitude ionosphere, in Extended Kriging the distance between ionospheric pierce points (IPPs) is replaced by the notion of distance between ray paths, or, what is equivalent, covariance between ray paths. Since the method has been described in [3], we will recall the main steps of the estimation process and leave the details for the Appendix and [3].

We consider a snapshot solution, that is, we estimate an unknown ionospheric delay for a given line of sight using *n* measured ionospheric delays taken at the same time at the reference stations. The first step is to compute the covariance matrix of the measurements due to the ionosphere (*n* by *n*) *C* which is a function of the assumed ionospheric decorrelation and of the geometry of the measurements and *M*, which is the covariance describing the measurement noise and interfrequency bias residuals; the second step is to compute the covariance between the unknown line of sight and the measurements, *c*; The third step is to compute the *G* matrix, which describes the relevant geometric parameters of the measurements; finally we compute *g*, which describes the geometric parameters of the line of sight of the delay to be estimated. (For the details on how to obtain all these parameters, please see the Appendix). Once we have these parameters, we compute the weighting matrix *W*:

$$W = \left(C + M\right)^{-1}$$

Then we compute the set of coefficients λ:

$$\lambda = \left(W - WG\left(G^T W G\right)^{-1} G^T W\right)c + WG\left(G^T W G\right)^{-1} g$$

The estimate for the ionospheric delay is given by:

$$I_{unknown} = \lambda^T I_{measured}$$

where $I_{measured}$ is the vector of measurements.

ERROR ANALYSIS

In this section the estimation algorithm is tested using truth data collected at 12 sites over Brazil on February 19 and 21, 2002 during 24 hours every 5 minutes. For more information about the pre-processing of the data and the location of the stations please refer to [3], [4]. Station-wise cross-validation was chosen to evaluate the algorithm. In [3] it was shown how Extended Kriging (like kriging) provides an error bound associated to the estimate. However, this bound is only valid under well behaved ionospheric conditions (when the random structure of the ionosphere is close to Gaussian). In this work, we are trying to compute error bounds which do not depend too much on the specific error distributions. In addition to that, the error analysis presented here tries to account for the correlation of the ionospheric induced errors. Motivated by these requirements the following was done for each user (a station excluded from the set used to form the estimate) at every epoch. First, the ionospheric delays were computed using Extended Kriging; then, using the truth data, the residual slant errors were formed. Instead of plotting these errors, we computed the difference between each pair of slant errors. Also, for each pair, we computed the angle between the lines of sight. The results of this process were plotted in a two dimensional histogram with the difference in slant error on the vertical axis and the angle between the lines of sight on the horizontal one. Figure 1 shows the results for all the stations over Brazil (even those in under sampled situations) for the two considered days.

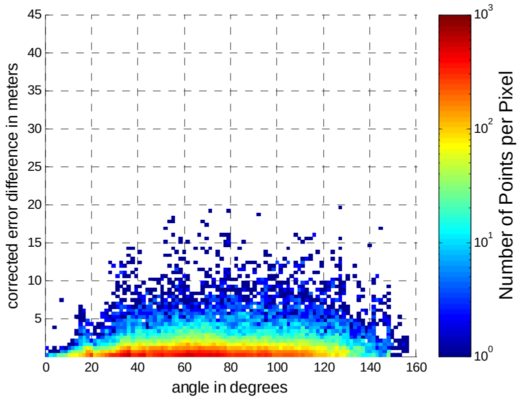

Figure 1. Cross-validation results using Extended Kriging over Brazil for February 19 and 21, 2002.

To measure the benefit of Extended Kriging compared to thin shell based algorithms, we show in Figure 2 the same plot obtained applying a planar fit using the thin shell at 350 km height.

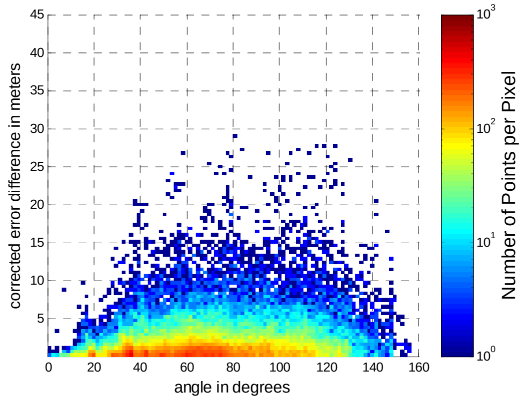

Figure 2. Cross-validation results using the planar fit over Brazil for February 19 and 21, 2002.

ERROR BOUNDING

This section introduces a new characterization of the error induced by the ionospheric delay. The idea is to assert that there exists a curve that bounds the two dimensional histogram formed in the previous section. Of course, such a curve would need a much more extensive validation than the one shown here, and should take into account all available data and, possibly, include physical considerations. Here, we will only take into account the data surveyed. The error bounding function is such that for a given user and two ionospheric residual errors ΔI_1 and ΔI_2 we have:

$$\left|\Delta I_1 - \Delta I_2\right| \le f(\theta)$$

where θ is the angle between the lines of sight. With the results shown in the previous section, we can take:

$$f(\theta) = \min(a + b\theta, c)$$

with a = 10 m, b = .3 m per degree, c = 25 m. The main advantage of this error description is that it does not rely on a specific shape of the error distribution, and that it only depends on the largest *observed* differential error delays.

VERTICAL POSITION LEVEL

One can evaluate the performance of an SBAS whose ionospheric errors are well described by the previous model. However, because the error model introduced in this work is not based on Gaussian statistics we need to modify the computation of the Vertical Position Level (VPL) to account for this new model.

In the current standards, the VPL is based on Gaussian statistics [6]. The user forms a diagonal weighting matrix W with the information coming from the SBAS message and computes the coefficients to be applied to the pseudoranges:

$$H = \left(G^T W G\right)^{-1} G^T W$$

With these coefficients, the covariance of the error position is given by:

$$Cov = \left(G^T W G\right)^{-1}$$

where G is the usual user geometry matrix (Notice that, although we use the same notation here, we refer to something different from the previous equations). If the third component of the G matrix is the vertical axis, the VPL equation is given by:

$$VPL = K\left[Cov\right]_{3,3}$$

where K=5.52. (Please refer to [7] for more details).

In this work, because we cannot assert that the ionospheric errors are well characterized by Gaussian statistics, we treat them as biases:

$$VPL = K\left[Cov\right]_{3,3} + bias$$

In this equation, the error covariance no longer includes the error caused by the ionosphere, as it is accounted in the second term. The bias term is given by:

$$bias = \max\left|\left[H\right]_{3,\cdot}\,\varepsilon_{iono}\right|$$

where ε_{iono} is the vector of ionospheric errors remaining after correction. This vector is subject to the linear constraints imposed by the function f introduced in the previous section. This bias can be computed using linear programming.

SBAS PERFORMANCE

To evaluate the performance of an SBAS over Brazil using Extended Kriging and the VPL equation outlined in earlier, we used the Matlab Algorithm Availability Simulation Tool (MAAST) [8]. The network of reference stations assumed coincides with the network where the ionospheric data has been collected [3]. MAAST predicts realistically the performance of an SBAS over a given period of time by computing at each location and time step the result of the VPL equation. In this work, MAAST was modified to account for the ionospheric bias. This term was computed using the MATLAB function linprog. Figures 3 and 4 respectively show the 95% percentile of the VPL and the HPL over a period of 24 hours (for the HPL, we took the square root of the sum of the squared lateral PL and longitudinal PL, which is pessimistic).

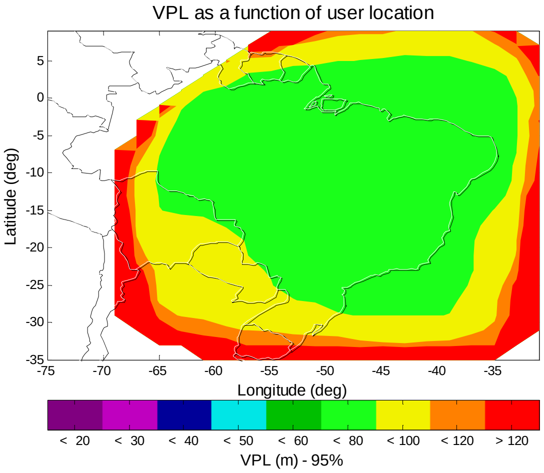

Figure 3. 95th percentile of the Vertical Protection Level over Brazil using Extended Kriging and the new PL equation.

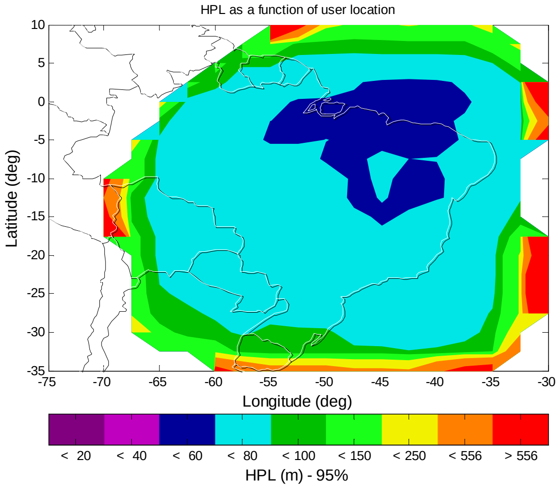

Figure 3. 95th percentile of the Horizontal Protection Level over Brazil using Extended Kriging and the new PL equation.

Although the analysis method is pessimistic (it does not rely on the statistics of the error distribution), the results are not very far from a 50 meters VPL, which is the Vertical Alert Limit for the LPV level of service.

CONCLUSION

This work has shown how Extended Kriging reduces by almost 30% the effect of a disturbed ionosphere for an SBAS user (Figures 1 and 2). This comparison was done using a new error analysis that captures the correlation of errors for a given user and the true magnitude of the slant errors (as opposed to errors projected in the vertical domain through the obliquity factor).

Because the errors induced by disturbed ionospheric conditions are not well described by Gaussian statistics, we have introduced a new characterization of the error as linearly constrained bias. Based on this error characterization and after modifying the Protection Level equation we see that it is possible to design an SBAS in Equatorial regions with a capability that is not too far from LPV (50 meters VAL).

APPENDIX

We first recall how to define the covariance two ray paths. We first discretize the ionosphere in several layers and assign a coefficient to each layer:

Height in km	300	350	400	450
φ_k	1/8.	1/8	1/8	1/8
Height in km	500	550	600	650
φ_k	1/8	1/8	1/8	1/8

For each ray path *i* and each layer *k*, we compute the corresponding Ionospheric Pierce Point $x_{k,i}$(location where the ray path crosses the shell) and the corresponding obliquity factor $ob_{k,j}$. The covariance between the ray path *i* and *j* is given by:

$$C_{ij} = \sum_{k=1}^{p} \varphi_k^2 ob_{k,i} ob_{k,j} \operatorname{cov}\left(x_{k,i}, x_{k,j}\right)$$

where we have:

$$\operatorname{cov}\left(x_{k,i}, x_{k,j}\right) = A \exp\left(-\frac{\left\|x_{k,i} - x_{k,j}\right\|}{d}\right)$$

with A=5 m^2 and d= 200 km.

The definition for G is:

$$G_i = \sum_{k=1}^{p} \varphi_k \cdot ob_{k,i}$$

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REFERENCES

[1] R. Lejeune, M.B. El-Arini. "Performance of a Tomographic Approach to SBAS Ionospheric Estimation in the Equatorial Region," *Proceedings of the Institute of Navigation National Technical Meeting*, San Diego, CA, 2004.

[2] L. Sparks, A. Komjathy, and A.J. Mannucci. "Estimating SBAS Ionospheric Delays Without Grids: The Conical Domain Approach," *Proceedings of the Institute of Navigation National Technical Meeting*, San Diego, CA, 2004.

[3] J. Blanch, T. Walter, P. Enge, "A New Ionospheric Estimation Algorithm for SBAS Combining Kriging and Tomography." *Proceedings of the Institute of Navigation National Technical Meeting*, San Diego, CA, 2004.

[4] A. L. Komjathy, L. Sparks, T. Mannucci, X. Pi. "Assessment of WAAS Ionospheric Correction Algorithms in the South American Region," *Proceedings of the Institute of Navigation GPS-02*. Portland, OR, 2002.

[5] J. Blanch. "An Ionospheric Estimation Algorithm for WAAS Based on Kriging," *Proceedings of the Institute of Navigation GPS-02*. Portland, OR, 2002.

[6] RTCA Special Committee 159, *Minimum Operational Performance Standards for Airborne Equipment Using Global Positioning System/ Wide Area Augmentation System*, RTCA/DO-229C, November 2001.

[7] T. Walter, P. Enge, A. Hansen. "An Integrity Equation for Use with Space Based Augmentation Systems**".** *GNSS-97* Munich, Germany, April 1997.

[8] S. Jan, W. Chan, T. Walter, P. Enge. "Matlab Simulation Toolset for SBAS Availability Analysis." *Proceedings of the Institute of Navigation GPS-01*. Salt Lake City, UT.