Analysis of GNSS Constellation Performance for Advanced RAIM

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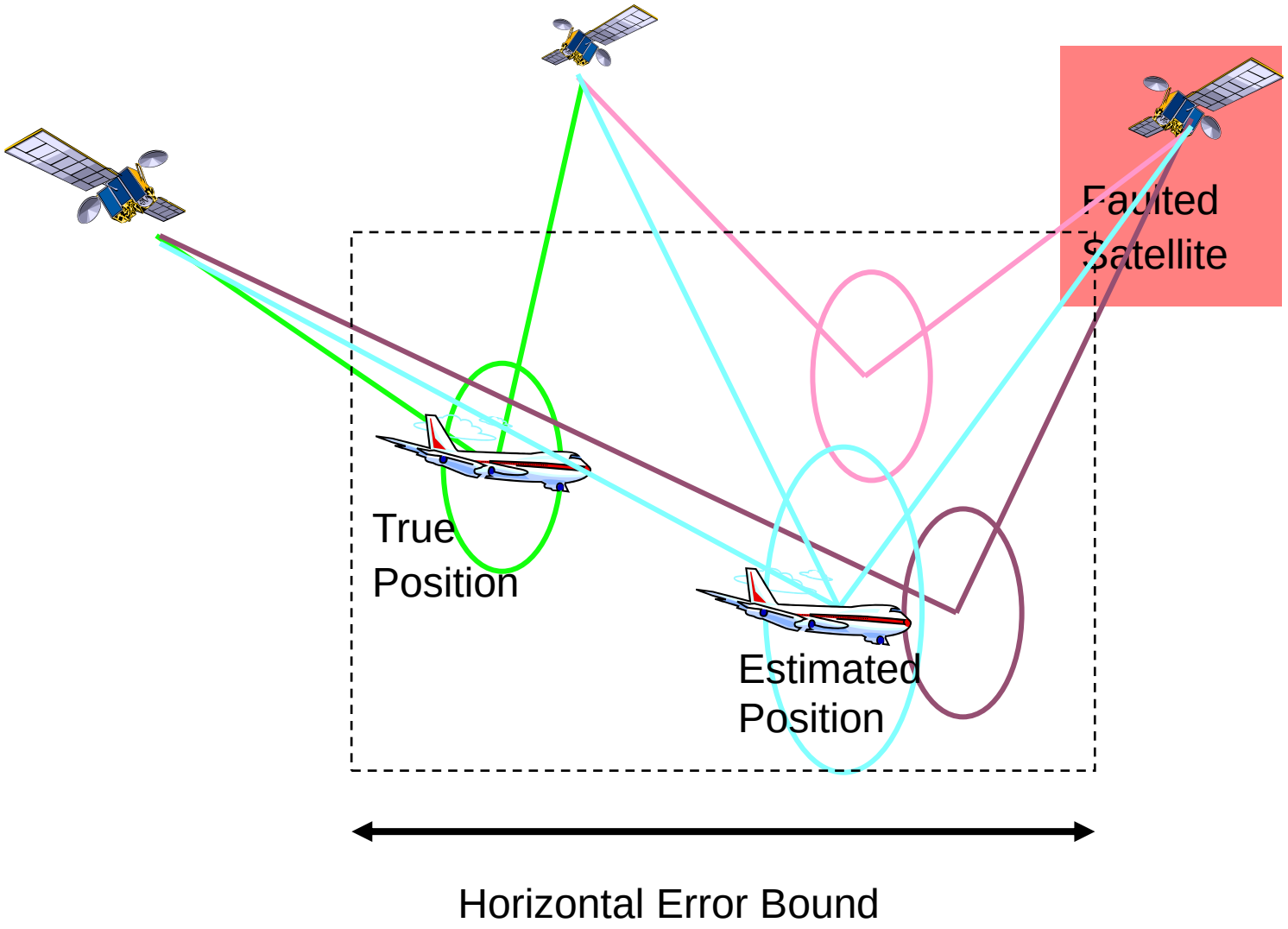
Motivation: ARAIM Safety Case



ARAIM safety case is dependent on:

- bounding the probability of occurrence of faults (P_{sat}, P_{const}, R_{sat}, R_{const}, MFD)
- overbounding the distribution of nominal errors (b_{nom}, σ_{URA})

Fault Rates and Fault Probabilities

- There are two related concepts: fault rate and fault probability
  - Fault rate is the probability that a fault will initiate per unit of time
  - Fault probability is the likelihood of experiencing a fault at a given time

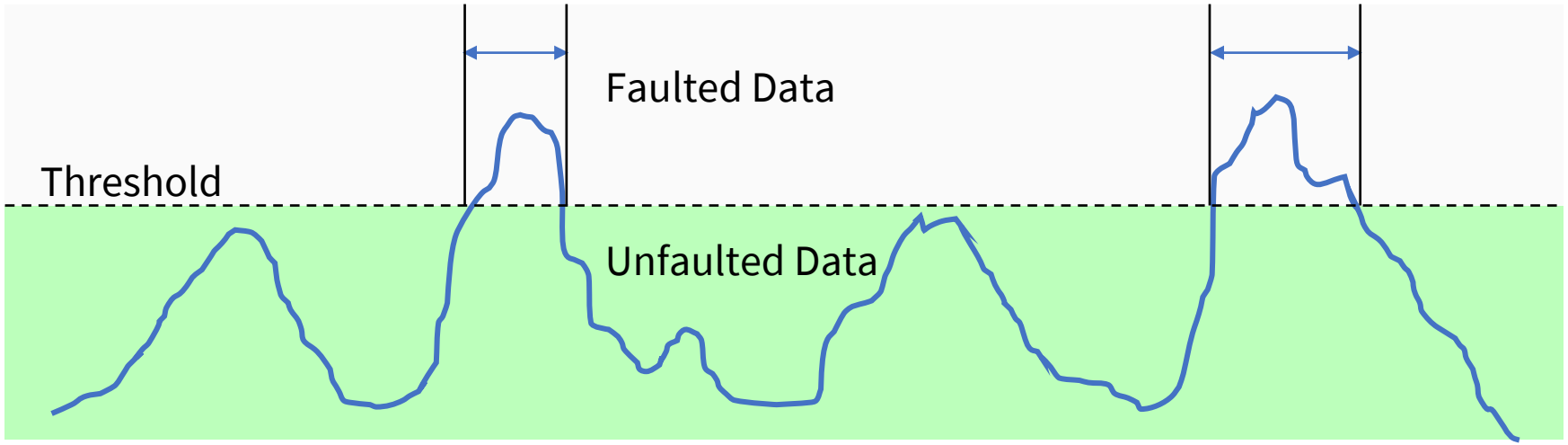
- These 2 concepts are related by the Mean Fault Durations (MFD)
  - $P_{sat} = MFD_{sat} \times R_{sat}$
  - $P_{const} = MFD_{const} \times R_{const}$

Satellite Performance Commitments

	GPS	Galileo	GLONASS	Beidou
URA	table	6 m DF, 7.5 m SF	18 m	table
Threshold	4.42 xURA	4.17xURA (25 m, 31.3 m)	70 m (3.89xURA)	4.42 xURA
R_{sat}	1e-5/hour	-	-	-
P_{sat}	1e-5	3e-5	1e-4	1e-5
P_{const}	1e-8†	2e-4	1e-4	6e-5
MFD	1 hour	-	-	-
TTA	10 seconds	-	10 seconds	300 seconds
source	SPS PS and NSP6_wp2	NSP6_wp4	NSP6_wp3	NSP6_wp5

† Recently proposed R_{const} = 1e-9/hour and MFD_{const} = 10 hours

R_{sat}, P_{sat} and MFD



R_{sat} counts the number of upward crossings of the threshold in a given time period

MFD is used to indicate the mean fault duration and is the total length of time that the errors are above the threshold divided by the number of upward crossings

P_{sat} is the fraction of time spent above the threshold and equals R_{sat} x MFD

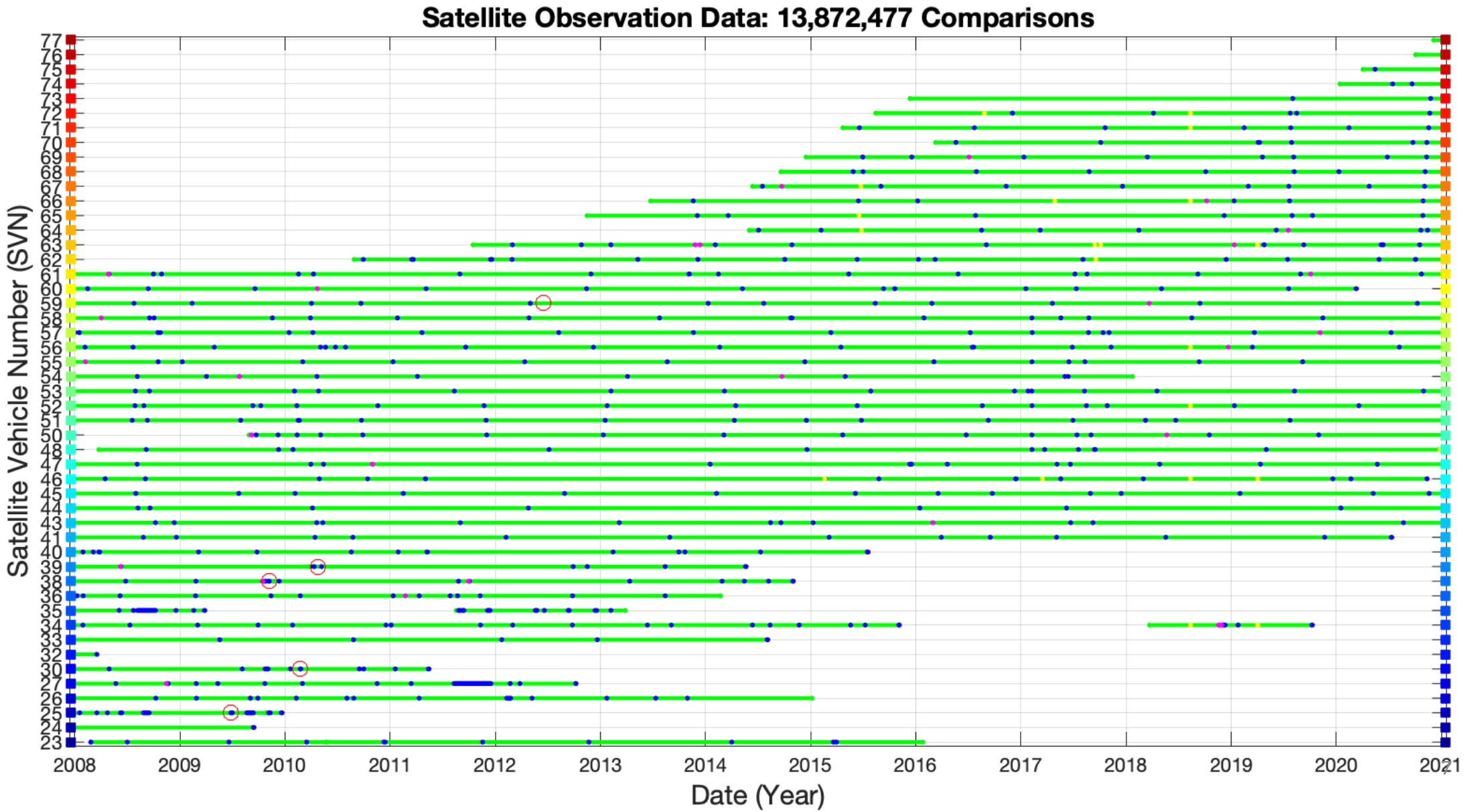
Rate estimate based on Poisson distribution For Fault Onset

- A fault occurring in one time interval does not affect the probability of it occurring in other time intervals (when the SV is set healthy), and
- The probability of a fault occurring does not change over time.

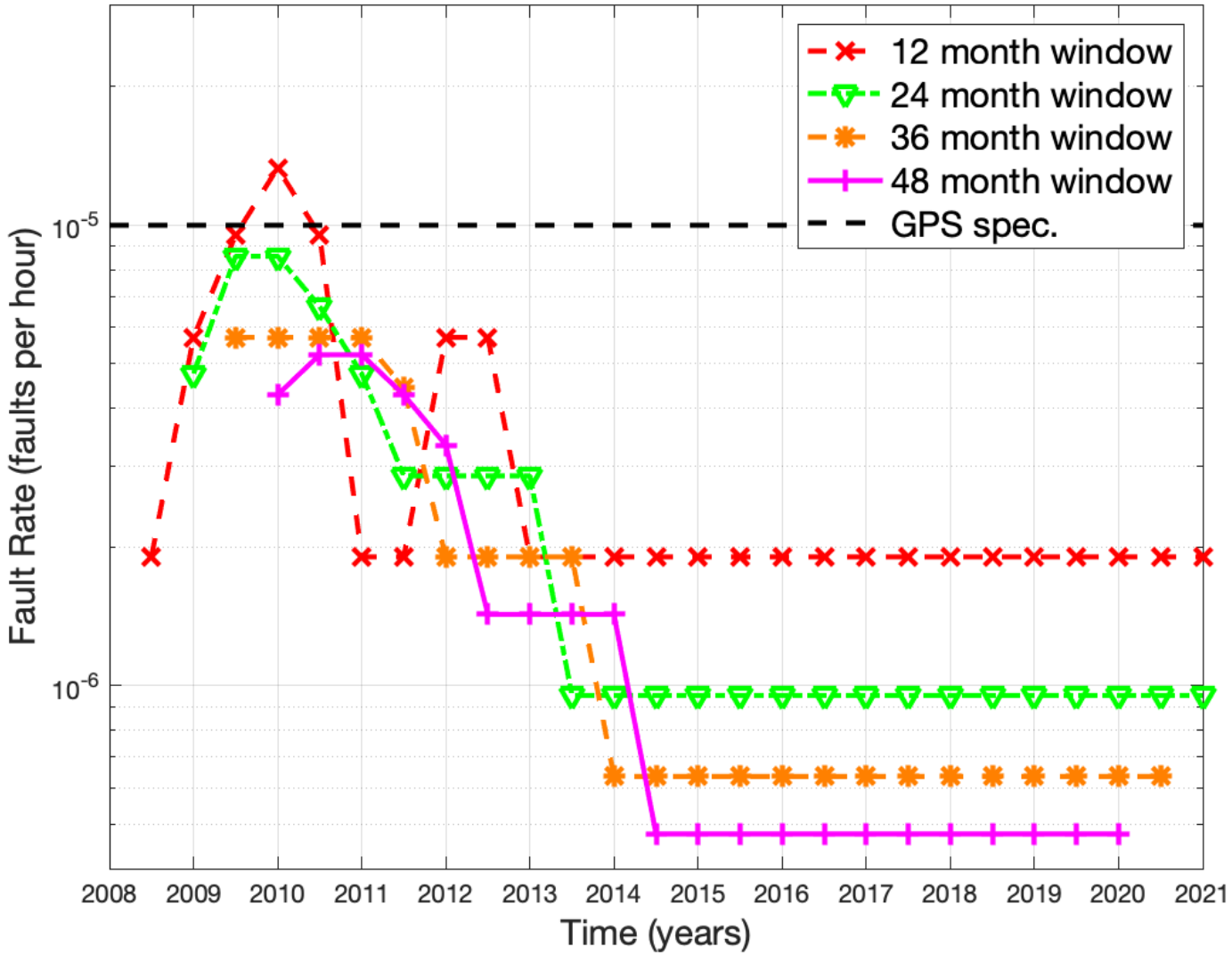
$$P(k|R) = \frac{(R\,T)^k\, e^{-R\,T}}{k!}$$

$$\hat{R} = \left(k + \frac{1}{2}\right)/T$$

GPS Performance Summary



Estimated Upper Bound on R_{sat}



- R_{sat} has been improving over time
- Can easily validate committed value of 10^{-5}/hour
- MFD is easily bounded by 1 hour
  - $P_{sat} < 10^{-5}$

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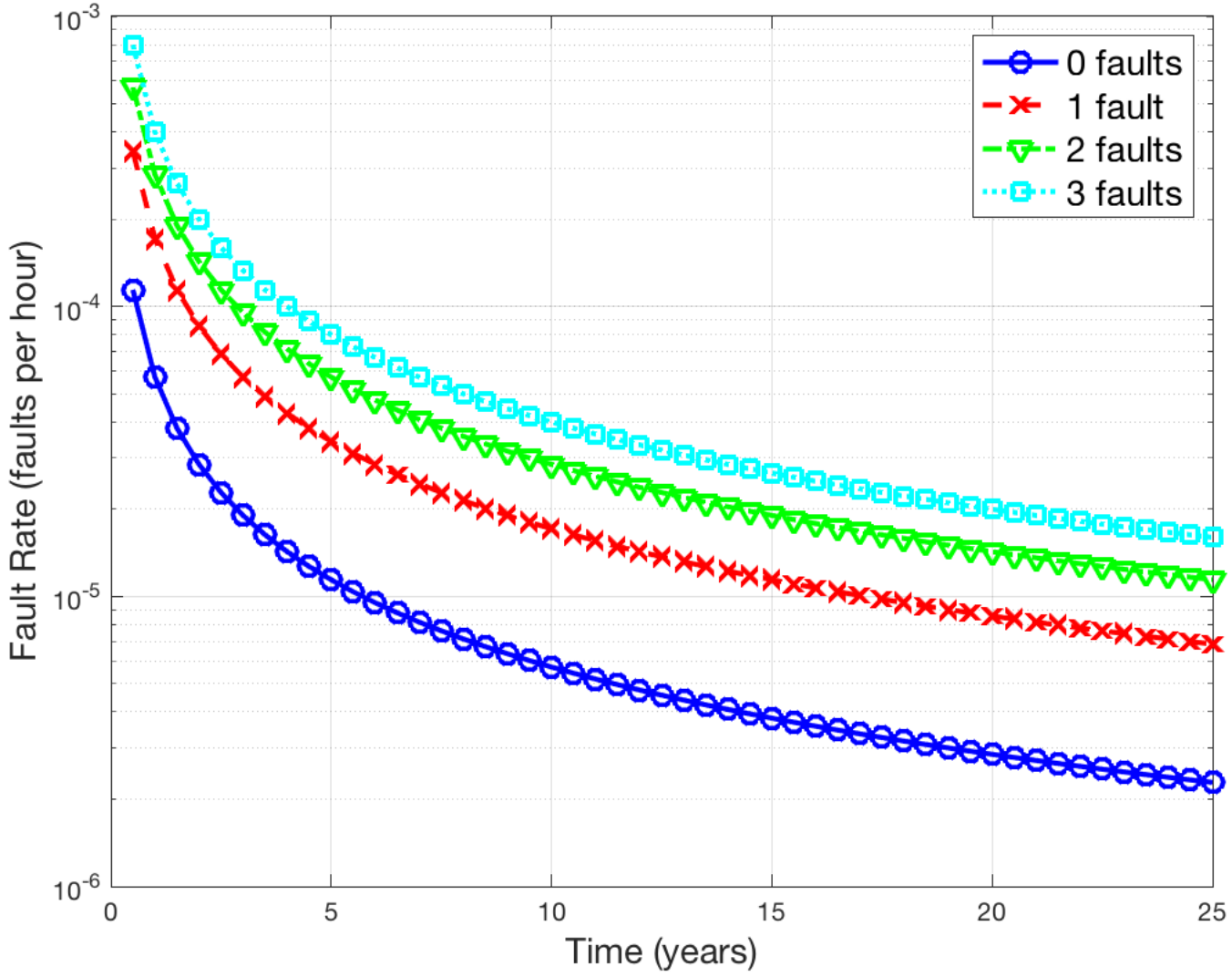
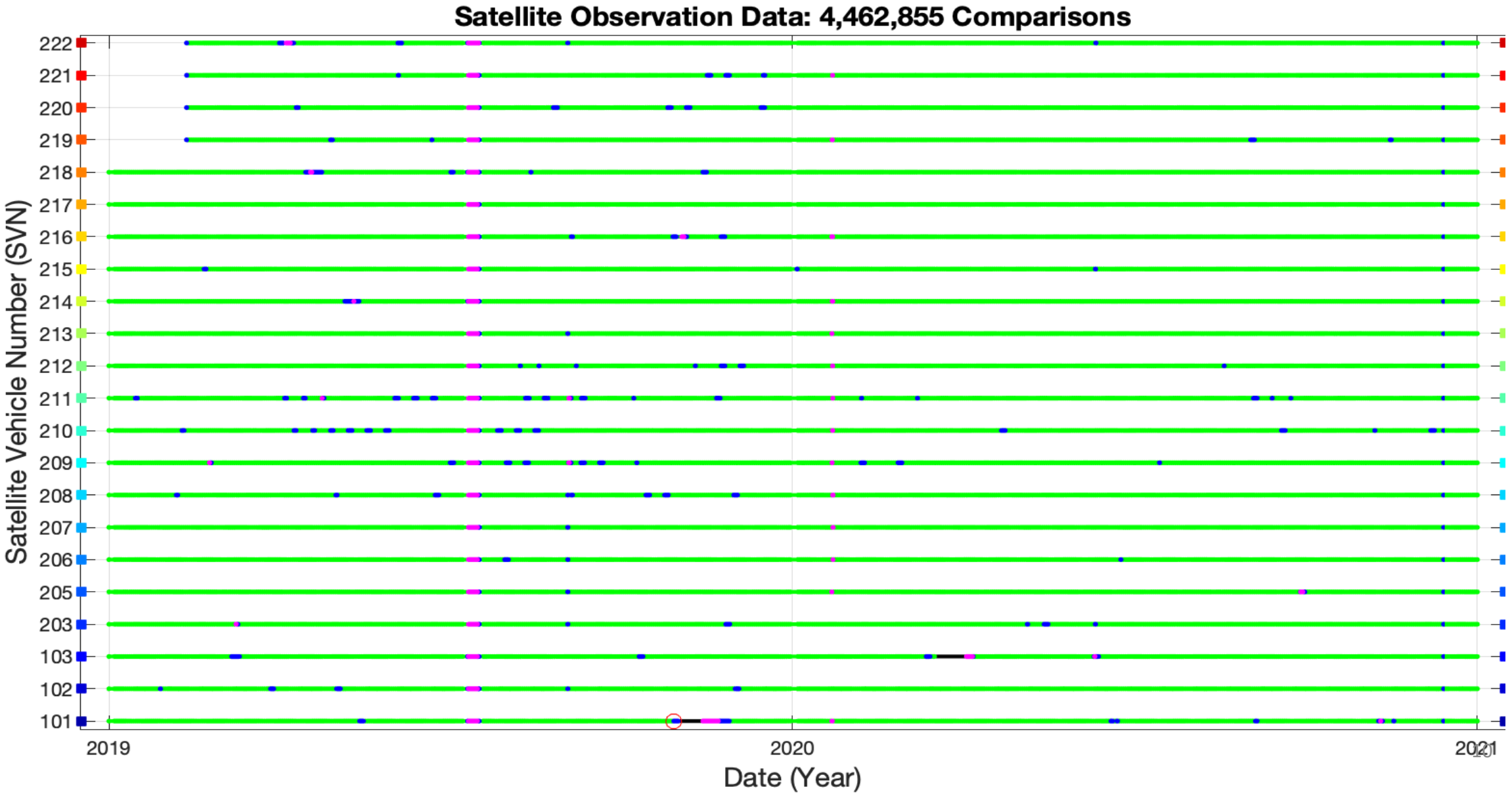
Estimated Upper Bound on R_{const}

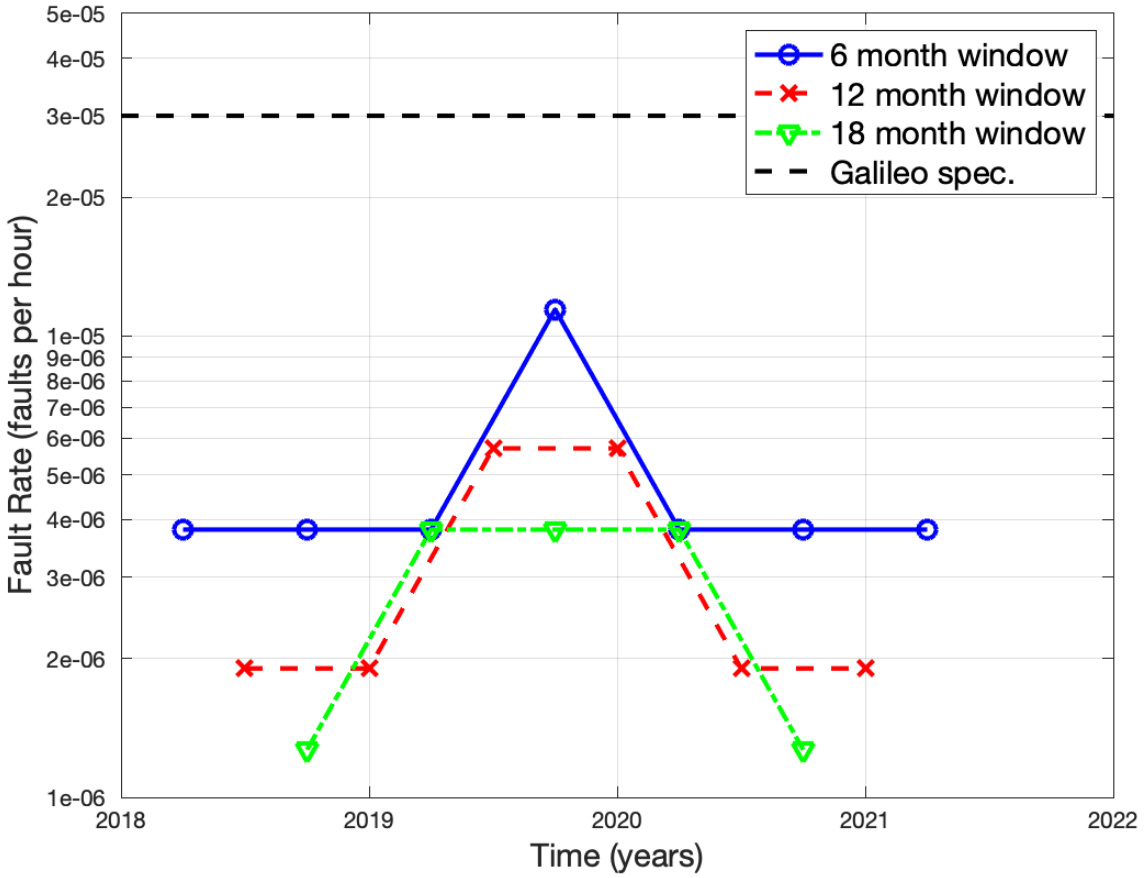


- Cannot bound probabilities much below 10^{-5} using only data validation
- Considering 25 years of GPS service, the smallest bound is ~2.5×10^{-6}/hour
- Must rely on commitments for smaller numbers

Galileo Performance Summary



Estimated Upper Bound on R_{sat}



- Can easily validate expected value of $3x10^{-5}$/hour
- Single observation is 36 minutes
  - $P_{sat} < 3x10^{-5}$
- Not yet reached IOC and fairly limited data set

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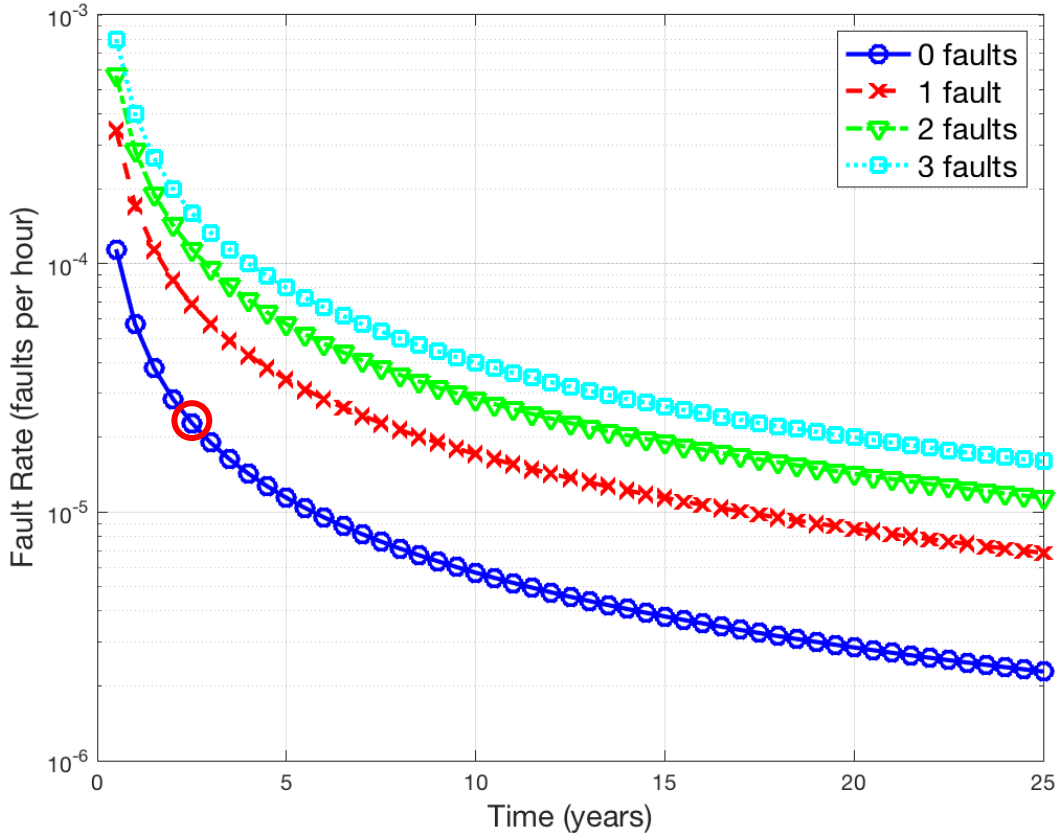
Estimated Upper Bound on R_{const}



- Considering 0 faults in 2.5 years of Galileo service, the smallest bound is ~2.3×10^{-5}/hour
- No data or commitment on MFD_{const}
  - Needs to be less than 9 hours to meet commitment now

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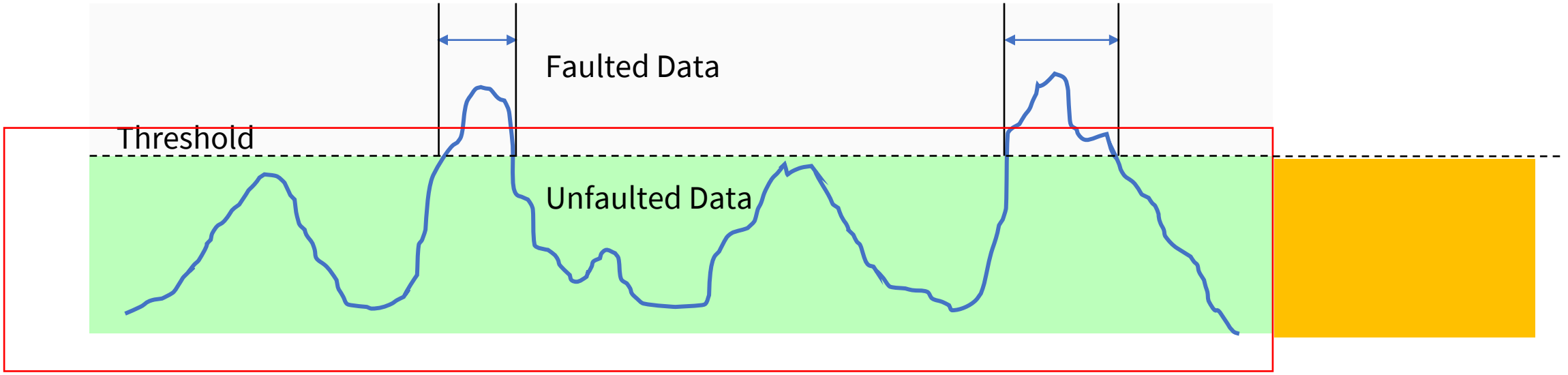
Summary on rates and durations

- The safety case for P_{sat} and P_{const} relies on a combination of commitments, similarity to previous systems, and data validation:
  - Published commitments for P_{sat} and P_{const} exist for all constellations
  - Data used to validate commitment values

- P_{sat} is easier to validate assuming common values across all satellites

- P_{const} cannot exclusively use data to validate below ~3×10^{-6}

Nominal performance



$$\text{range error} \leq N\left(b_{nom}, \sigma_{URA}\right) \ ?$$

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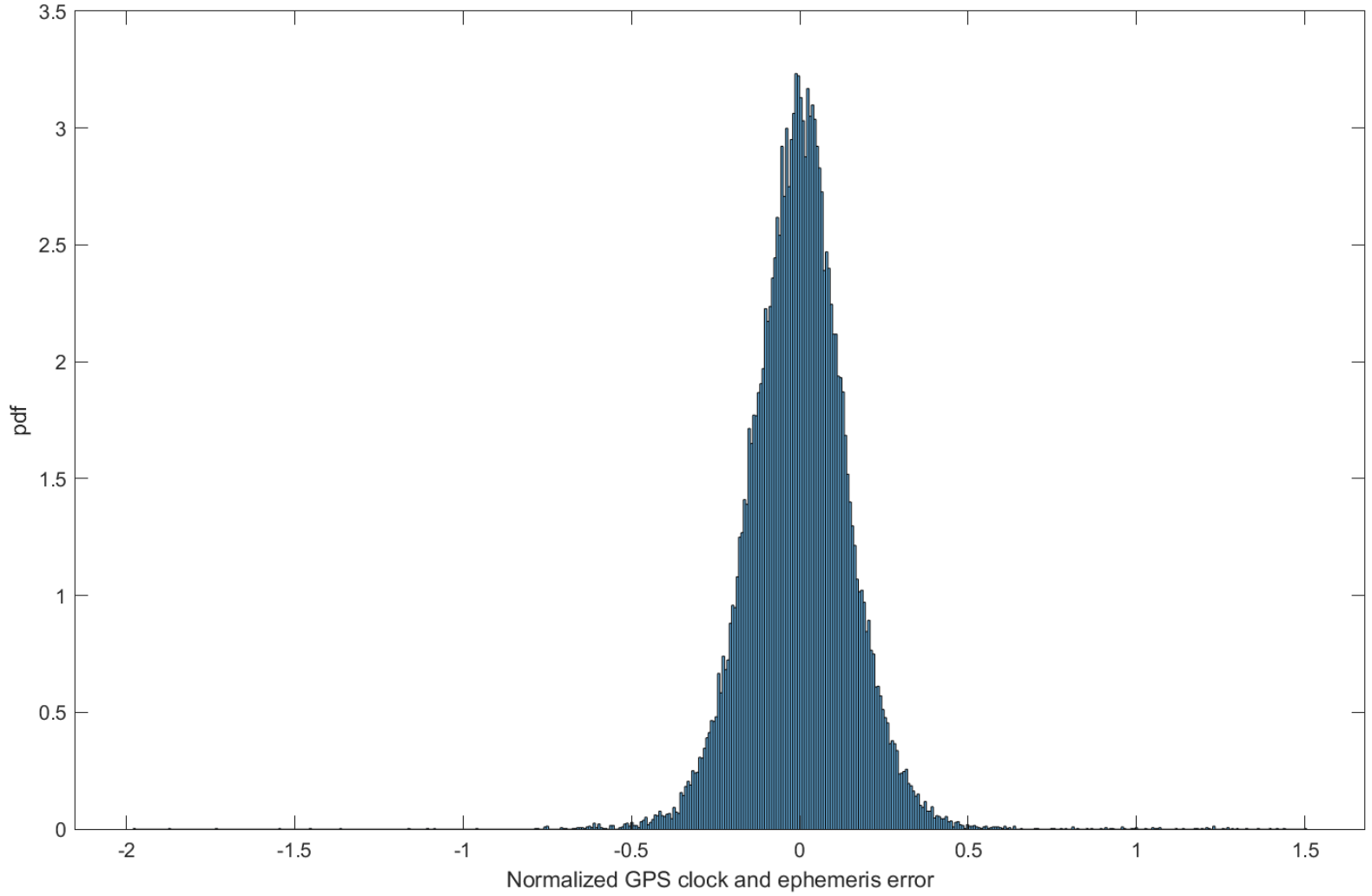
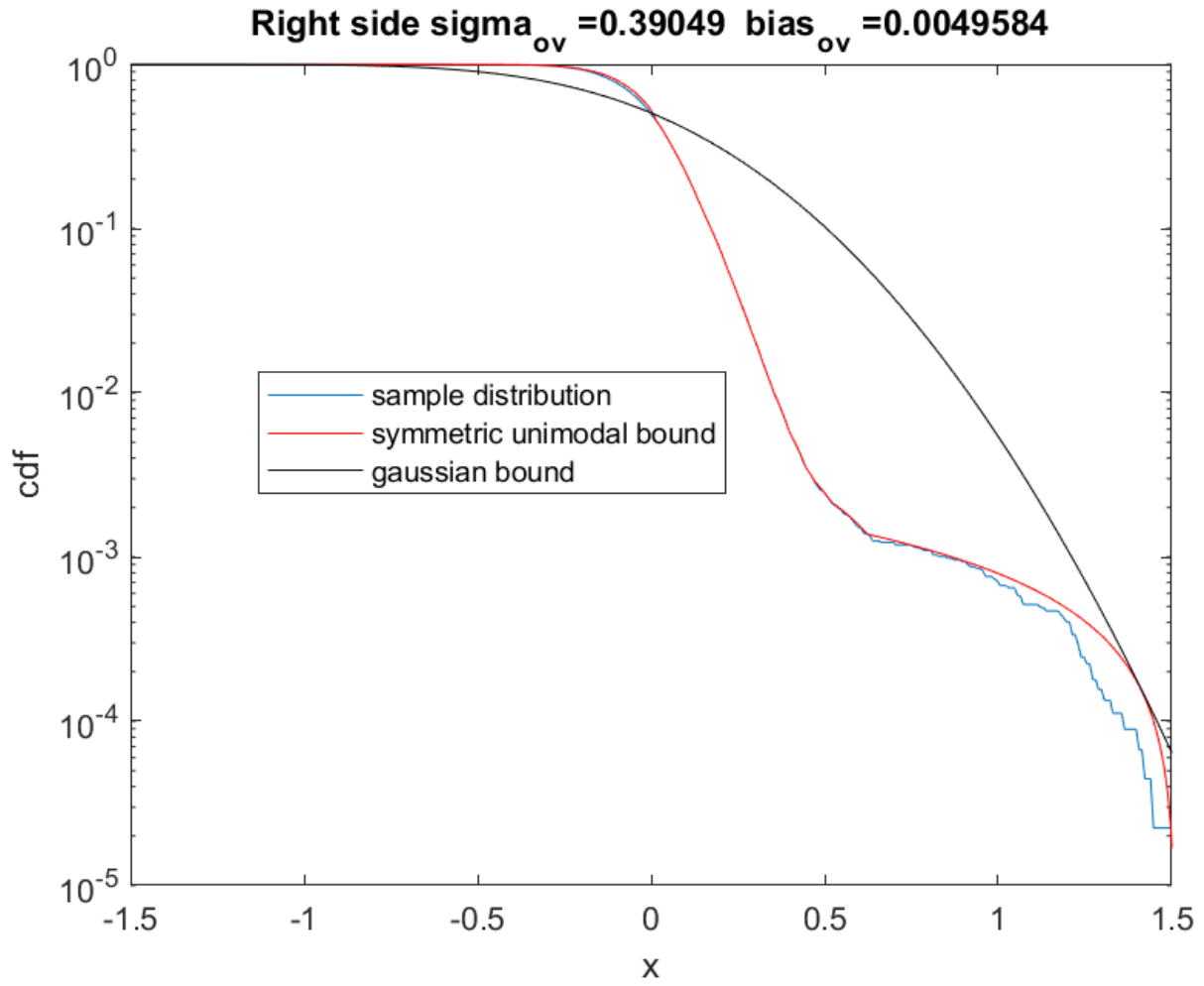
Normalized clock and ephemeris errors



$$\chi = \frac{\varepsilon}{\sigma_{URA}}$$

Bounding nominal performance

Users must be able to safely use a Gaussian model characterized by a normal distribution



$$F(x) \le F_{bound}(x) = Q\left(\frac{x-\gamma}{\sigma}\right)$$

Gaussian tail cdf

$$F\left(\sum_i s_i x_i\right) \le F_{bound}\left(\sum_i s_i x_i\right) = Q\left(\frac{\sum_i s_i x_i - \sum_i s_i \gamma_i}{\sqrt{\sum_i s_i^2}}\right)$$

Bounding correlated errors

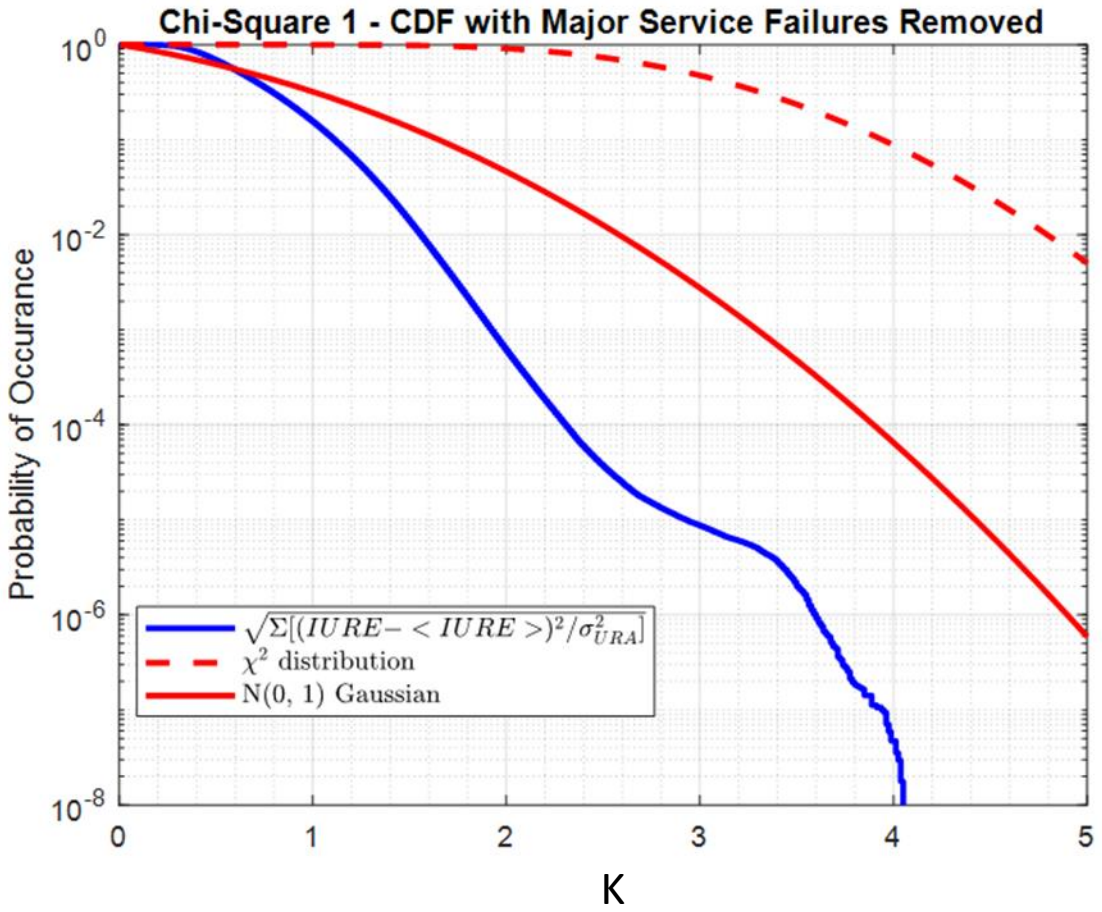
$$\text{user error}^{\,2} = \left| \sum_{i=1}^{n} s_i x_i \right|^2 \le \left| \sum_{i=1}^{n} s_i^2 \right| \left| \sum_{i=1}^{n} x_i^2 \right|$$

$$P\left(\text{user error} > K\sigma_{user}\right) \le P\left(\sum_{i=1}^{n} x_i^2 > K^2 \right) \le Q\left(K\right)$$

User will be safe if this distribution is bounded by a normal distribution

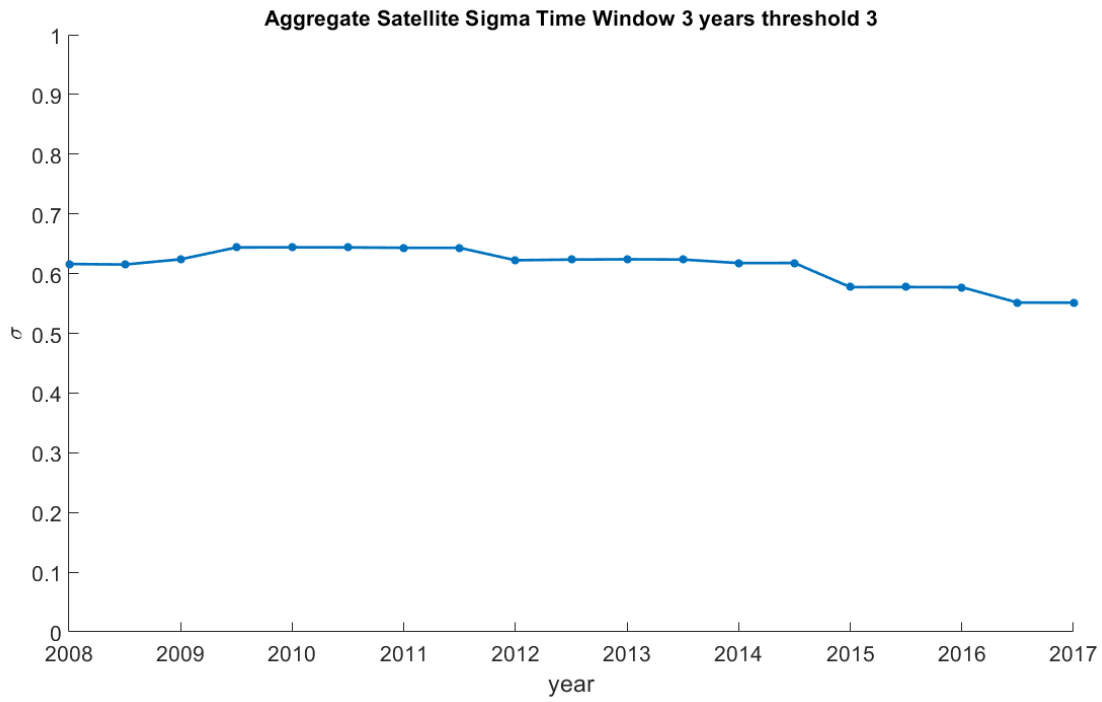
Bounding the correlation of errors

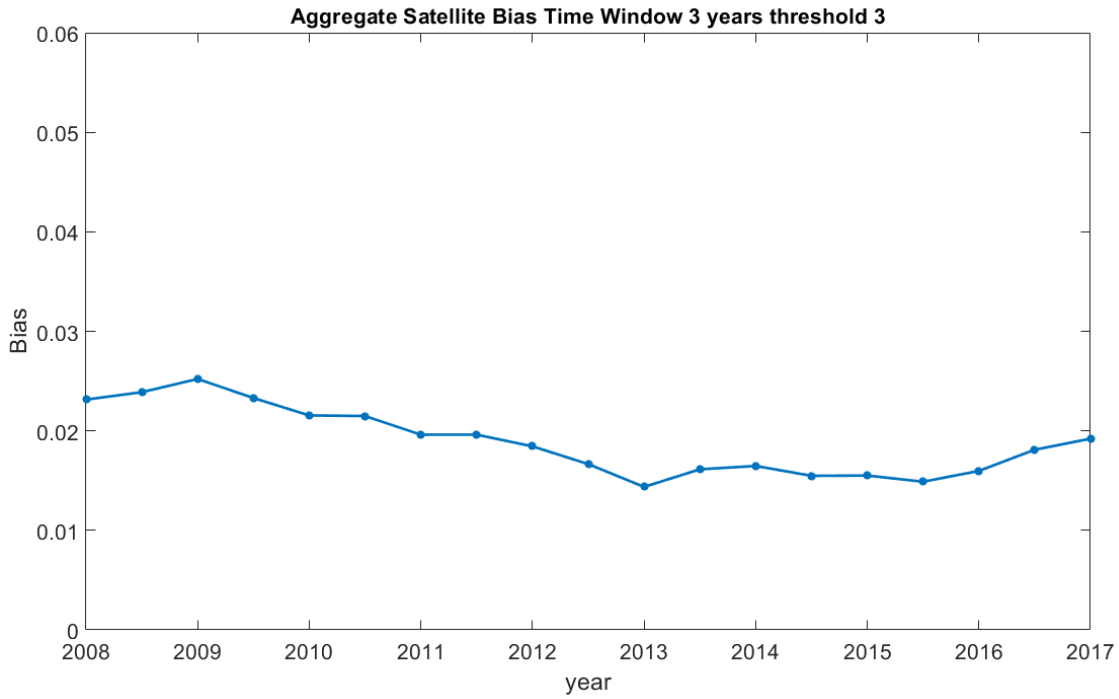
$$P\left(\sum_{i=1}^{n} x_i^2 > K^2\right)$$



For both GPS and Galileo, we find that correlated errors are well bounded by the model that assumes independence

GPS nominal bounding results for all satellites over last 12 years



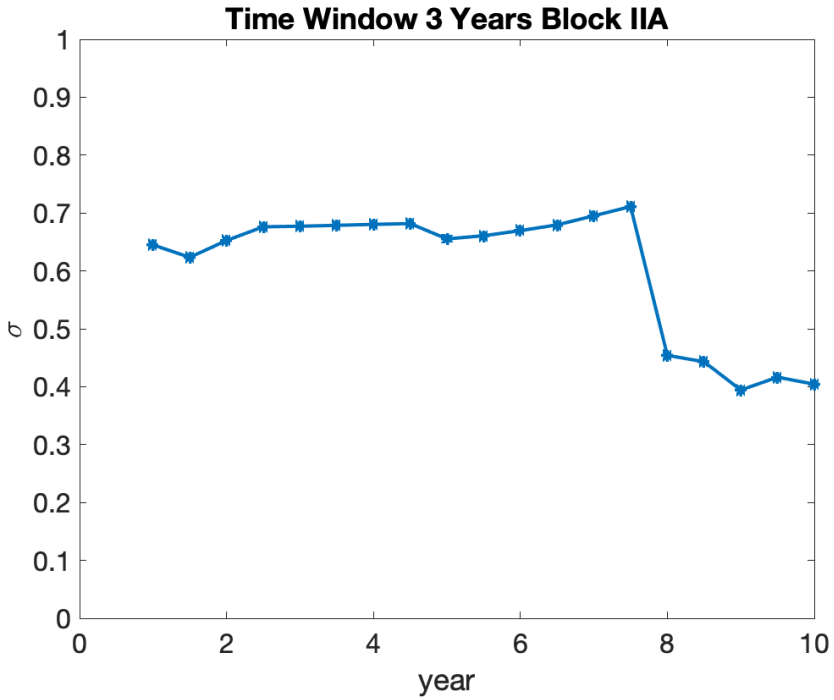


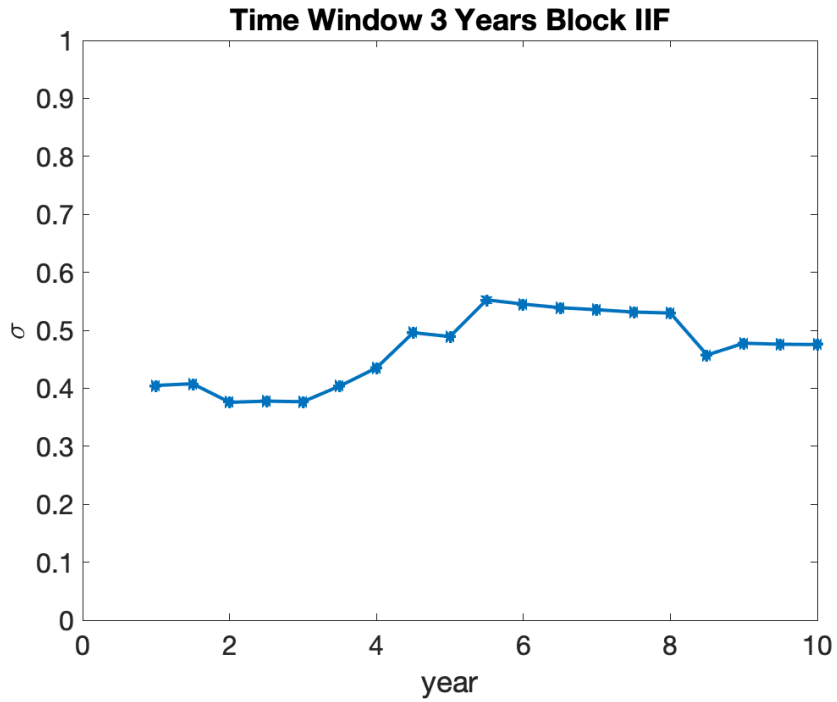
Partitioning the data

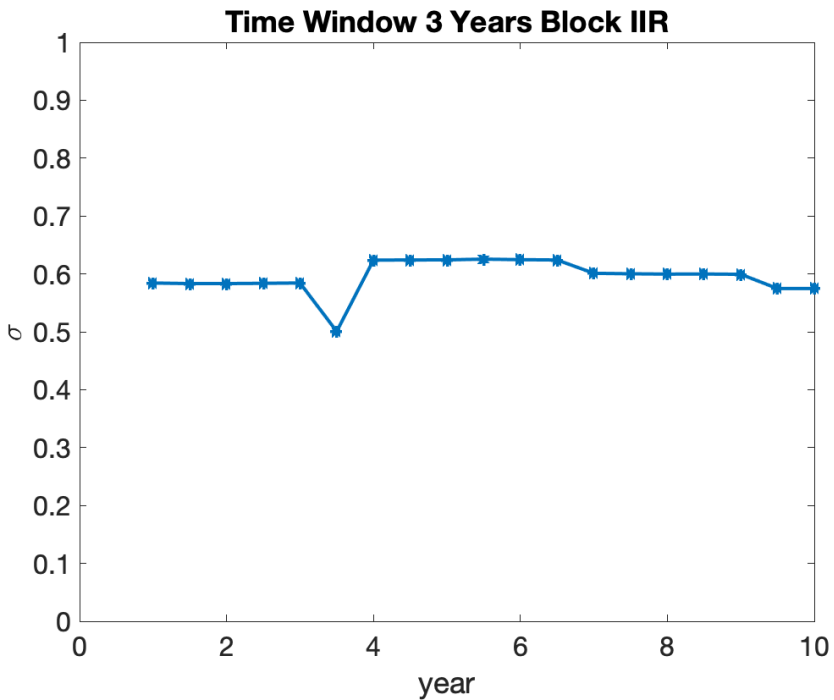
To a certain extent, we must protect the user against specific risk (for conditions that are knowable):

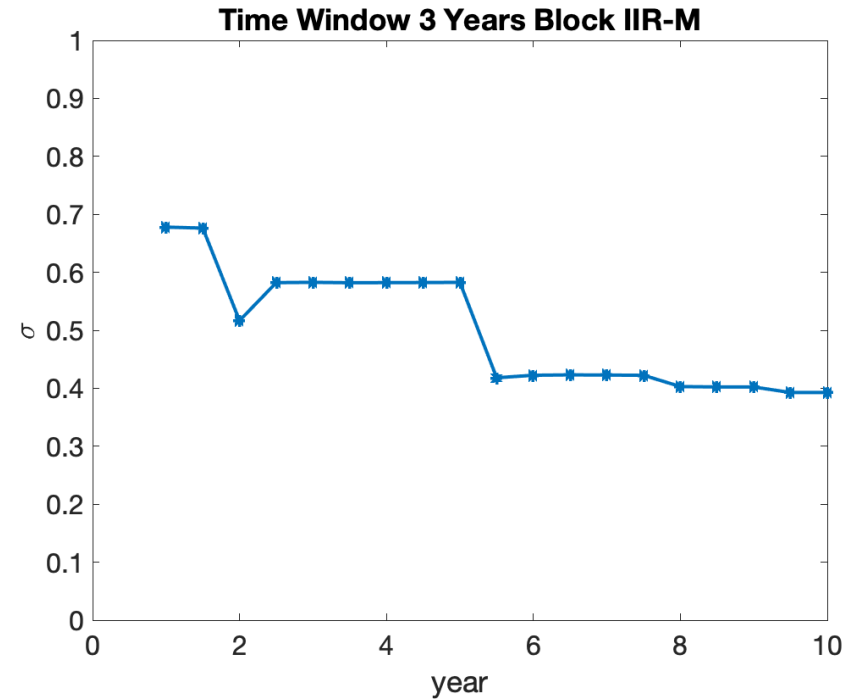
- Individual satellites
- Satellite block type (including clock type)
- Time (by year, by season, by month, or by day)
- Satellite age
- Age of navigation data

Partition per block

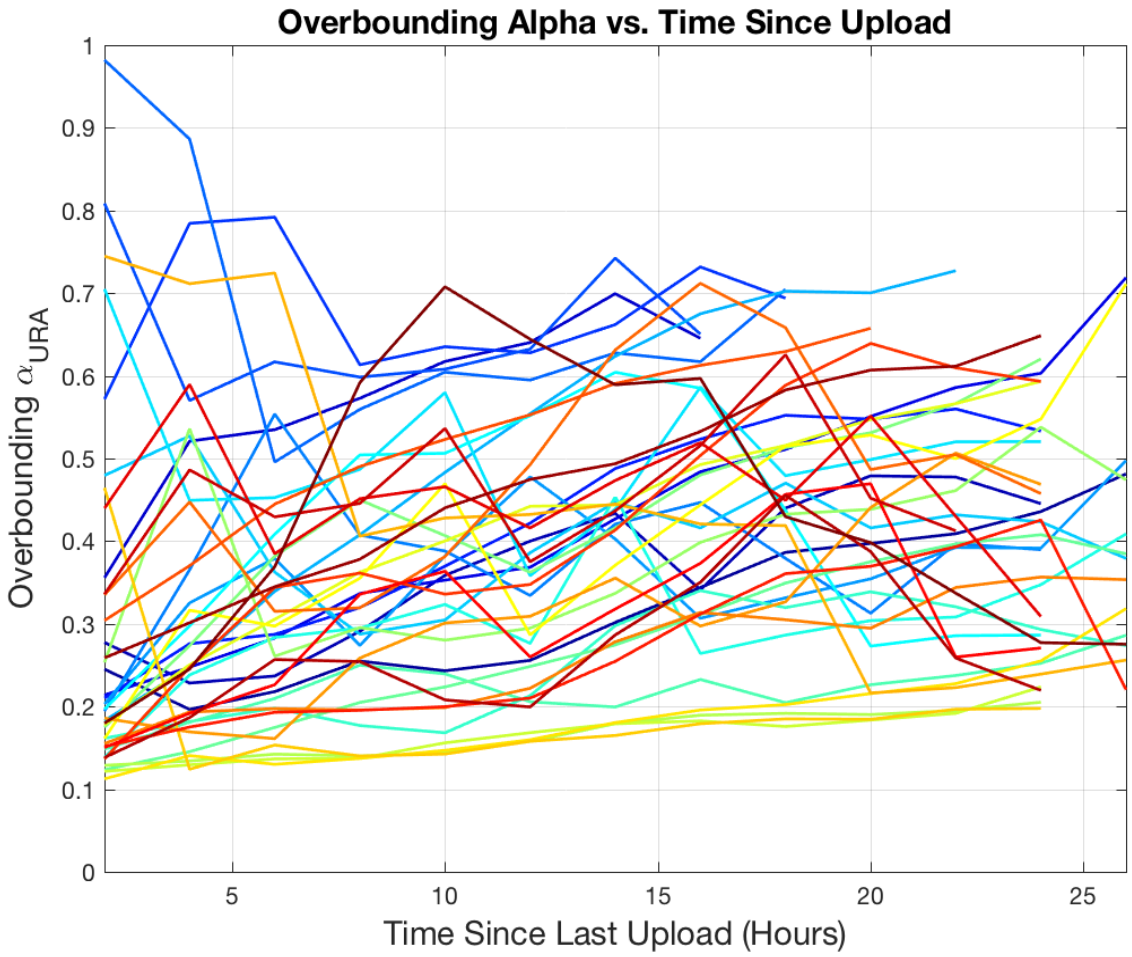


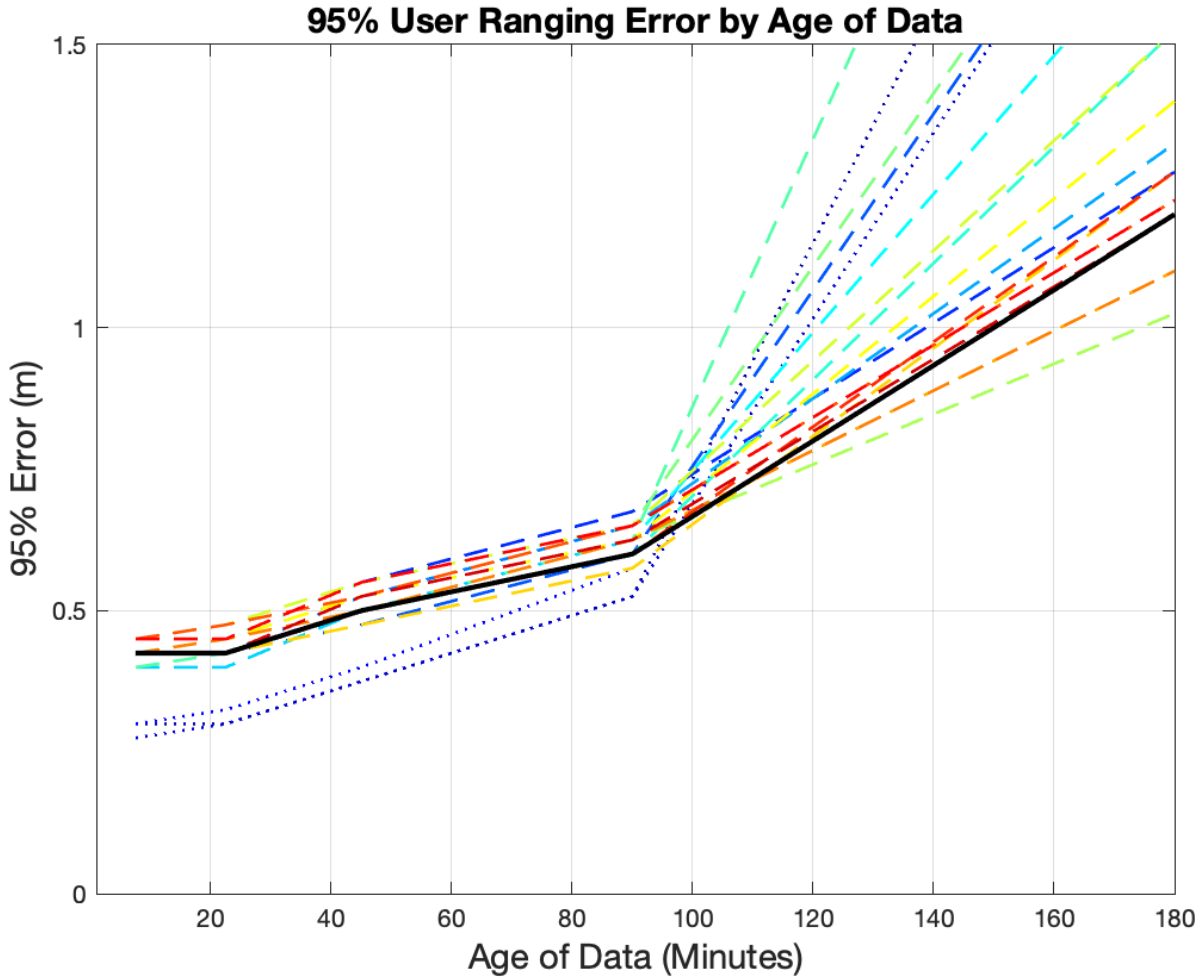




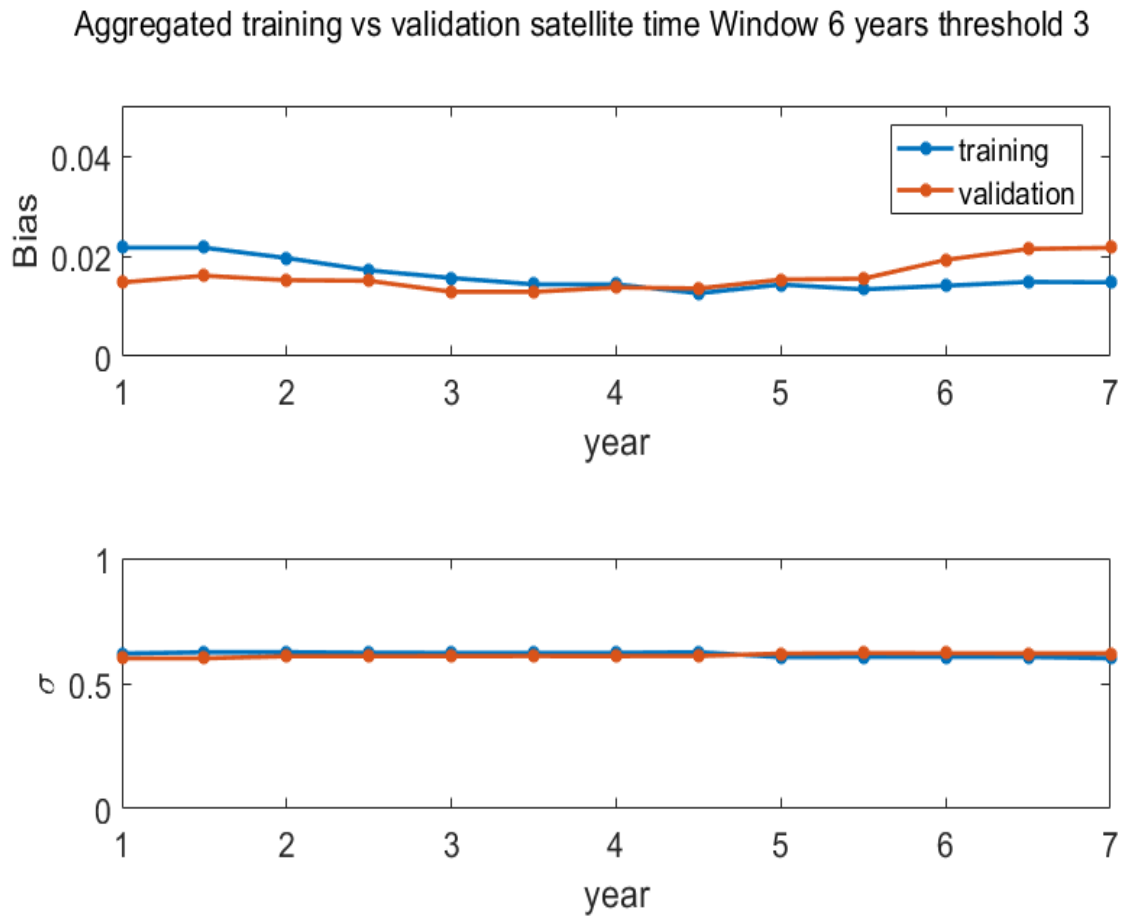


Example: by age of data





Cross-validation and bootstrap methods to determine confidence in overbounds



Summary

- We have described some of the techniques and approaches that can be used to analyze GNSS constellation performance for Advanced RAIM:
  - Estimation of fault rates
  - Overbounding of nominal performance
  - Correlation of ranging errors
  - Partitioning the nominal data

References

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