A Simple Satellite Exclusion Algorithm for Advanced RAIM

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ABSTRACT

Advanced Receiver Autonomous Integrity Monitoring is a concept that extends RAIM to multi-constellation and dual frequency that could provide worldwide coverage of vertical guidance [1], [2]. A baseline algorithm for the user receiver has been described in [3]. This algorithm includes the computation of the Protection Levels, the Effective Monitor Threshold, the accuracy, and the description of a preliminary exclusion algorithm, which was refined in [4]. The exclusion function helps maintain availability through the duration of a fault or to meet the continuity requirements.

In our previous work [4], we had proposed an exclusion algorithm for ARAIM whose goal was to provide a set of conditions under which a fault (satellite or group of satellites, for example) can be excluded and treated as if it was flagged by the ground. This property allows the receiver to treat the satellites remaining after exclusion as an all-in-view situation, which greatly simplifies the processing. To achieve this goal, that algorithm used exclusion confirmation tests. The thresholds of the exclusion confirmation tests were tied to the Protection Level equation. There were two minor drawbacks in this algorithm. First, the exclusion confirmation tests, although not computationally expensive, require a complex description and a somewhat complicated logic. Second, when the exclusion confirmation tests do not pass, (that is, in practice when there are other consistent sets,) there can be a momentary loss of performance due to the ambiguity in which is the correct exclusion.

In this paper we present an evolution of the exclusion scheme presented in [4] that, while retaining its qualities, solves the first issue and mitigates the second one. To address the first one, the logic of the algorithm has been simplified: it now uses the same function to compute the Protection Levels whether exclusion has been performed or not. The second issue, that is, the loss of performance, is mitigated by exploiting the ambiguity as to which satellite must be excluded. In this new algorithm, the exclusion confirmation tests are now only an output of the all-in-view Protection Level calculation.

We will present a step by step description of the algorithm, and the associated analytical proof of integrity. To show the performance of the algorithm in a realistic setting, we will test the algorithm using GPS and GLONASS measurements (both with real faults and injected faults).

INTRODUCTION

The purpose of the exclusion function for ARAIM described in this paper is to recover availability after a detection event, with the following limitations:

- The receiver is not aware of the Alert Limit
- The receiver does not need to characterize each of the exclusion options prior or after fault detection

In addition, it would be desirable that:

- There is a negligible impact on the detection only performance
- The protection level after detection and exclusion is equal to the protection level as if the excluded measurements had been flagged by the ground monitoring (or as if it was an outage) if the fault is unambiguous (this will be defined)

We note that the first two limitations prevent the receiver from guaranteeing performance when a fault is present. A performance guaranteed to a given probability would require the receiver to characterize all exclusion options and outage conditions. This is done by making sure that the probability that there is at least one subset that is consistent is below the requirement, and by accounting for all the exclusion options when computing the integrity risk. This approach was implemented in [5], (although it did not account for outage conditions other than faults).

EXCLUSION ALGORITHM

Finding exclusion candidates

The exclusion algorithm consists in finding a subset of measurements that is consistent. (A subset is determined to be consistent if it passes the solution separation tests [3]). As shown in [3], it is possible to avoid testing all possible subsets by checking the chi-square statistic of each of the subsets. It is defined by:

$$q_i = y^T \left(W^{(i)} - W^{(i)} G \left(G^T W^{(i)} G \right)^{-1} G^T W^{(i)} \right) y \qquad (1)$$

where:

y is the vector of measurement residuals as defined in [3]

$W^{(i)}$ is the weighting matrix for subset i corresponding to the integrity error model [3]

G is the geometry matrix as defined in [3]

The set of candidates is given by $\left\{ i \mid q_i \le T_{exclusion,i} \right\}$ where the threshold $T_{exclusion,i}$ will be defined below.

Selecting measurements or fault mode to be excluded

Among the subsets that pass the test, we find one subset that also passes the consistency checks. There may be several subsets that pass the consistency check. In that case, the receiver can either choose the first one that passes the consistency check, or compute all possibilities and choose the one providing the lowest protection level.

POST EXCLUSION PROTECTION LEVEL EQUATION

Let us suppose that fault mode j has been selected for exclusion. We show that the following protection level meets the integrity requirements:

$$PL_{j,exclusion} = \max \begin{pmatrix} PL_{j,baseline}, \\ \max \left\{ PL_{0,baseline} - T_{0,i} + \left| \hat{x}_j - \hat{x}_i \right| \mid q_i \le T_{exclusion,i} \right\} \end{pmatrix} \qquad (2)$$

where:

$PL_{j,exclusion}$ is the protection level calculated after subset j has been excluded.

$PL_{j,baseline}$ is the protection level calculated assuming the set of measurements j is unavailable (0 means no exclusion). It is defined as in [3].

$T_{0,i}$ is the threshold for the solution separation test on fault mode i as defined in [3]

$T_{exclusion,i}$ is the threshold for the chi-square statistic i

$\hat{x}_i$ is one of the coordinates of the position fix that is not affected by fault mode i [3]

The interpretation of this equation is the following: the set of measurements j has been selected for exclusion, but the subsets excluding i are also consistent, which could mean that the fault might be on the set of measurements i. Since the estimate $\hat{x}_i$ is free of the error in case the fault i is present, we make sure that the protection level includes it (as well as a margin). As the error grows, there will be fewer sets within the exclusion statistic. If there are none left, the protection reverts to the baseline protection level, as if there was an outage.

This protection level is defined for each coordinate. The HPL can be obtained by combining the PL in the East and North coordinates as explained in [3].

Threshold computation

The exclusion thresholds must be chosen such that:

$$\sum_{i=1}^{N_{\text{fault modes}}} p_{fault,i} \chi^2_{k_i} \left(T_{exclusion,i} \right) = P_{HMI,exclusion} \qquad (3)$$

where:

$N_{fault\ modes}$ is the number of fault modes as defined in [3]

$p_{fault,i}$ is the probability of fault mode i

$P_{HMI,exclusion}$ is the integrity risk budget accounting for the exclusion. This parameter should be set so that nominal performance is not affected. In this paper we used a preliminary value of 10^{-8}.

$\chi^2_{k_i}$ is the distribution of the chi-square statistic assuming that there is no fault.

One way to choose the thresholds $T_{exclusion,i}$ to fulfil Equation is to set:

$$T_{exclusion,i} = \chi^2_{k_i}\left(\frac{P_{HMI,exclusion}}{N_{\text{fault\_modes}}\, p_{fault,i}}\right) \tag{4}$$

INTEGRITY PROOF

The integrity risk includes all the possible exclusion options. The equation can be written as:

$$\text{Integrity Risk} = P\left\{\bigcup_i\left(\bigcup_j\left(\left|\hat{x}_j - x\right| > PL_{j,exclusion}, \text{exclusion of } j\right), \text{fault } i\right)\right\} \tag{5}$$

The protection level *PLj,exclusion* can have different values depending on the results of the exclusion tests, so we write:

$$\text{Integrity risk from fault } i = P\left\{\bigcup_j\begin{pmatrix}\left|\hat{x}_j - x\right| > PL_{j,\text{exclusion}}, \text{exclusion of } j, q_i > T_{excl,i}\\ or\\ \left|\hat{x}_j - x\right| > PL_{j,\text{exclusion}}, \text{exclusion of } j, q_i \le T_{excl,i}\end{pmatrix} \mid \text{fault } i\right\} p_{fault,i} \tag{6}$$

We have the upper bound:

$$\text{Integrity risk from fault } i \le P\left\{\bigcup_j\begin{pmatrix} q_i > T_{excl,i}\\ or\\ \left|\hat{x}_j - x\right| > PL_{j,\text{exclusion}}, q_i \le T_{excl,i}\end{pmatrix} \mid \text{fault } i\right\} p_{fault,i} \tag{7}$$

If $q_i \le T_{excl,i}$, we impose the constraint:

$$PL_{j,exclusion} \ge PL_{0,baseline} - T_{0,i} + \left|\hat{x}_i - \hat{x}_j\right| \tag{8}$$

This inequality implies:

$$\left|\hat{x}_j - x\right| - \left|\hat{x}_i - \hat{x}_j\right| \ge PL_{0,baseline} - T_{0,i} \tag{9}$$

As a consequence, we have:

$$\left|\hat{x}_i - x\right| \ge PL_{0,baseline} - T_{0,i} \tag{10}$$

We therefore have the following inequality:

$$\text{Integrity risk from fault } i \le P\left\{\bigcup_j\begin{pmatrix} q_i > T_{excl,i}\\ or\\ \left|\hat{x}_i - x\right| > PL_{0,baseline} - T_{0,i}\end{pmatrix} \mid \text{fault } i\right\} p_{fault,i} \tag{11}$$

We can now remove the union over the index *j*, as the event is the same for all indices. We finally have:

$$\text{Integrity risk from fault } i \le P\left\{\begin{pmatrix} q_i > T_{excl,i}\\ or\\ \left|\hat{x}_i - x\right| > PL_{0,baseline} - T_{0,i}\end{pmatrix} \mid \text{fault } i\right\} p_{fault,i} \tag{12}$$

An upper bound of the probability of having either of these two events is given by the sum of the probabilities of each event:

$$\text{Integrity risk from fault } i \le P\left\{q_i > T_{excl,i} \mid \text{fault } i\right\} p_{fault,i} + P\left\{\left|\hat{x}_i - x\right| > PL_{0,baseline} - T_{0,i} \mid \text{fault } i\right\} p_{fault,i} \tag{13}$$

The second term in Equation (13) is the integrity risk when no fault has been excluded. Therefore, the integrity risk added by the exclusion function is given by:

$$\text{Integrity risk from exclusion} = \sum_{i=1}^{N_{\text{fault modes}}} p_{fault,i} P\left(q_i \le T_{exclusion,i}\right) \tag{14}$$

The statistic q_i is independent of the fault I, so as long as the thresholds fulfil Equation (3), we will have:

$$\text{Integrity risk from exclusion } \leq P_{HMI,exclusion} \tag{15}$$

RESULTS WITH REAL DATA

The main property of this scheme is that, as long as the error is large enough, the protection level will revert to the baseline protection level with very little penalty (over the fault detection only performance). The examples shown here are not exhaustive and are only meant to illustrate this property.

The first example (Figure 1) was generated with simulated data. The nominal noise is fixed. A ramp error is injected in one of the measurements. At first the fault is too small to be detected. Then, the fault is detected, but there is some uncertainty as to which measurement should be excluded. Finally, when the error is large enough, the VPL reverts to the VPL obtained by assuming that the excluded satellite is not available.

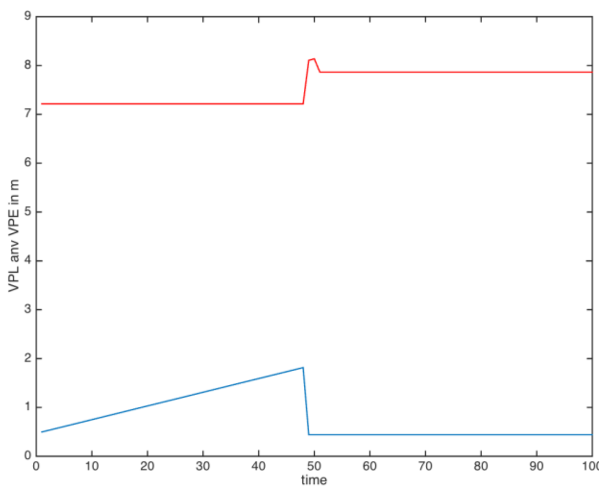

Figure 1. *VPL (in red) and Vertical Position Error (in blue) when a fault is injected.*

In Figures 2 through 5 we examine the behavior of a L1-L2 GPS – GLONASS ARAIM prototype (as described in [6]) with injected faults. We assumed the following Integrity Support Message content [3]:

GPS		
$Mask_i$	All 1	All 1
$P_{const,i}$	**10^{-8}**	**10^{-4}**
$P_{sat,j}$	**10^{-5}**	**10^{-5}**
$\alpha_{URA,j}$	**1**	**1**
$\alpha_{URE,j}$	**1**	**1**
$b_{nom,j}$	**0.0**	**0.0**

Table 1. *Integrity Support content assumed in the GPS – GLONASS ARAIM prototype*

Figures 2 through 6 shows both the Horizontal Position Error (HPE) and the HPL. In Figure 2, no fault was injected. In Figures 3 through 5, a constellation wide fault consistent with an East West rotation of 25, 50, and 100 m was injected in all GLONASS measurements. Figure 6 shows the results assuming that GLONASS is not available. As the error grows, there are fewer periods were the fault was undetected (or not excluded correctly). When the error is large enough (Figure 5), the fault is always detected and excluded, and the HPL reverts to the GPS only HPL (this can be seen by comparing Figure 5 with Figure 6). In all cases the HPL was above the HPE.

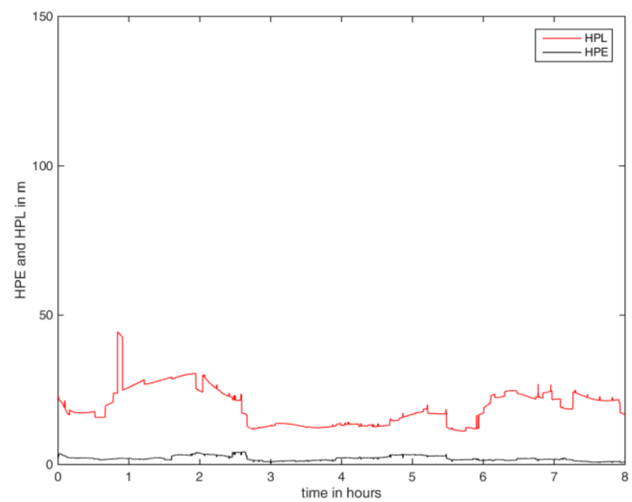

Figure 2. *HPE and HPL of the L1-L2 GPS – GLONASS prototype (no fault).*

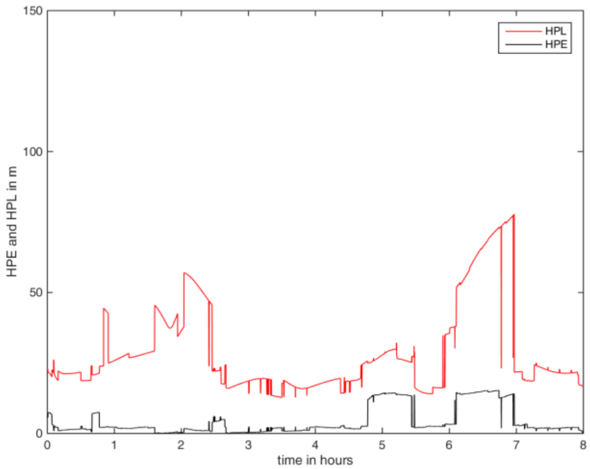

Figure 3. *HPE and HPL of the L1-L2 GPS – GLONASS prototype with an injected constellation wide GLONASS fault (25 m)*

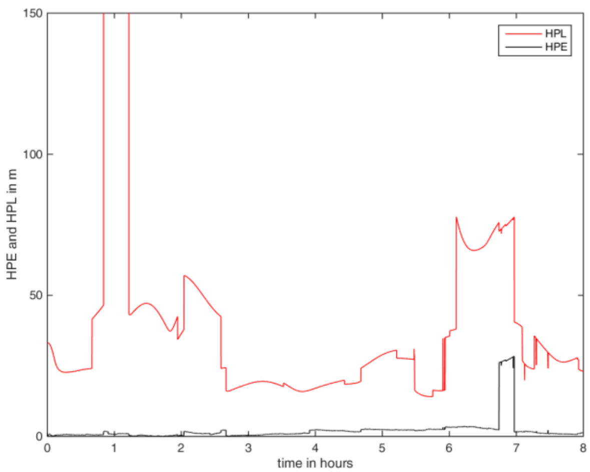

Figure 4. *HPE and HPL of the L1-L2 GPS – GLONASS prototype with an injected constellation wide GLONASS fault (50 m)*

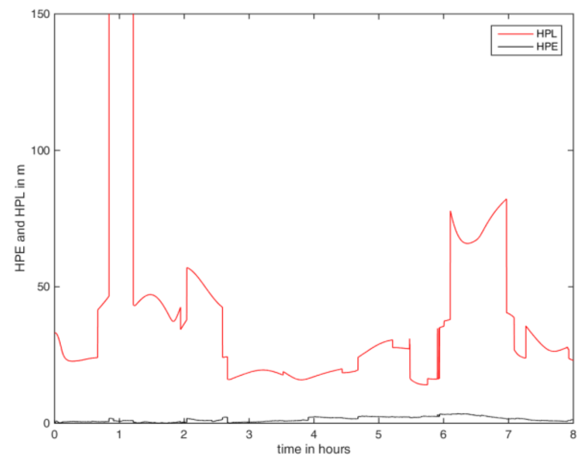

Figure 5. *HPE and HPL of the L1-L2 GPS – GLONASS prototype with an injected constellation wide GLONASS fault (100 m)*

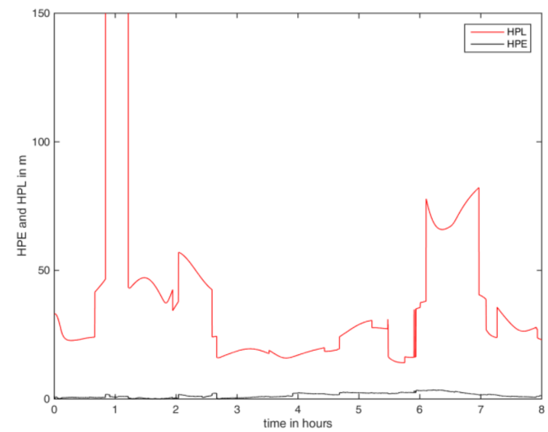

Figure 6. *HPE and HPL of the L1-L2 GPS – GLONASS prototype assuming that GLONASS measurements are not available*

SUMMARY

We have presented a simple exclusion algorithm and the associated protection levels for Advanced RAIM. This algorithm does not require the characterization of all exclusion options. The protection level after exclusion depends on outputs of the function that computes the all-in-view protection level and the protection level computed assuming that the fault is actually an outage. This property makes the exclusion computational load low. When the fault magnitude is large enough, the protection level is simply the protection level that would have been obtained if the fault had been an outage.

ACKOWLEDGEMENTS

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