Application of Spatial Statistics to Ionosphere Estimation for WAAS

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ABSTRACT

Now that the Wide Area Augmentation System (WAAS) has finalized the algorithms providing Lateral Navigation/Vertical Navigation (LNAV/VNAV) service, an increasing level of attention is being given to the feasibility of a GNSS Landing System (GLS) service. This would reduce the required Vertical Alarm Limit (VAL) from 50 meters to 20 meters or below. One of the algorithms that would be required to increase performance substantially is the ionospheric correction algorithm. WAAS incorporates information from reference stations to create a correction map of the ionosphere. More importantly, this map contains confidence bounds describing the integrity of the corrections. The confidence bounds must be large enough to describe the error in the correction but tight enough to allow the operation to proceed. The difficulty in generating these corrections is that the reference station measurements are not co-located with the aviation user measurement. For any estimation algorithm, a very sensitive parameter in the real time estimation of the ionospheric delay is the quality of the coverage by the Ionospheric Pierce Points (IPPs) measurements. Because the IPPs are scattered irregularly over the region of interest, the measure of coverage, or metric, is not an easy parameter to define, and has lead to many difficulties in the current WAAS system.

Geostatistics, a field that was originally developed for ore reserve estimation and is part of the broader field known as spatial statistics, has created a number of models and techniques to treat estimation problems involving spatial data. In particular the method called kriging is nowadays popular in many fields of science and industry where there is a need for evaluating spatially or temporally correlated data.

After summarizing the geostatistical method, this paper examines the worthiness of kriging for ionospheric estimation. It is explained in particular how to generate a family of metrics measuring the quality of sampling of a given region, useful for any estimation method. Extensive validation of the algorithms presented, using past WAAS ionospheric measurements for both quiet and disturbed periods, suggest that kriging provides useful insights and solutions to the ionospheric estimation problem, and could help WAAS to achieve GLS capability.

INTRODUCTION

The vast majority of the development effort for the Wide Area Augmentation System (WAAS) is devoted to designing algorithms that generate in real time protection limits for the residual position error in the differential correction to GPS. The ionospheric correction relies upon the estimation of the ionospheric delay at any location from the measurements obtained by the network of (WAAS) reference stations [1]. These reference station receivers have dual frequency capability, which enables them to measure the total electron content crossed by the raypath [2]. The estimation process is simplified by the assumption that the total electron content is concentrated at a given altitude. In this model, called thin shell model [3], each measurement corresponds to a location in the shell, labeled Ionospheric Pierce Point (IPP). Figure 1 shows a map of the CONUS region with the IPPs at a given time.

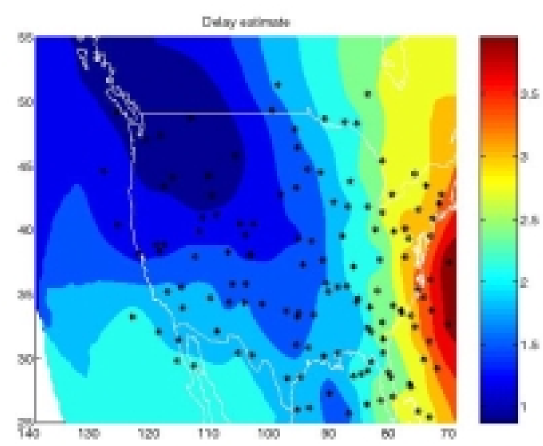

Figure 1. *IPPs over the CONUS region for a quiet day (July 2nd, 2000).*

Ideally, each user would use the IPP measurements to estimate the ionospheric delay from the user receiver to each of the satellites in view, by estimating the delay at each corresponding IPP as well as the confidence bound on the estimate. Unfortunately, the available bandwidth to transmit the corrections is not sufficient to transmit both the location of the measurement –which changes over time - and the magnitude. Instead, the WAAS message specifies the ionospheric delay and the confidence bound for a set of points disposed in a regular grid, the Ionospheric Grid Points (IGPs)[4]. In this paper we are interested in the limitations of ionospheric delay estimation due to ionospheric behavior and due to the location of the measurements (IPPs), within the thin shell model. For this reason, we assume here that each user has complete knowledge of all IPP locations and ionospheric delays, as if all the necessary bandwidth were available. The problem then becomes the estimation of the delay and the protection limit at the location x_0, knowing the delay at the locations $x_1, \ldots, x_n$.

Previous work [5] strongly suggests that the degree of correlation of the measurements is a function of the distance between the locations, even after removing a planar trend. For example, as we arbitrarily remove points from the fit, the empirical distribution of the residuals (error-confidence bound ratio) gets worse. This phenomenon is not harmful for the actual level of capability (LNAV/VNAV), since this effect is small for the very conservative confidence bounds sent. Nevertheless, in the actual algorithm the measure of coverage is taken into account through a corrective term labeled σ 'undersampled'. This term measures very conservatively the quality of coverage. The method to obtain σ 'undersampled' in the current algorithms is fully described in [6].

The next goal for WAAS is to achieve GLS capability. It would require in particular to reduce the ionospheric confidence bound by at least a factor of 2 (from 1.5 meters to .60 meters. For this reason, either much more sensitive 'metrics' will be needed, or an algorithm whose performance is not affected by poor coverage. The purpose of this work is to show how we could obtain such an algorithm by using the models and techniques developed in geostatistics. We will also show how we can use those concepts to produce a family of 'metrics'. After a brief introduction of the technique called 'ordinary kriging', the actual algorithm is described; this is followed by the results of the algorithm on the supertruth data [2]. Finally, a possible metric measuring coverage of a region based on kriging is suggested.

GEOSTATISTICS

One of the most important problems in mining is to predict the ore grade in a mining block from observed samples at irregularly spaced locations. To treat this problem geostatistics uses models comprising one random variable per location to model incomplete knowledge. However, the different random variables are correlated, with varying degrees depending on the distance. The collection of random variables is a random function :

$\{Z(x, \omega) \mid x \in \mathbb{R}^2, \omega \in \Omega\}$

For a fixed location x_i, $Z(x_i, \omega)$ is a function of ω, that is, it is a random variable. For a fixed ω_j in Ω, $Z(x, \omega_j)$ is a deterministic function of x. Our assumptions about what Ω is will determine the estimation algorithm. For example, that Z(x) is a linear function of x with independent noise the algorithm should be based on a planar fit.

A classical assumption in geostatistics is the intrinsic stationarity. It states that for any x and in a neighborhood of x :

$$E(Z(x+h) - Z(x)) = 0 \qquad (1)$$

$$\mathrm{var}(Z(x+h) - Z(x)) = 2\gamma(|h|) \qquad (2)$$

Equation (1) states that the expectation of Z is locally stationary and (2) states that the variance of a measurement to another measurement is only a function of the distance between the measurements. The IPP measurements violate the first assumption (since they follow a clear trend -shown in Figure 1). A variable trend can also be assumed, but we will assume this model for the sake of simplicity in the presentation.

Kriging provides a solution to the problem of estimation based only on the above assumptions and on the knowledge of the function γ, called the variogram. We summarize below the most common type of kriging, known as ordinary kriging [7], [8]. It assumes that the mean is unknown. We write the estimate at x_0 as a linear combination of the function at known locations:

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

Since we want the estimate to be unbiased and using the first assumption, we get:

$$\sum_{i=1}^{n} \lambda_i = 1$$

We want to minimize the difference between the estimate and the true value, that is:

$$\mathrm{var}((\hat{Z}(x_0) - Z(x_0))^2) =$$
$$2\sum_{i=1}^{n}\lambda_i\gamma(x_i, x_0) - \sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j\gamma(x_i, x_j)$$

The minimization of this function with the constraint on the sum of the coefficients is straightforward using the method of Lagrange. Using the following notations:

$$A = \begin{bmatrix} \gamma(x_1,x_1) & \dots & \gamma(x_1,x_n) & 1 \\ \dots & \gamma(x_i,x_j) & \dots & \dots \\ \gamma(x_n,x_1) & \dots & \gamma(x_n,x_n) & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} \gamma(x_1,x_0) \\ \dots \\ \gamma(x_n,x_0) \\ 1 \end{bmatrix} \qquad \lambda = \begin{bmatrix} \lambda_1 \\ \dots \\ \lambda_n \\ \varphi \end{bmatrix}$$

the equation defining the minimizer is:

$$A\lambda = b$$

The resulting estimation variance is:

$$\hat{\sigma}^2(x_0) = b^T\lambda = b^T A^{-1} b$$

Under the above assumptions the kriging estimator is the best unbiased linear estimator in the sense that it minimizes the estimation variance, also called kriging variance. The main feature of kriging is the explicit use of spatial decorrelation to optimize the estimation, resulting in two properties that are desirable in an estimation algorithm. First, near points carry more weight than distant ones. Their relative proportions depend on the position of the sampling points and on the variogram γ. Second, there is a declustering effect: measurements that are close together will carry less weight individually than isolated ones at the same distance. At the same time the kriging variance is a function of the geometric location that closely follows our intuition: points that are 'far' from the measurements will have a larger kriging variance. In Figure 2 a map of the kriging variance has been generated using a linear variogram.

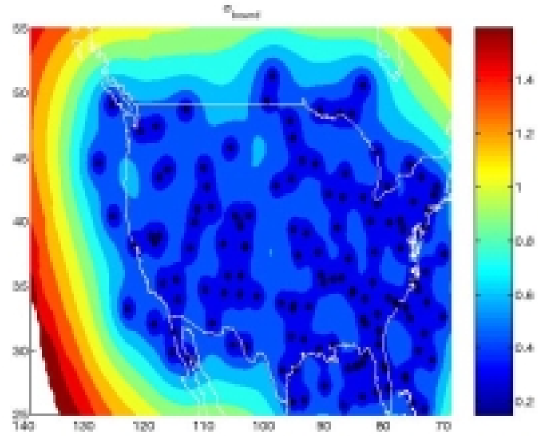

Figure 2. *Kriging variance map over CONUS region for a quiet day.*

The critical point in kriging is the variogram γ.γ is not known and needs to be estimated from the IPP measurements. There are several ways of estimating it. The classical formula for the experimental variogram is:

$$\hat{\gamma}(h_j) = \frac{1}{2m(h_j)}\sum_{i=1}^{m(h_j)}(Z(x_i) - Z(x_j))^2$$

where $m(h_j)$ is the number of pairs of measurements separated by a distance h comprised in the interval $[h_j, h_{j+1}]$. However, one cannot use the experimental variogram in the formulas above. γ has to be such that the estimation variance is always positive. More precisely, γ needs to be Conditional Negative Semidefinite. For the purposes of this work, we retain that any variogram γ of the form $\gamma(h)=a+b(\mathrm{distance})^{\alpha}$ is admissible provided that a and b are positive and α is between 0 and 2. Once the experimental variogram is determined we match an admissible variogram to it.

ALGORITHM

In order to apply kriging to ionospheric estimation, we need to know what variogram should be used, that is, which one describes the random behavior of the ionosphere conservatively. Figure 3 shows the variogram of the ionosphere for quiet days computed using the formula above, taking the width of each bin to be 200 km.

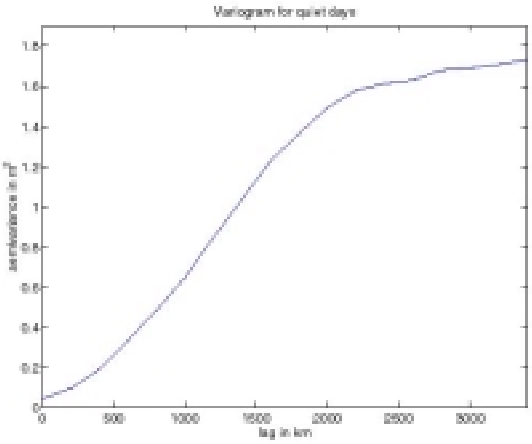

Figure 3. *Variogram of ionospheric measurements for quiet days.*

The shape of this variogram is mainly determined by the planar trend of the ionosphere. A deterministic planar trend produces a parabolic variogram with a flat derivative at the origin. If that was the true structure of the ionosphere, kriging would not be adapted to ionosphere estimation. However one can see that the derivative at the origin is not zero, which means that even after removing the planar trend the decorrelation decreases with distance. We also see that for distances larger than 1500 km the behavior is not planar anymore.

In this study it we chose to estimate the variogram in real time. For each location where the ionosphere needed to be computed, the experimental variogram was computed with the IPPs located within 2000 km. The theoretical variogram –an admissible variogram close enough to the experimental variogram- was chosen to be linear $\gamma(h)=a+b*distance$. The constant term was taken to be the intercept of the experimental variogram $\hat{\gamma}$ and the slope was such that $\hat{\gamma}$ would lie under γ. All those choices were arbitrary, mainly because the algorithm performance was robust against changes, provided they were reasonable.

Once the variogram was computed, the kriging equations were applied to find the estimate and the variance. Figure 1 and 2 show the map of the kriging estimates and the map of the kriging variances at a given time respectively.

RESULTS

The evaluation of the algorithm was done through cross-validation: for each measurement we compute the estimate produced by the remaining measurements at the same location as well as the kriging variance. We then form the normalized residual:

$$K = \frac{\hat{I} - I}{\sigma_{kriging}}$$

Figure 4 shows such a distribution for July 2nd 2000.

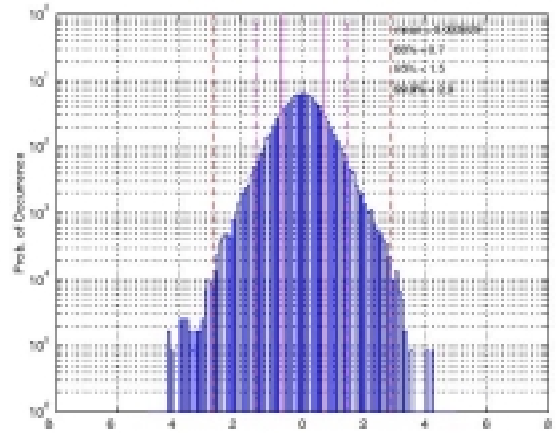

Figure 4. *Histogram of the normalized residuals for July 2nd 2000.*

The performance of the algorithm was measured through the characteristics of the resulting distribution: mean, variance, maximum normalized residual, gaussian overbound. The gaussian overbound is the tighter normal distribution such that it always bounds the empirical cumulative distribution function. It is expressed as a factor indicating by how much we would have to inflate the bound. If this factor is larger than 1 it means that there might be integrity failures. The algorithm based on kriging was compared to one based on a planar fit where the decorrelation is determined in real time [9]. Three days worth of supertruth data where analyzed: a nominal day, July 2nd 2000, a quiet day showing some unusual behavior without being stormy, May 25th 2000 and a storm day, July 15th 2000. Tables 1, 2 and 3 summarize the results obtained.

	KRIGING	PLANAR FIT
Variance of norm. residuals	.51	.86
Gaussian overbound	1.01	1.14
Maximum norm. residual	4.33	4.93
Mean of σ_{bound}	.28 meters	.23 meters

Table 1. *Results for July 2nd 2000.*

	KRIGING	PLANAR FIT
Variance of norm. residuals	.37	.78
Gaussian overbound	.91	1.44
Maximum norm. residual	3.78	6.14
Mean of σ_{bound}	.38 meters	.28 meters

Table 2. *Results for May 25th 2000.*

	KRIGING	PLANAR FIT
Variance of norm. residuals	.52	.80
Gaussian overbound	.95	1.21
Maximum norm. residual	3.76	5.40
Mean of σ_{bound}	.82 meters	.69 meters

Table 3. *Results for July 15th 2000.*

We first notice that kriging provides results that are comparable in performance to the planar fit. Besides this, kriging also seems to provide more integrity than the planar fit since its gaussian overbound is systematically under the one corresponding to the planar fit. However, these results depend heavily on parameters that can be tuned.

It is far more informative to look at the behavior of these algorithms under data deprivation schemes. The estimation algorithms are more likely to fail in providing integrity when there are few measurements. These undersampled situations do not occur often but can be responsible for integrity failures. It is thus essential to test the robustness of the estimation algorithms by simulating these undersampled situations. For this study, we implemented a disk data deprivation scheme: in the estimation process we only use measurements that are at a distance larger than a given radius R. The cross-validation process was carried out for radii going from 0 km to 900 km every 100 km. For each radius an empirical distribution was obtained. Instead of displaying the 10 different empirical distributions we show in Figure 5 their variance as a function of the radii.

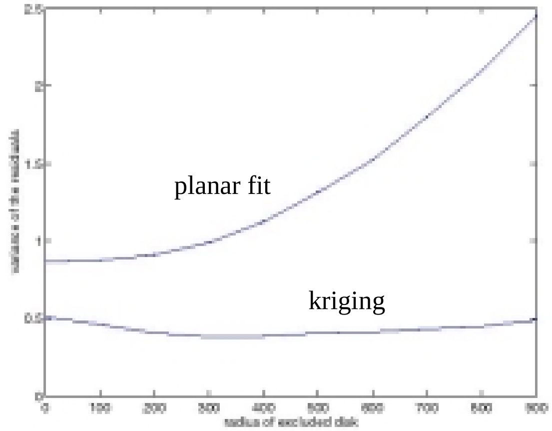

Figure 5. *Variance of the normalized residuals of both algorithms under a data deprivation scheme (July 2nd ,2000).*

In the planar fit the parameter that has a major impact on the estimated bound is the number of measurements considered for the fit. One can see that this parameter does not describe the degree of coverage very well: the distribution of the normalized residuals widens as the excluded radius increases. It is for this reason that a metric had to be added in the current WAAS algorithm to account for this deterioration without losing too much availability. The curve corresponding to kriging has a much more satisfactory behavior: the distribution of the normalized residuals remains the same, regardless of the radius. Because of the explicit use of the variogram, the algorithm takes into account the loss of information due to the scarcity of data, or due to its clustering.

METRICS BASED ON THE KRIGING VARIANCE

There are still many questions that need to be studied to bring full integrity to an algorithm based on kriging. In this section we propose a short term application of kriging that could enhance the capabilities of the current algorithms with few modifications. The shape of the kriging variance map shown in Figure 2 suggests that kriging could be readily applied to measure the degree of coverage of a given region, regardless of the estimation algorithm. The regions where the kriging variance is large correspond to regions where visually we would assign a larger uncertainty. This feature makes the kriging variance a good candidate for a metric, that can be used to determine σ 'undersampled' following the method presented in [6]. For this purpose, we do not need to estimate the variogram in real time. Instead, we need to find a fixed variogram that produces a good correlation between error and kriging variance.

CONCLUSION

Since the measure of uncertainty is given by the variogram, it is essential to have a very high confidence on the variogram used in a given situation. Although here a real time estimation of the variogram gave excellent results, we need to further study the stationarity of the ionosphere.

Kriging does not try to fit a given shape to the ionosphere; instead, it only uses the correlation between measurements to take into account the loss of information due to distance. It includes explicitly the random behavior of the ionosphere through the variogram. It is this capacity to model spatial uncertainty that distinguishes kriging from other estimation techniques. By applying kriging under data deprivation schemes we have shown its ability to measure uncertainty. Because of the good results obtained and the attractive features of kriging we believe that the undersampled problem in WAAS could be greatly mitigated using ideas based on kriging.

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REFERENCES

[1] Enge P.,Walter T., Pullen S., Kee C., Chao Y.C., and Tsai Y.-J, ”Wide Area Augmentation of the Global Positioning System,” Proceedings of the IEEE, vol. 84, no 8, pp. 1063-1088,1996.

[2] Hansen, A., Walter T., Blanch, J., and Enge P., ”Ionospheric Correlation Analysis for WAAS,” in proceedings of ION GPS, Salt Lake City, UT, September 2000.

[3] Global Positioning System Standard Positioning Service Signal Specification, June 1995.

[3] Walter T., Hansen, A., Blanch, J., and Enge P., ”Robust Detection of Ionospheric Irregularities,” in proceedings of ION GPS, Salt Lake City, UT, September 2000.

[4] WAAS MOPS, RTCA SC 159 DO-229B.

[5] Blanch J., Walter T., and Enge P., ”Ionospheric Threat Model Methodology for WAAS,” in proceedings of ION 57th NTM, Albuquerque, NM, June 2001.

[6] Sparks L., Mannucci A.J., Altshuler E., Fries R., Walter T., Hansen A., Blanch J., Enge P. “The WAAS Ionospheric Threat Model,” in proceedings of the Beacon Satellite Symposium, Boston, MA, June 2001.

[7] Webster R., Oliver M., (2001) Geostatistics for Environmental Scientists. John Wiley and Sons, New York.

[8] Cressie N. A. C. (1993) Statistics for Spatial Data, revised edition. John Wiley and Sons, New York.

[9] Lejeune R.,Bakry El-Arini M., Altschuler E., Walter T. “Trade Study of Improvements to the WAAS Ionospheric Integrity function for GNSS Landing System (GLS)”, in proceedings of ION 57th NTM, Albuquerque, NM, June 2001.