Adapting Kriging to the WAAS MOPS Ionospheric Grid

Juan Blanch, Todd Walter, Per Enge. *Stanford University.*

ABSTRACT

The most productive way to increase the availability of single frequency users of the Wide Area Augmentation System (WAAS) is by decreasing the Grid Ionospheric Vertical Error (GIVE). Currently the GIVE's are very conservative, since WAAS has to protect against the worst possible case of ionospheric behavior given the measurements. By characterizing more accurately the vertical ionospheric delay model in nominal conditions and by better defining the 'well sampled' regions we can be less conservative while maintaining integrity.

It has been shown previously that an ionosphere estimation algorithm based on kriging could address these two issues efficiently. In quiet conditions, the kriging method produces at each location an estimate of the Vertical Ionospheric Delay and a confidence bound on the estimate. The confidence bounds obtained seem to be close to the lower limit of what is possible within the thin shell model –before the storm detector is applied. In addition to that, the particular behavior of the kriging variance at the edge of coverage can be used to mitigate the non-stationarity of Total Electron Content (TEC) during storms. To do that, one can either define a 'well sampled' region or use it as a metric of coverage.

As it has been described, the algorithm would require the user to know at all times the location of all Ionospheric Pierce Points (IPPs). Unfortunately, there is not enough bandwidth for the user to receive all of this information nor would this be efficient. Instead, the user receives a grid of points, the Ionospheric Grid Points (IGPs), which is updated every 5 minutes. For each satellite, the user interpolates both the delay and the GIVEs from the four closest IGPs. However, kriging gives an optimal estimate only at the IGPs, and the kriging variance is only valid at the IGPs. Therefore the delay computed by the user is not optimal, and the confidence bound will necessarily change as we depart from the IGP.

In this study, we present a calculation of the GIVE for the kriging method that protects the user at any location. Results of the algorithm will be shown using ionospheric data collected from the WAAS reference stations. We will show that, even after the increase in confidence bound caused by the grid, we can still generate GIVEs below .6 m inland and 1 m in coastal regions.

INTRODUCTION

The most important attribute of the Wide Area Augmentation System (WAAS) is integrity [1]. Along with the corrections broadcast to the user, WAAS sends strict confidence bounds on those corrections under all conditions. For example, the ionospheric information included in the WAAS message enables the user to correct for the ionospheric delay in each pseudo-range and know accurately the interval in which the true delay lies. Unfortunately, the vast range of ionospheric behavior [2] and the fact that the ionosphere is irregularly sampled have forced these confidence intervals to be very conservative [3]. Now, to reduce the conservativeness, we need to understand better the spatial characteristics of the ionosphere.

Within the thin shell approximation [4], each ray path has a corresponding Ionospheric Pierce Point (IPP) and each measurement is converted to an equivalent vertical ionospheric delay. It has been shown that the nominal ionosphere can be well characterized by a planar trend and a random gaussian field with a covariance depending on distance [5]. The minimum mean square estimator corresponding to this structure is called kriging [6]. For each location, kriging provides a confidence bound. In [5], an algorithm for WAAS based on kriging was sketched. This work showed that the confidence bounds where both safe and significantly smaller than the current confidence bounds. This algorithm reused extensively elements of the current WAAS algorithm, in particular the storm detector [7]. A problem with the straight forward implementation of the kriging algorithm is that it supposes the user has all the IPP measurements.

In fact, the WAAS user receives the corrections according to the WAAS Minimum Operational Standard (MOPS) which specifies that the ionosphere information be sent in a grid of 5 by 5 degrees in the thin shell at a height of 350 km [7]. At each node of the grid the user receives a vertical ionospheric grid delay (IGD) and a grid ionospheric vertical error (GIVE). The user calculates each of the ionospheric delays corresponding to the satellites in sight as well as the confidence bounds from this grid, according to an algorithm which is also set in the WAAS MOPS.

In order to make available the benefits of kriging to WAAS, we need to modify the algorithm presented in [5] such that it can be fit into the ionospheric grid. In the first part, we will review kriging assuming full knowledge of the IPP measurements; we will then show how to compute a GIVE for a kriging algorithm and, finally, we will show the gain in performance that kriging could provide.

KRIGING ALGORITHM

In this paper, we will skip the discussion about the storm detector. Here, it is sufficient to notice that the storm detector results on an inflation of the confidence bound in the case that the chi-square test passes. More information on the storm detector can be found in [7] and in [9] for a more detailed account of the influence of measurement noise on the chi-square detector.

It was shown in [5] that a good model for the vertical ionospheric delay on the thin shell is a planar trend to which a random gaussian field [6], [10] is added, that is:

$$I(x) = a_0 + a_1 x^{(east)} + a_2 x^{(north)} + r(x)$$

In this formula, *I(x)* is the vertical ionospheric delay at location x, the three coefficients a_0, a_1, and a_2 describe the planar trend and *r(x)* is the random gaussian field. A nominal ionosphere is such that the field *r(x)* has a covariance *C(x,y)* that depends on the distance between x and y:

$$E\left(r(x)\,r(y)\right) = C(x,y) = C\left(\|x-y\|\right)$$

Please refer to [5] to see how C was modeled and determined, and the appendix for the analytical expression. Now let us suppose that we have n IPP measurements. Each measurement has a noise pattern which is supposed to be known.

$$\tilde{I}(x_k) = a_0 + a_1 x_k^{(east)} + a_2 x_k^{(north)} + r(x_k) + n(x_k)$$

Unlike *r*, *n* is uncorrelated from one location to another. Kriging gives the best linear unbiased estimate of the field *I(x)* for each point. The expression:

$$E\left(I(x) - \sum_{k=1}^{n} l_k \tilde{I}(x_k)\right)^2$$

is minimized with respect to the weights on the measurements ?. Please refer to the appendix and [5] for additional details. What is important here is that the weights ? can be expressed as a function of the measurement noise covariance N and the covariance C:

$$l = PC(x, x_k) + QX$$

P and *Q* are matrices that do not depend on the location and only on measurement information. They are defined in the appendix. $C(x,x_k)$ is a vector of the covariance between each of the measurements and the location to be estimated. X is equal to $[1\ x^{(east)}\ x^{(north)}]^T$.

GIVE FORMULA

As discussed before, the user does not have the IPP measurements. Therefore, the algorithm outlined above cannot be applied directly. Let us now describe how the user computes both the vertical ionospheric delay and the confidence bound, called User Vertical Ionospheric Error (UIVE), at each of the IPPs. For each of the IPPs, the user determines the box in which the IPP is contained. Then the user interpolates the data of the four IGPs which form the box:

$$\hat{I}_{user} = \sum_{i=1}^{4} a_i \hat{I}_{IGP,i}$$
$$s_{UIVE}^2 = \sum_{i=1}^{4} a_i s_{GIVE}^2$$

The weights a_i are computed according to a bilinear interpolation scheme [7]. Assuming that all the random variables are gaussian, the integrity requirement can be written as:

$$E\left(\hat{I}_{user} - I_{real}\right)^2 \le s_{UIVE}^2$$

All the IGDs and GIVEs (in this paper GIVE and s_{GIVE} are the same) have to be such that this inequality always holds.

Now, in order to develop a formula for the GIVE, we need to make some assumptions:

- all IGPs use the same set of IPPs

- we ignore the curvature of the earth in the 10 by 10 degree box in where the IGP has influence
- the GIVEs are computed at the same time
- the ionosphere is in the same state for contiguous IGPs

First of all we need to decide what delay value to use at the IGPs. A natural choice consists in computing there the kriging estimate at the grid points. For each IGP we have:

$$I_{IGP,i} = l_i^T \tilde{I}\left(x_k\right)$$

where the coefficients $?_i$ have been expressed above. The user estimate is then:

$$\hat{I}_{user} = \sum_{i=1}^{4} a_i l_i^T \tilde{I}\left(x_k\right)$$

We can now express the user estimation as a function of the model parameters:

$$E\left(\hat{I}_{user} - I_{real}\right)^2 = C_0 + a^T S a - a^T T a$$
$$+\sum_{i=1}^{4} a_i l_i^T \left(\sum_{i=1}^{4} a_i C\left(x_i, x_k\right) - C\left(x, x_k\right) \right)$$

For more details on this derivation, please see the appendix. C_0 is a constant and S and T are two positive definite quadratic forms. The idea now is to find four quantities s_i (one for each IGP) such that:

$$E\left(\hat{I}_{user} - I_{real}\right)^2 \le \sum_{i=1}^{4} a_i s_i^2$$

For each IGP there will be four such s_i corresponding to each of the quadrants where the IGP has influence. s_{GIVE}^2 (before applying the storm detector inflation) will be the maximum of these four quantities. We now focus on getting the s_i. Because of the convexity of S, the first two terms are easy to bound. The third term can be bound by finding a linear function such that:

$$-a^T T a \le b^T a$$

For the fourth term we bound independently each of the terms:

$$l_i^T \left(\sum_{i=1}^{4} a_i C\left(x_i, x_k\right) - C\left(x, x_k\right) \right)$$

In the appendix we explain how to obtain e and d>0 such that for any measurement and any location within the box we have:

$$-d \le \sum_{i=1}^{4} a_i C\left(x_i, x_k\right) - C\left(x, x_k\right) \le e$$

Therefore we have:

$$l_i^T \left(\sum_{i=1}^{4} a_i C\left(x_i, x_k\right) - C\left(x, x_k\right) \right)$$
$$\le d \sum_{l_{ik}<0} \left| l_{ik} \right| + e \sum_{l_{ik}>0} \left| l_{ik} \right| = h_i$$

We can now overbound the user estimation variance by:

$$E\left(\hat{I}_{user} - I_{real}\right)^2 \le C\left(0\right) + \sum_{i=1}^{4} a_i S_{ii}$$
$$+\sum_{i=1}^{4} a_i b_i + \sum_{i=1}^{4} a_i h_i = \sum_{i=1}^{4} a_i s_i^2$$

where:

$$s_i^2 = C\left(0\right) + S_{ii} + b_i + h_i$$

As said before, for each IGP there are 4 s_i corresponding to each quadrant where the IGP has influence. The 'pre-storm detector' broadcast GIVE will be the maximum of these four values.

We now need to check that the difference between optimal estimate and user estimate are not too different. We can measure this by comparing the optimal estimation variance at the IGP with the computed s_{GIVE}. In Figure 1 we show the percentage increase of s_{GIVE} with respect to the optimal estimation variance. This plot indicates how much performance is lost due the grid and the bounding process for the nominal covariance and the current measurement noise level. It is always an overbound and is never 10% more than it needs to be.

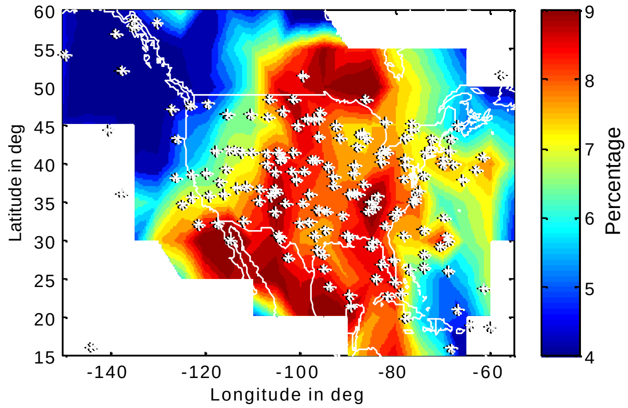

Figure 1. *Comparison between s_{GIVE} and optimal variance at each IGP. The plot shows the percentage increase.*

RESULTS

Now that we have a way of computing the GIVE at each IGP we can evaluate the gain in performance provided by this algorithm. In Figure 2 we show, for one epoch, s_{GIVE} for each IGP. Between the IGPs an interpolation scheme mimics the user bilinear interpolation.

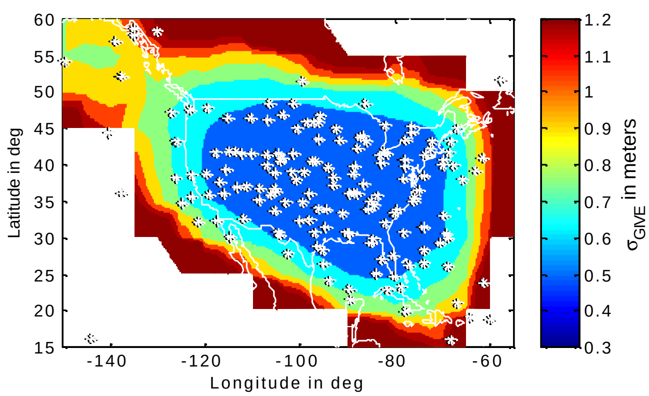

Figure 2. *Map of s_{GIVE} for a quiet day using kriging.*

The search radius was fixed to 1500 km and the parameters of the model are specified in the appendix. The GIVEs computed here take into account the storm detector. Since the threat model has not been applied, we could expect some degradation in the coastal regions. In order to compare these results with the current algorithm, we plot in Figure 3 the difference in percentage between the GIVEs computed using kriging and the GIVEs using a planar fit (where we try to mimic the current algorithm). We see that in most of the CONUS region, we get a reduction of 20%. One can also notice that, where the measurements become sparser, there is less reduction. This is in fact an advantage of kriging over a simple planar fit: it means that the threat model (which accounts for the undersampled threats) will not increase, and might even decrease substantially.

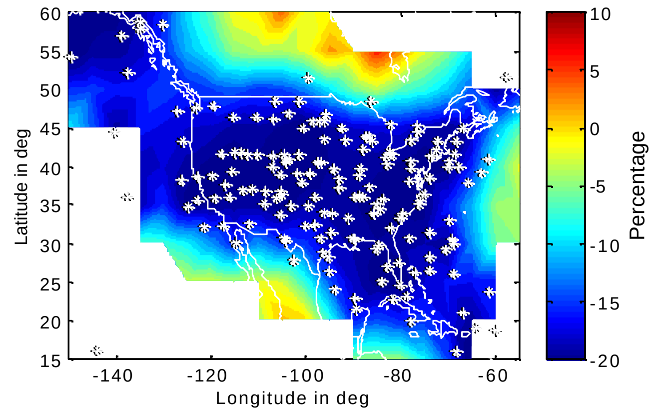

Figure 3. *Reduction in GIVE provided by kriging over the current algorithm (the threat model is not included).*

CONCLUSION

Because it is a local method, kriging needs a few modifications in order to be adapted to The WAAS MOPS ionospheric grid. However, the modifications do not mean a significant increase in computation nor do they dramatically reduce the benefits compared to the optimal solution. With the bounding method used here, an algorithm based on kriging produces GIVEs that are 20% below the ones produced by a simple planar fit. In the results showed in this paper, the threat model has not been applied. The relative improvement map shows that it might not increase, and might even be reduced.

APPENDIX

- The covariance model used in this work (found by fitting an admissible model to real data [5]) is:

$$C\left(\|x-y\|\right)=a\exp\left(-\frac{\|x-y\|}{c}\right)\text{ if }\|x-y\|>0$$

$$C\left(\|x-y\|\right)=b\text{ if }\|x-y\|=0$$

with $a=.15\text{ m}^2\quad b=.2\text{ m}^2\quad c=5000\text{ km}$

- The optimal coefficients ? are given by the formula:

$$l = PC(x, x_k) + QX$$
$$P = W - WG\left(G^T WG\right)^{-1} G^T W$$
$$Q = WG\left(G^T WG\right)^{-1}$$

where we have:

$W = \left(C(x_k, x_l) + N(x_k, x_l)\right)^{-1}$ (n by n matrix) and

$$G = \begin{bmatrix} 1 & x_1^{(east)} & x_1^{(north)} \\ \vdots & \vdots & \vdots \\ 1 & x_n^{(east)} & x_n^{(north)} \end{bmatrix}$$

- The user estimation variance is:

$$E\left(\sum_{i=1}^{4} a_i I_{IGP,i} - I(x)\right)^2 = E\left(\sum_{i=1}^{4} a_i r_{IGP,i} - r(x)\right)^2$$
$$= E\left(\sum_{i=1}^{4} a_i l_i^T \left(r(x_k) + n(x_k)\right) - r(x)\right)^2$$

The trend is filtered because of the unbiased nature of the user's interpolation scheme. We can now express this quantity as a function of the covariance:

$$E\left(\sum_{i=1}^{4} a_i l_i^T \left(r(x_k) + n(x_k)\right) - r(x)\right)^2 =$$
$$C(0) - 2\sum_{i=1}^{4} a_i l_i^T C(x, x_k)$$
$$+\left(\sum_{i=1}^{4} a_i l_i\right)^T (C+N)\left(\sum_{i=1}^{4} a_i l_i\right)$$

We can develop it more:

$$= C(0) + \left(\sum_{i=1}^{4} a_i \left(X_i - G^T WC(x_i, x_k)\right)\right)^T \left(G^T WG\right)^{-1}$$
$$\left(\sum_{i=1}^{4} a_i \left(X_i - G^T WC(x_i, x_k)\right)\right)$$
$$-\left(\sum_{i=1}^{4} a_i C(x_i, x_k)\right)^T W\left(\sum_{i=1}^{4} a_i C(x_i, x_k)\right)$$
$$+2\sum_{i=1}^{4} a_i l_i^T \left(\sum_{i=1}^{4} a_i C(x_i, x_k) - C(x, x_k)\right)$$
$$= C_0 + a^T S a - a^T T a$$
$$+\sum_{i=1}^{4} a_i l_i^T \left(\sum_{i=1}^{4} a_i C(x_i, x_k) - C(x, x_k)\right)$$

where S and T are 4 by 4 positive definite matrices:

$$S = \left[X_i - G^T WC(x_i, x_k)\right]^T \left(G^T WG\right)^{-1}$$
$$\left[X_i - G^T WC(x_i, x_k)\right]$$

$$T = \left(C(x_i, x_k)\right)^T W\left(C(x_i, x_k)\right)$$

- Because of the convexity of positive quadratic forms we have:

$$a^T S a \le \sum_{i=1}^{4} a_i S_{ii}$$

which takes care of the second term. This term is typically very small, and it increases as the IPP distribution looses symmetry (in the edge of coverage regions)

- We cannot treat the third term in the same way. But we have:

$$(a - a_0)^T T (a - a_0) \ge 0$$
$$-a^T T a \le -2a^T T a_0 + a_0^T T a_0 = b^T a$$

where:

$$b = -2a_0^T T + a_0^T T a_0 * [1 \quad 1 \quad 1 \quad 1]^T$$

here we choose a_0 such that the 4 IGPs compute the same overbound so a_0=[.25 .25 .25 .25].

- The fourth term takes into account the fact that within the 4 IGPs the covariance from a measurement to any other point in the box is very close to concave. We consider now one measurement. It is easy to find –but cumbersome to write- a concave function such that within the 4 IGPs the following is true:

$$0 \le C'(x, x_k) - C(x, x_k) \le e$$

with a very small *e*. Since $x = \sum_{i=1}^{4} a_i x_i$ and C' is concave we have:

$$\sum_{i=1}^{4} a_i C'(x_i, x_k) - C'(x, x_k) \le 0$$

As a result:

$$\sum_{i=1}^{4} a_i C(x_i, x_k) - C(x, x_k) \le e$$

For the lower bound we just notice that:

$$C(x_i, x_k) - C(x, x_k) \ge a \exp\left(-\frac{d_{\max}}{c}\right) = d$$

where d_{max} is the maximum distance within a box (~700 km). As a consequence:

$$\sum_{i=1}^{4} a_i C(x_i, x_k) - C(x, x_k)$$
$$= \sum_{i=1}^{4} a_i \left(C(x_i, x_k) - C(x, x_k)\right) \ge d$$

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