Analysis of GNSS Constellation Performance for Advanced RAIM

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Motivation: ARAIM Safety Case

ARAIM safety case is dependent on:

- bounding the probability of occurrence of faults ($P_{sat}$, $P_{const}$, $R_{sat}$, $R_{const}$, MFD)

- overbounding the distribution of nominal errors ($b_{nom}$, $\sigma_{URA}$)
Fault Rates and Fault Probabilities

• There are two related concepts: fault rate and fault probability
  • Fault rate is the probability that a fault will initiate per unit of time
  • Fault probability is the likelihood of experiencing a fault at a given time

• These 2 concepts are related by the Mean Fault Durations (MFD)
  • \( P_{\text{sat}} = MFD_{\text{sat}} \times R_{\text{sat}} \)
  • \( P_{\text{const}} = MFD_{\text{const}} \times R_{\text{const}} \)
# Satellite Performance Commitments

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>Galileo</th>
<th>GLONASS</th>
<th>Beidou</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>URA</strong></td>
<td>table</td>
<td>6 m DF, 7.5 m SF</td>
<td>18 m</td>
<td>table</td>
</tr>
<tr>
<td><strong>Threshold</strong></td>
<td>4.42 xURA</td>
<td>4.17xURA (25 m, 31.3 m)</td>
<td>70 m (3.89xURA)</td>
<td>4.42 xURA</td>
</tr>
<tr>
<td><strong>$R_{sat}$</strong></td>
<td>1e-5/hour</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>$P_{sat}$</strong></td>
<td>1e-5</td>
<td>3e-5</td>
<td>1e-4</td>
<td>1e-5</td>
</tr>
<tr>
<td><strong>$P_{const}$</strong></td>
<td>1e-8†</td>
<td>2e-4</td>
<td>1e-4</td>
<td>6e-5</td>
</tr>
<tr>
<td><strong>MFD</strong></td>
<td>1 hour</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>TTA</strong></td>
<td>10 seconds</td>
<td>-</td>
<td>10 seconds</td>
<td>300 seconds</td>
</tr>
<tr>
<td><strong>source</strong></td>
<td>SPS PS and NSP6_wp2</td>
<td>NSP6_wp4</td>
<td>NSP6_wp3</td>
<td>NSP6_wp5</td>
</tr>
</tbody>
</table>

† Recently proposed $R_{const} = 1e-9$/hour and $MFD_{const} = 10$ hours
\(R_{\text{sat}}\), \(P_{\text{sat}}\) and MFD

\(R_{\text{sat}}\) counts the number of upward crossings of the threshold in a given time period.

MFD is used to indicate the mean fault duration and is the total length of time that the errors are above the threshold divided by the number of upward crossings.

\(P_{\text{sat}}\) is the fraction of time spent above the threshold and equals \(R_{\text{sat}} \times \text{MFD}\).
Rate estimate based on Poisson distribution
For Fault Onset

• A fault occurring in one time interval does not affect the probability of it occurring in other time intervals (when the SV is set healthy), and
• The probability of a fault occurring does not change over time.

\[ P(k|R) = \frac{(R T)^k e^{-RT}}{k!} \]

\[ \hat{R} = \left( k + \frac{1}{2} \right) / T \]
GPS Performance Summary
Estimated Upper Bound on $R_{sat}$

- $R_{sat}$ has been improving over time
- Can easily validate committed value of $10^{-5}$/hour
- MFD is easily bounded by 1 hour
  - $P_{sat} < 10^{-5}$
Estimated Upper Bound on $R_{\text{const}}$

- Cannot bound probabilities much below $10^{-5}$ using only data validation.
- Considering 25 years of GPS service, the smallest bound is $\sim 2.5 \times 10^{-6}$/hour.
- Must rely on commitments for smaller numbers.
Galileo Performance Summary

Satellite Observation Data: 4,462,855 Comparisons
Estimated Upper Bound on $R_{sat}$

- Can easily validate expected value of $3 \times 10^{-5}$/hour
- Single observation is 36 minutes
  - $P_{sat} < 3 \times 10^{-5}$
- Not yet reached IOC and fairly limited data set
Estimates Upper Bound on $R_{\text{const}}$

- Considering 0 faults in 2.5 years of Galileo service, the smallest bound is $\sim 2.3 \times 10^{-5}$/hour

- No data or commitment on $\text{MFD}_{\text{const}}$
  - Needs to be less than 9 hours to meet commitment now
Summary on rates and durations

• The safety case for $P_{sat}$ and $P_{const}$ relies on a combination of commitments, similarity to previous systems, and data validation:
  • Published commitments for $P_{sat}$ and $P_{const}$ exist for all constellations
  • Data used to validate commitment values

• $P_{sat}$ is easier to validate assuming common values across all satellites

• $P_{const}$ cannot exclusively use data to validate below $\sim 3 \times 10^{-6}$
Nominal performance

\[ \text{range error} \leq N\left(b_{\text{nom}}, \sigma_{\text{URA}}\right) \]
Normalized clock and ephemeris errors

\[ \chi = \frac{\varepsilon}{\sigma_{URA}} \]
Bounding nominal performance

Users must be able to safely use a Gaussian model characterized by a normal distribution

$$F(x) \leq F_{\text{bound}}(x) = Q\left(\frac{x-y}{\sigma}\right)$$

Gaussian tail cdf

$$F\left(\sum_i s_i x_i\right) \leq F_{\text{bound}}\left(\sum_i s_i x_i\right) = Q\left(\frac{\sum_i s_ix_i - \sum_i s_i y_i}{\sqrt{\sum_i s_i^2}}\right)$$
Bounding correlated errors

\[
\text{user error}^2 = \left| \sum_{i=1}^{n} s_i x_i \right|^2 \leq \left| \sum_{i=1}^{n} s_i^2 \right| \left| \sum_{i=1}^{n} x_i^2 \right|
\]

\[
P(\text{user error} > K \sigma_{\text{user}}) \leq P\left(\sum_{i=1}^{n} x_i^2 > K^2 \right) \leq Q(K)
\]

User will be safe if this distribution is bounded by a normal distribution
Bounding the correlation of errors

For both GPS and Galileo, we find that correlated errors are well bounded by the model that assumes independence.
GPS nominal bounding results for all satellites over last 12 years
Partitioning the data

To a certain extent, we must protect the user against specific risk (for conditions that are knowable):

- Individual satellites
- Satellite block type (including clock type)
- Time (by year, by season, by month, or by day)
- Satellite age
- Age of navigation data
Partition per block
Example: by age of data
Cross-validation and bootstrap methods to determine confidence in overbounds
Summary

• We have described some of the techniques and approaches that can be used to analyze GNSS constellation performance for Advanced RAIM:
  • Estimation of fault rates
  • Overbounding of nominal performance
  • Correlation of ranging errors
  • Partitioning the nominal data
References


• Blanch J., Walter, T., and Enge, P., “Gaussian Bounds of Sample Distributions for Integrity Analysis”. IEEE Transactions on Aerospace and Electronic Systems.” PP. 1-1. 10.1109/TAES.2018.2876583

