Optimization of a Vertical Protection Level Equation for Dual Frequency SBAS

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ABSTRACT

The advent of dual frequency Satellite Based Augmentation Systems (SBAS) will allow the aviation community to address some of the shortcomings of the current SBAS Minimum Operational Performance Standards (MOPS), in particular of the Vertical Protection Level (VPL), and also to adapt it to threats for a dual frequency user. First, satellite faults will become the dominant source of error – because the ionospheric delay threat is canceled. Second, it was discovered that there are nominal biases that cannot be corrected by the ground and that cause the current VPL to be inflated. Finally, actual data collected in support of system performance has shown that the statistics exhibit non-gaussian behavior. A Vertical Protection Level equation that addresses these points was proposed in [1]. This equation accounts explicitly for possible nominal biases and for the fact that the position error bound is dominated by one possible fault at a given time. The equation was evaluated using a Service Volume analysis tool and it was shown that it could provide significant benefits over the current equation. These simulations computed user coefficients (the coefficients that project the pseudorange onto the user position) based on a Least Squares (LS) approach. The LS approach is not optimal for the proposed VPL equation.

In this paper we show that choosing optimally the coefficients that project the pseudoranges onto the position domain could significantly increase the performance of an SBAS based on the equation described in [1]. To this purpose, we will describe an algorithm that minimizes the Vertical Protection Level. Then, we will evaluate the availability benefits that can be obtained using the optimal algorithm using a Service Volume Analysis tool.

INTRODUCTION

The addition of a new civil signal in L5 [2], [3], [4] will greatly expand the capabilities of Satellite Based Augmentation Systems (SBAS), because the largest source of uncertainty, the ionospheric delay, will be removed by the receivers. Also, there will be an opportunity to adopt a different approach to providing corrections in L5. A new approach should integrate both the new dual frequency situation and the lessons learned from single frequency SBAS [1]. Taking this into account, a new Vertical Protection Level (VPL) equation was proposed and evaluated in [1]. This paper is a continuation of [1], where a more detailed justification of the proposed VPL equation is given. The goal here is to evaluate the potential of this equation, since, as was suggested in the conclusion of [1], it is possible to lower the VPL by choosing the set of coefficients that project the ranges onto the position to minimize the VPL.

In the first part of the paper, we will recall the expression of the VPL equation. Then, we will show that is possible to find the minimum of the expression by casting the problem under a canonical form, which can be solved efficiently. Once the methodology is explained, we will evaluate the improvement with respect to the baseline. Finally, we will discuss the effects that the optimization has on the position accuracy, and how accuracy constraints can be enforced within the optimization program.

PROTECTION LEVEL EQUATION

The VPL equation proposed in [1] covers a nominal fault-free situation, and a separate faulted condition. This approach is similar to that taken by the Ground-Based Augmentation System (GBAS) [5] and recently proposed Advanced Receiver Autonomous Integrity Monitoring (ARAIM) equations [6] [7]. The un-faulted term takes the form of:

$$VPL_0 = K_{v,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2 \sigma_{ff,i}^2} + \sum_{i=1}^{N} \left| S_{3,i} b_i \right| \quad (1)$$

where $K_{v,PA}$ corresponds to the Gaussian tail and is expected to be 5.33, $S_{3,i}$ is the third element of the i^{th} row of the matrix that projects the pseudorange measurements onto the position. $S_{3,i}$ represents the effect on the vertical position error due to an error on the ranging measurement to the i^{th} satellite. $\sigma_{ff,i}^2$ is the fault-free or nominal error variance and b_i is the nominal bias bound.

The VPL under the faulted condition is similar except that it adds a faulted bias term

$$VPL_1 = K_{v,md}\sqrt{\sum_{i=1}^{N} S_{3,i}^2\sigma_{ff,i}^2} + \sum_{i=1}^{N}\left|S_{3,i}b_i\right| + \max_i\left|S_{3,i}B_i\right| \qquad (2)$$

Because the occurrence of a fault is unlikely, the value of $K_{v,md}$ can be below 5.33 and we expect it to take a value between 3 and 4 depending on the assigned probability. The term B_i is an upper bound on the magnitude of the fault on the ith pseudorange. The user VPL is the maximum of the two terms:

$$VPL = \max\left(VPL_0, VPL_1\right) \qquad (3)$$

For the definition of $\sigma_{ff,i}^2$, B_i, and b_i, a detailed discussion is given in [1]. A summary is provided here. We define:

$$\sigma_{ff,i}^2 = \sigma_{ff,flt,i}^2 + \sigma_{ff,tropo,i}^2 + \left(2.6\times\sigma_{ff,air,i}\right)^2 \qquad (4)$$

$\sigma_{ff,\,flt}$ is set to 30% of σ_{UDRE}, [8], the tropospheric term is set to 5 cm (times the mapping function specified in [8]), and the airborne term was based on measurements reported in [9]. The nominal biases were set to 0.5 m and the faulted bias term was set to 5.33 times σ_{flt}. This term is a function of $\delta UDRE$ and σ_{UDRE}. The $\delta UDRE$ term is used to describe the uncertainty associated with satellite clock/ephemeris faults and is described elsewhere [8], [10].

OPTIMIZATION

In [1], the projection matrix S is given by a least squares solution where the weighting matrix W is determined using the overbounding standard deviations. Here, instead of choosing a set of coefficients resulting from Least Squares, we attempt to find the set that minimizes the VPL. This problem can be expressed as follows.

$$\min_{S_{3,\cdot}^T G=[0\ \ 0\ \ 1\ \ 0]} \max\left\{\begin{array}{l} K_{v,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2\sigma_{ff,i}^2} + \sum_{i=1}^{N}\left|S_{3,i}b_i\right| \\ K_{v,md}\sqrt{\sum_{i=1}^{N} S_{3,i}^2\sigma_{ff,i}^2} + \sum_{i=1}^{N}\left|S_{3,i}b_i\right| + \max_i\left|S_{3,i}B_i\right| \end{array}\right\} \qquad (5)$$

To lighten the notations we define:

$$C = \begin{bmatrix} \sigma_{ff,1}^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{ff,N}^2 \end{bmatrix} \qquad (6)$$

$$A = \begin{bmatrix} I_n \\ -I_n \end{bmatrix},\ b = \begin{bmatrix} b_1 \\ \vdots \\ b_N \\ b_1 \\ \vdots \\ b_N \end{bmatrix},\ \mathrm{B} = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & B_n \end{bmatrix} \qquad (7)$$

To remove the absolute values, we make the following standard change of variable:

$$S_{3,\cdot} = \lambda^T A \qquad (8)$$

The problem is then equivalent to:

$$\begin{array}{ll} \text{minimize} & VPL \\ \text{subject to} & G^T A^T\lambda = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} \\ & \lambda \geq 0 \\ & t\underline{1} \geq \begin{bmatrix}B & B\end{bmatrix}\lambda \\ & K_{v,md}\mu + b^T\lambda + t \leq VPL \\ & K_{v,PA}\mu + b^T\lambda \leq VPL \\ & \sqrt{\lambda^T ACA^T\lambda} \leq \mu \end{array} \qquad (9)$$

Under this form, we can see that we are optimizing a linear expression over the intersection of a simplex and a second order cone. This type of problems is known as a Second Order Cone program (SOCP). SOCPs are a class of convex problems that have been extensively studied in convex optimization theory [11] and for which there exist very efficient solvers (although not as widespread as Linear Program solvers) that use interior point methods. These iterative solvers have the very important following properties:

- they converge in very few steps to the global optimal (typically less than 20)
- they provide an upper bound on the distance to optimality
- they converge in polynomial time to a given accuracy

This approach was already used in a simpler setting [12]. As in [12], we used a MATLAB based toolbox to solve the Second Order Cone Programs. We chose to use the free MATLAB package SeDuMi [13], interfaced with YALMIP [14]. Once installed, these tools are almost transparent for a MATLAB user as there are only three new commands to learn. These tools have been typically developed for large numbers of variables (typically hundreds). Here, we only have tens of variables. The solver reached the optimal solution with 10 digits precision in about 10 iterations (which took less than a second).

AVAILABILITY ANALYSIS

The goal of this section is to evaluate the benefit that can be obtained by using the optimal VPL instead of a sub-optimal one. We used MAAST to determine the satellite geometries and expected clock and ephemeris bounds, σ_{flt}, given the WAAS network. MAAST also calculated two VPLs on a grid of users around North America. Both VPLs use Equations (1),(2), and (3). In the first one, the coefficients S are determined using least squares:

$$S = \left(G^T W G\right)^{-1} G^T W \qquad (10)$$

The weighting matrix W is based on the standard deviation of the overbounding distribution, as defined in [1]. In the second one, the optimal VPL is computed using the method presented above.

The simulations conditions are similar to the ones in [1], that is, a 24 satellite optimal constellation is assumed, users are spaced every 5 degrees in latitude and longitude, and, for each user the VPL is computed every 300 s for 24 hours.

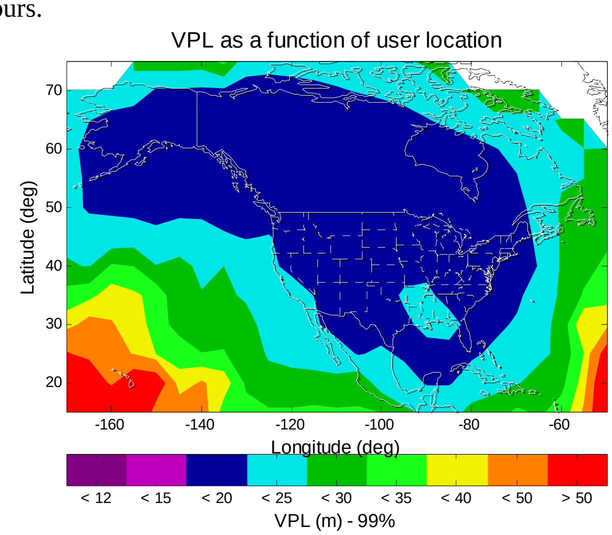

Figure 1. *The 99% maximum VPL as a function of user location for the sub-optimal coefficients (Equation* (10) *)*

In Figures 1 and 2, the colored contours indicate a value that is larger than or equal to 99% of the VPLs that would be obtained at that location during the course of the day. Figure 1 shows the VPLs that can be obtained with the least squares coefficients (which were already presented in [1]), and Figure 2 shows the optimal VPLs. As can be seen, a significant improvement can be achieved with the optimal VPL. For example, the areas of CONUS that did not have a 20 m VPL are now fully covered with VPLs below 20 m.

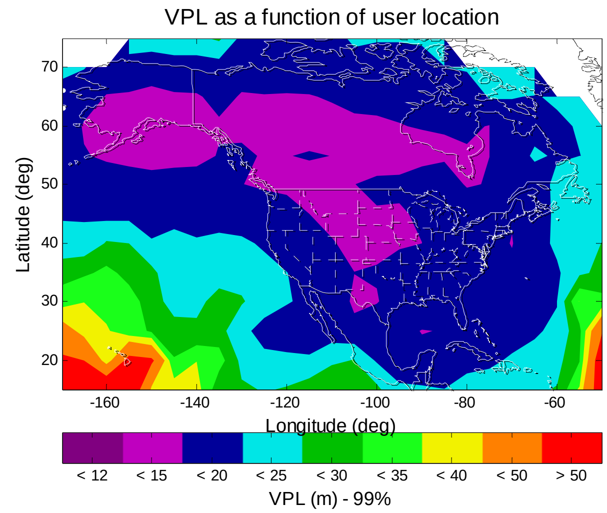

Figure 2. *The 99% maximum VPL as a function of user location for the optimal coefficients*

Figure 3 shows a histogram of the ratio of the optimal VPL to the sub-optimal VPL. The VPL is reduced up to 30% and 15 % in average.

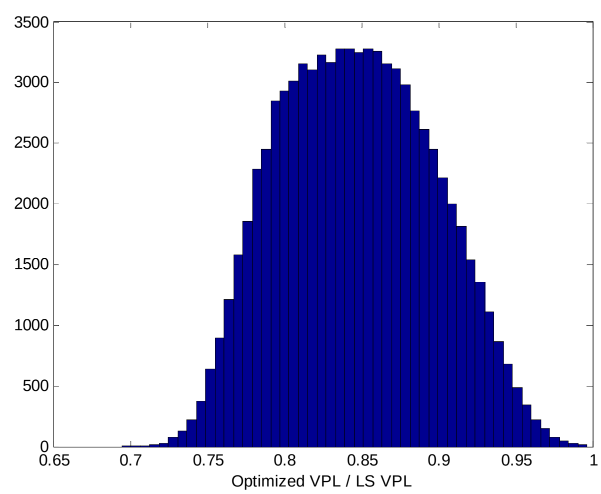

Figure 3. *Histogram of the ratio between the optimal VPL and the sub-optimal VPL*

ACCURACY CONSTRAINTS

As opposed to the Least Squares position fix, the position fix resulting in the minimum VPL will not necessarily result in the solution with the best accuracy. Following [1], the accuracy is defined as the standard deviation of the position using the characteristics of the core of the error distributions. Here, it means using a zero mean Gaussian with a standard deviation of $\sigma_{ff,i}^2$ for the pseudorange errors:

$$\sigma_{accuracy} = \sqrt{\sum_{i=1}^{N} S_{3,i}^2 \sigma_{ff,i}^2} \qquad (11)$$

Figure 4 shows a histogram of the ratio of the accuracy using the optimal coefficients to the accuracy using the sub-optimal coefficients. As can be seen, in most of the cases the accuracy is not degraded significantly, but there are some cases where the standard deviation of the error is 70% more with the optimal VPL coefficients.

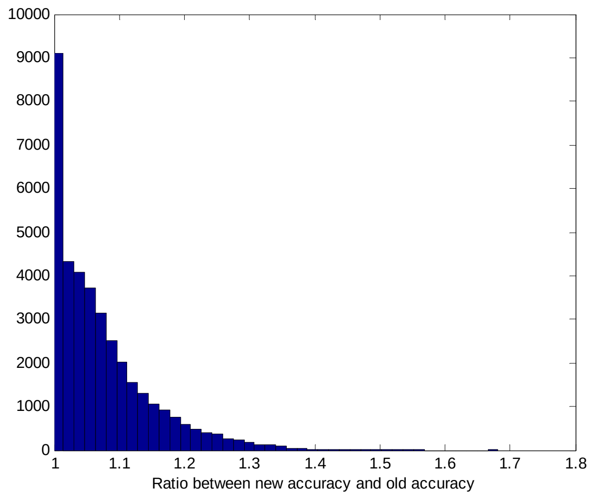

Figure 4. *Histogram of the ratio between the accuracy resulting from the optimal VPL and the accuracy using the sub-optimal VPL*

For LPV 200 [15], in addition to the 35 m Vertical Alert Limit, there is a requirement of 4 m on the 95% accuracy and a 10 m requirement on the 10^{-7} fault free VPL, which can be interpreted as a 3.76 m bound on the 95% accuracy [1]. For the simulations above, all user geometries with a VPL below 35 m had a 95% accuracy below 3.6 m, which means that although the accuracy was degraded, it was still sufficient for LPV-200.

It is also possible to include the accuracy requirement in the SOCP program, so that the VPL is minimized given a certain accuracy requirement. This can be done by including the additional constraint:

$$\sum_{i=1}^{N} S_{3,i}^2 \sigma_{ff,i}^2 \leq \sigma_{acc,req}^2 \qquad (12)$$

where $\sigma_{acc,req}^2$ is the required accuracy. The SOCP is then written:

$$\begin{aligned} \text{minimize} \quad & VPL \\ \text{subject to} \quad & G^T A^T \lambda = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \\ & \lambda \geq 0 \\ & t\underline{1} \geq \begin{bmatrix} B & B \end{bmatrix} \lambda \\ & K_{v,md}\mu + b^T\lambda + t \leq VPL \\ & K_{v,PA}\mu + b^T\lambda \leq VPL \\ & \sqrt{\lambda^T ACA^T \lambda} \leq \mu \\ & \sqrt{\lambda^T ACA^T \lambda} \leq \sigma_{acc,req}^2 \end{aligned} \qquad (13)$$

By slightly changing the problem, one could optimize for the accuracy while maintaining the VPL within a given requirement.

CONCLUSION

This paper proposes a methodology to maximize the benefits of the Protection Level equation that was proposed in [1] for dual frequency SBAS. It is shown that it is possible to minimize the Protection Level by optimally choosing the coefficients that project the measurements onto the position. These coefficients are computed optimally by casting the problem as a Second Order Cone Program, a type of convex problem for which efficient and guaranteed to converge solutions exist. It is demonstrated that additional reductions in Protection Level of up to 30% can be obtained with this method. We also show that it is possible to either include an accuracy constraint or minimize the accuracy while maintaining the Protection Level below a given threshold.

If the equation is adopted in future standards, it will be worth examining the feasibility of this method in an airborne receiver, or developing sub-optimal techniques that are easier to implement.

ACKNOWLEDGEMENTS

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