High Integrity Multipath Mitigation Techniques for Ground Reference Stations

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Biography

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Dr. J. David Powell is a Professor in the Department of Aero-Astro at Stanford University, where he has been on the faulty for 26 years. Prof. Powell's research deals with land, air, and space vehicle navigation and control. He is a coauther of two control system textbooks.

Abstract

There is an increasing demand for accuracy, integrity, continuity and availability in the ongoing development of differential GPS (DGPS) techniques. DGPS performance relies on the elimination of errors that are common to both users and ground reference stations. A multitude of error sources affect DGPS measurements. Most of them can be effectively eliminated with differencing techniques. Unfortunately, multipath and receiver thermal noise cannot be canceled in this manner. Code phase noise is reduced very effectively by carrier smoothing, however, this method is less effective against multipath. The bias-like multipath remains the dominant error source for differential GPS.

This paper presents a spectral decomposition-based multipath mitigation technique, which combines carrier smoothing, carrier Signal-to-Noise Ratio (SNR) and repeatability. Multipath stochastic modeling, SNR and repeatability aiding schemes are investigated in detail from an integrity viewpoint. Applications in WAAS and LAAS reference station receivers are demonstrated through analysis and data from National Satellite Test Bed (NSTB) reference stations.

1. Introduction

As the development of differential GPS (DGPS) techniques continues, there is an increasing demand for accuracy, integrity, continuity and availability. A multitude of error sources affect the DGPS system operation performance. Fortunately, most of these can be effectively eliminated using difference techniques. DGPS performance relies on the elimination of errors that are common to both users and ground reference stations. Unfortunately, multipath and receiver thermal noise cannot be canceled in this manner. Code phase noise is reduced very effectively by carrier smoothing, however, this method is less effective against multipath. The bias-like multipath re-

mains the dominant error source for differential GPS.

Multipath mitigation techniques have been an open question for half a century. Much effort has been put into solving this problem. Th majority of achievements could be summarized as follows:

Siting Siting is considered as the most critical issue for ground reference stations due to the highly geometric related nature of multipath.

Antenna Ground plane and choke ring are most widely used by survey and ground reference station receivers. Adaptive antenna array beam-forming is another effective way to mitigate multipath and interference as well.

Delay Lock Loop (DLL) Several successful DLL designs have been demonstrated to date. Narrow correlator, Multipath Elimination Technology (MET) and Multipath Estimation DLL (MEDLL) are typical strategies.

Signaling Technique Some improvement could be done by signaling, for example, longer C/A code could do a better job in alleviating multipath errors.

Data Processing Even though most users can take advantage of above techniques, there is still multipath error in the measurements. Further improvements have to be done by data processing schemes. There are a wide variety of mitigation techniques which employ data processing schemes. **Carrier smoothing** takes advantage of the fact that carrier phase measurement errors typically are negligible compared to code multipath. Optimal combination of carrier and code phase measurements can efficiently reduce the code multipath to centimeter level and are widely used. Other schemes have been investigated to mitigate multipath. Typical methods are focusing on taking advantage of **SNR** measurements [Com96, Bre96], **repeatability** of multipath at ground reference stations or **multiple receivers** used for canceling spatial correlated multipath. Performance improvements are limited and it is difficult to guarantee robust performance in practical applications.

This paper will investigate multipath mitigation techniques from the data processing viewpoint. Detailed analysis of carrier smoothing techniques are provided for more insight on optimal combination of code and carrier measurements. Furthermore, starting from physical mechanisms and spectral analysis of GPS multipath errors, a spectral decomposition based multipath mitigation strategy, taking advantage of SNR and repeatability is proposed. Statistical and integrity analysis of SNR and repeatability are investigated carefully. Applications in WAAS and LAAS reference receivers are demonstrated in detail.

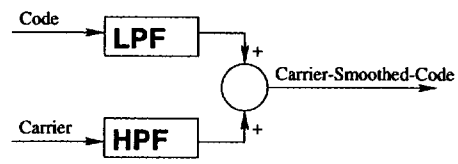

Figure 1: Carrier-Code Combination scheme block diagram.

2. Carrier Smoothing Schemes

Dual frequency pseudorange and carrier phase measurements are:

$$\begin{aligned}
\tau_{L1} &= \rho + I_{L1} + M\varepsilon_{\tau 1} \\
\tau_{L2} &= \rho + \gamma I_{L1} + M\varepsilon_{\tau 2} \\
\phi_{L1} &= \rho - I_{L1} + N\lambda_1 + M\varepsilon_{\phi 1} \\
\phi_{L2} &= \rho - \gamma I_{L1} + N\lambda_2 + M\varepsilon_{\phi 2}
\end{aligned}$$

Ionosphere-free measurement could be obtained by combining dual-frequency measurements:

$$\begin{aligned}
\tau_c &= \frac{\gamma\tau_{L1} - \tau_{L2}}{\gamma - 1} = \rho + M\varepsilon_{c,\tau} \\
\phi_c &= \frac{\gamma\phi_{L1} - \phi_{L2}}{\gamma - 1} = \rho + N\lambda_c + M\varepsilon_{c,\phi} \\
\phi_c - \tau_c &= N\lambda_c + \Delta M\varepsilon_c
\end{aligned}$$

The code measurement is noisy, while the carrier phase is precise but biased due to the integer ambiguity. So-called carrier smoothing schemes represent ways to take advantage of both.

The most intuitive explanation for combination of both measurements is shown in Figure 1, where a low pass filter (LPF) and a high pass filter (HPF) are conceptually used from the "Complementary Filtering" viewpoint. Fundamentally, all the carrier smoothing strategies are based on this concept using different low pass and high pass filter structures.

2.1. Hatch Filter

The Hatch filter is one of the most well-known and simplest schemes. It estimates the carrier phase bias, $N\lambda_c$, averaging the difference of code and carrier measurements [Wu92]:

$$N\hat{\lambda}_c(n) = \frac{1}{n}\sum_{i=1}^{n}[\phi_c(i) - \tau_c(i)] \qquad (1)$$

where, n is the length of the data set. This time averaging is just a LPF. If $\Delta M\varepsilon$ were white, zero-mean gaussian, $N\hat{\lambda}_c$ would be the maximum likelihood estimator (mle) of the carrier bias. Then pseudorange

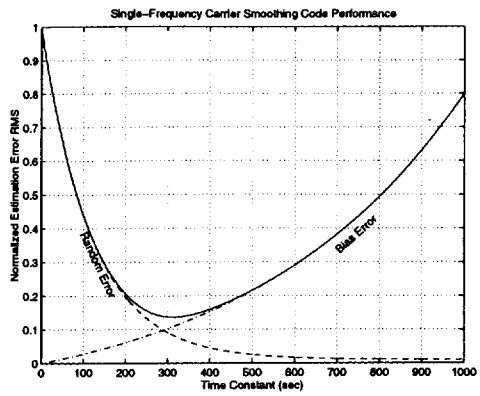

Figure 2: Carrier smoothing code performance as a function of time constant in presence of random errors (receiver noise and multipath) and bias error (ionospheric divergence).

could be estimated recursively by:

$$\hat{\rho}(n) = \frac{(n-1)}{n}[\hat{\rho}(n-1) + \Delta\phi_c(n)] + \frac{1}{n}\tau_c(n) \quad (2)$$

where, $\Delta\phi_c(n) = \phi_c(n+1) - \phi_c(n)$. This is the well-known Hatch filter.

For the single frequency case, the same concept gives a similar equation:

$$\hat{\rho}(n) = \frac{(k-1)}{k}[\hat{\rho}(n-1) + \Delta\phi_{L1}(n)] + \frac{1}{k}\tau_{L1}(n) \quad (3)$$

Here, this smoothing scheme is actually a moving average window of width k. Code and carrier ionospheric delay divergence causes the carrier bias to be unobservable from ionospheric effects. The performance of this scheme changes with the time constant (moving average window width). The estimation error of pseudorange as a function of the time constant is shown in Figure 2. The random errors include receiver noises and multipath components. Bias errors are introduced by the ionospheric divergence effects. Obviously, trade off is necessary for optimal performance.

The Hatch filter is an efficient carrier smoothing scheme. Unfortunately, error covariance information, which is critical for DGPS correction generation algorithms, is not defined. To compensate for this drawback, other filtering schemes are investigated here.

2.2. Multipath Stochastic Modeling and Filtering Schemes

As mentioned above, the critical point of code and carrier combination is to resolve the carrier integer ambiguity. Unfortunately, the carrier bias observables are corrupted by receiver noise and multipath, which are bandlimited, non-zero mean and change with elevation, azimuth angle and SNR measurements. It is well-known that the efficiency of any filtering scheme strongly depends on the nature of disturbances, dynamics of the process and error models. Here, different filtering schemes implemented for different stochastic multipath models are investigated to solve the same bias estimation problem for performance and integrity comparison.

2.2.1 Recursive Least Square (RLS): RLS method here is just a weighted version of the Hatch filter, a special case of Kalman filter. The improvement of RLS over the Hatch filter is that error covariance bounds are clearly defined. The estimates error of carrier bias and covariance bounds of RLS (Weighted Hatch filter) is shown in Figure 3. RLS act as a maximum likelihood estimator (mle) if the disturbances are white gaussian and zero-mean, and the error covariance bound shrinks down extremely fast and tight, as shown in the figure. Unfortunately, the multipath is a typical colored disturbance, so the covariance bounds given by RLS fall short from the integrity viewpoint due to the incorrect assumption in the disturbance model.

2.2.2 Kalman Filter (KF): Now we form the standard Kalman Filter. The system states include carrier bias, which is modeled by a random constant, augmented with the multipath model: a prior time-invariant second order Gaussian-Markov model or Autoregressive (AR) model.

The Kalman Filter estimation error and covariance bounds are also shown in Figure 3. Different from the RLS case, a Kalman filter with above multipath model gives more reasonable definition of the confidence interval. Obviously, a better statistical knowledge of the system disturbances is critical for both integrity and performance. The Kalman filter gives the closed form optimal solution in H_2 sense, which will be treated as a benchmark for comparison between different filtering schemes.

2.2.3 Extended Kalman Filter (EKF): An enhanced version can be implemented by using the Extended Kalman Filter(EKF), which takes the time-variant effects of multipath error into account. In EKF, the model parameters are adapted and oriented on-line. The EKF provides excellent adaptive performance when the process and measurement disturbance models are reasonably close to physical systems. Carrier phase bias estimates using this algorithm are given in Figure 3. The performance is comparable with KF results with more conservative covariance bounds, which is desirable from the in-

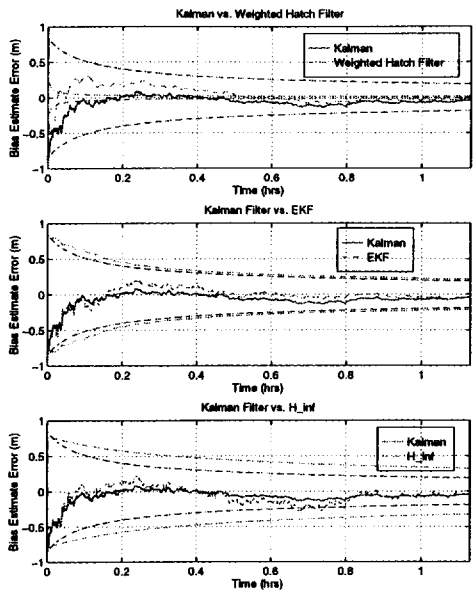

Figure 3: Comparison of different carrier smoothing schemes: RLS (Weighted Hatch filter), KF, EKF and H_∞ filter

tegrity viewpoint, but may be too conservative and numerically less robust to support sufficient availability.

2.2.4 H_∞ Filter: The optimum observers described above all require an accurate knowledge of the statistical properties of the disturbances. However, practical model uncertainties and lack of statistical information on the exogenous signals lead to interests in minimax estimators, with the belief that the resulting so-called H_∞ algorithms will be more robust and less sensitive to parameter variations and modeling assumptions. Based on H_∞ framework, a carrier bias estimation result is shown in Figure 3. Obviously, robust H_∞ filter designed for worst case gives most conservative covariance bounds with the overall performance degraded as expected.

2.2.5 Comments for Risk-Sensitive Filtering Schemes: From the risk-sensitive viewpoint, the above filtering schemes, including RLS, KF, EKF and H_∞, are ordered by increased noise model uncertainty, or increased risk-averse preference. RLS models the noise as white gaussian, which is one of the most ideal and cooperative disturbance and rarely happens in reality. Under this assumption, the error covariance shrinks too fast to hold integrity when colored multipath error dominates the disturbance budget. KF models the multipath as a time-invariant Gaussian-Markov process, which gives more reasonable covariance bound with integrity. EKF becomes more conservative, and time-variant multipath model is used. Hence the increase of risk-averse gives a wider confidence interval. For H_∞ case, the multipath and noise are assumed to work as exogenous disturbances, and can be combined in the worst case, which makes the filtering most conservative, the risk-averse preference expressed in extreme. Trade off between optimality and integrity is critical in practical applications. Multipath AR model with KF implementation is a reasonable choice.

3. Spectral Decomposition Based Multipath Mitigation Strategy

Differing from the above filtering schemes which mode the multipath from the stochastic viewpoint with integrity guaranteed, the following section investigates deterministic knowledge about multipath error seeking for optimality improvement. Based on spectral analysis of multipath error, a spectral decomposition based mitigation scheme is proposed. SNR and repeatability information could be combined naturally with carrier smoothing schemes in characterizing and mitigating multipath error based on this platform.

3.1. Spectral Analysis and Multipath Decomposition

Spectral analysis of multipath error is the fundamental concept for this technique. As described before, multipath errors are bandlimited and can be subdivided into bias-like and noise-like components. The frequency bands for those components are different. Time constants of multipath errors can have values from 1 sec to hours. The scheme to deal with various bandpass errors should be quite different to guarantee efficiency. Hence, effective decomposition of multipath error in the frequency domain is critical in multipath mitigation techniques.

An effective decomposition is illustrated in Figure 4. The multipath errors are decomposed into three components:

(1) **High Frequency (HF) Component: Noise and fast fading** mainly due to diffuse multipath, fast fading and receiver tracking errors. Its energy ranges about 40% of total residuals, with time constant 1 sec$< \tau <$1 min.

(2) **Medium Frequency (MF) Component: Multipath dominanted component** mainly due to specular dominated multipath errors. It usually contains another 40% of total energy, with time constant $1< \tau <8$ min.

(3) **Low Frequency (LF) Component: Slow fading** due to long-term bias-like multipath and drift errors, usually contains 20% total energy, $\tau >8$ min.

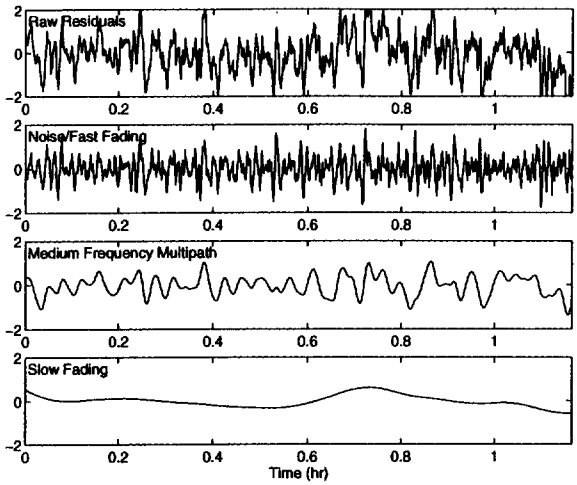

Figure 4: Measurement Residual Decomposition into three components: (1) Noise and fast fading (due to diffuse, scattering, atmosphere and unknown error sources, τ <1 min) (2) Multipath dominanted component (due to specular multipath error, which is correlated with SNR, 1< τ <8 min) (3) Slow fading (due to long-term bias-like multipath and drift errors, τ >8 min). Data from Stanford University, PRN15, January 15, 1997

This decomposition is based on the physical mechanisms of multipath errors and presents a simple way to combine the strengths from different schemes in mitigating multipath: carrier smoothing, SNR and repeatability. The following section will further detail this decomposition and the mitigation strategy follows naturally.

3.2. Multipath Calibration Strategy

Now we will further characterize each component extracted from the above decomposition scheme, and propose corresponding calibration strategies.

3.2.1 High Frequency Component (Noise and Fast Fading): Noise and fast fading are mainly due to ground and atmosphere introduced fast fading, scattering and diffuse multipath, receiver tracking errors etc., with time constant 1 sec< τ <1 min. This component presents noise-like statistics, which can be effectively averaged out by carrier smoothing schemes. Due to its bandlimited but zero-mean properties, AR model or Gaussian-Markov model is used to guarantee both performance and integrity.

3.2.2 Medium Frequency Component (Multipath Dominated Component) and SNR Calibration: Medium frequency component is mainly due to specular multipath dominated error, which is highly correlated with SNR variation, with time constant 1< τ <8 min. A typical correlation between this component and SNR is further illustrated in Figure 5, where both multipath error and SNR values are extracted within the same frequency window and put together for comparison. Obviously, the SNR bandpass components and multipath components are most in phase and present similar patterns. The relationship between them can be modeled as a linear system, but with time-variant property.

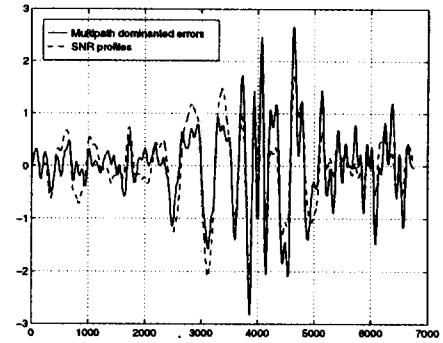

Figure 5: Multipath dominated error components vs. SNR calibration profile. Data from Stanford University, PRN14, January 15, 1997

The reason we highlight the correlation between SNR values and multipath errors within this specific frequency window band is also based on the spectral analysis of SNR. The higher frequency SNR component is corrupted by quantization errors, while the lower frequency component is dominated by antenna nominal gain pattern. The SNR at these frequencies can not be used as a reference to calibrate code multipath errors. By comparing multipath errors and SNR variations with specific frequency region, the correlation becomes clear. After identifying the linear relation between SNR and multipath error, we can effectively calibrate out this multipath component with benefits to the filtering scheme bandwidth. High bandwidth filtering is also highly desired for integrity purposes.

3.2.3 Low Frequency Component (Slow Fading) and Spline Function Calibration: Slow fading is mainly due to long-term bias-like multipath errors, with τ >8 min. Stochastic filter schemes fall short due to the long time constant of disturbances. An effective solution could benefit from the multipath repeatability at the ground reference stations.

GPS satellites are semisynchronous, with a period of one half sidereal day. Ground track geometry of each satellite at a reference station repeats with a fixed time shift of 236 seconds every day, due to the difference between sidereal day and mean sun day.

The spectral analysis shows that the most repeatable multipath errors are the low frequency components. By taking advantage of repeatability, most slow fading error can be reduced systematically. To model the repeatable multipath, spherical harmonics [Coh92], a set of linearly independent, orthogonal (without lossing generality) spline function in spherical coordinate, are specified as base functions for the reconstruction subspace, with the form:

$$s(\theta,\phi) = \sum_{l=1}^{n}\Big[J_l P_l cos(\theta) + \sum_{k=1}^{l} P_{lk} cos(\theta)(C_{lk} cos(k\phi) + S_{lk} sin(k\phi))\Big]$$

where, θ, ϕ are the elevation and azimuth angle, the accuracy is governed by the model order, n.

The effective result is because the base function only depends on satellite geometry and is less sensitive to exact orbit time shift than that in [Bis94]. This scheme provides more flexibility and robustness, which are critical for automatic data processing.

3.2.4 Synthesize Spectral Decomposition Based Multipath Mitigation: Based on spectral decomposition, carrier phase measurements, SNR measurements and repeatability are naturally connected to different multipath components. Frequency domain interpretation is illustrated by a typical example in Figure 6. This gives the fundamental platform for synthesizing multipath mitigation scheme, which can be readily implemented within a Kalman filter framework.

4. Statistical Analysis and Integrity

4.1. Multipath Mitigation Scheme Risk Analysis

The spectral decomposition based multipath mitigation scheme models deterministic multipath errors by taking advantage of their cooperative relationships with SNR and repeatability. The cooperative assumption is the baseline of this multipath mitigation scheme. From the risk-sensitive viewpoint, better performance is a natural result of the above risk-seeking assumption. The more we know about the given property (deterministic modeling), the closer we come to optimality.

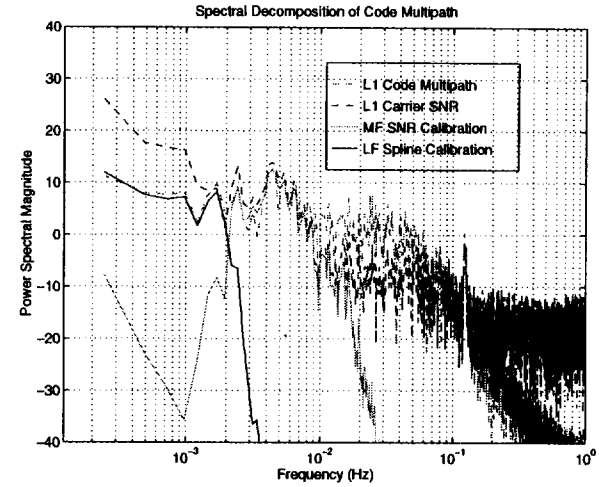

Figure 6: Spectral decomposition based multipath mitigation scheme frequency domain interpretation

The integrity of this multipath mitigation scheme relies heavily on the statistical analysis and integrity concern of SNR and repeatability in practical applications.

4.2. SNR Statistical Analysis and Integrity

Receiver SNR measurement reports carrier signal-to-noise ratio, which is highly receiver and environment dependent. Receiver dependency is dominated by antenna gain pattern and receiver processing gain, while environment dependency actually contains the information we want regarding multipath errors.

The high frequency component of SNR measurements is corrupted by quantization, and the low frequency component characterizes the receiver gain pattern. Only the medium frequency component is dominated by environmental geometry effects, which have strong correlation with multipath error. Both statistical and physical analysis support the concept presented in the decomposition based multipath mitigation scheme. To justify SNR calibration methodology, there are some questions that need answering.

4.2.1 SNR Multipath-Correction Profile Generation: SNR multipath-correction generation scheme includes the following procedures:

a. Bandpass Filtering Obtaining medium frequency SNR variation pattern is the first step. Bandpass filtering is needed to take off both nominal gain pattern and quantization error effects. Both infinite impulse response (IIR) filter or Kalman filter provide equivalent performance.

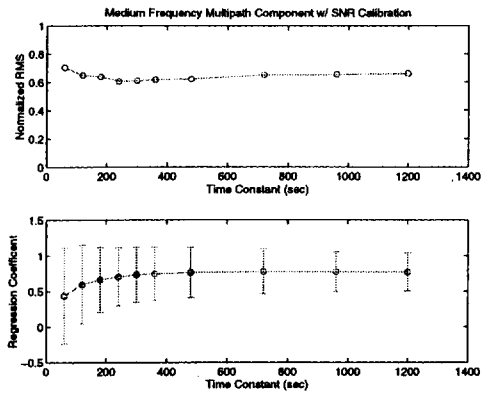

Figure 7: Medium frequency SNR-multipath correction regression coefficients and performance. Data from Stanford University, PRN06, April 15, 1997

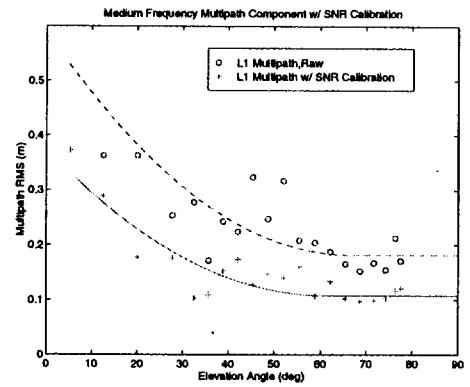

Figure 8: Medium frequency multipath mitigation performance by SNR-multipath correction scheme. Data from Stanford University, PRN06, April 15, 1997

b. Linear Regression Ratio Correlation between SNR variation and multipath error is typically a non-collocated, non-unique time-variant function. A linear regression model provides the most flexibility while maintaining simplicity:

$$\hat{\beta} = \frac{\hat{\mathbf{Cov}}(M\varepsilon, SNR)}{\hat{\mathbf{Var}}(SNR)}$$

In estimating $\hat{\mathbf{Cov}}(M\varepsilon, SNR)$ and $\hat{\mathbf{Var}}(SNR)$, the mean estimation of code multipath is of limited bandwidth, determined by the time constant. Performance and regression coefficient range are shown in Figure 7. A time constant of about 300 seconds gives optimal performance. Considering the time-variance, the ratio is identified in real-time. Typical improvements in multipath error versus elevation angles are shown in Figure 8. About 30-40% accuracy improvements are commonly observed.

For the single-frequency case, on-line regression coefficient identification is not available. A fixed prior SNR correction ratio should be selected and carefully justified. As shown in Figure 7, regression coefficient mean value approaches steady state when time constant increased. This could provide the basis for single-frequency application.

c. Integrity Protection Threshold From the integrity viewpoint, the SNR correction defined above should be bounded by a specific protection threshold for the worst case. This threshold can usually be set from prior information. Prior standard deviation of the multipath error could be used as baseline for threshold selection.

Sensitivity analysis, an example given in Figure 9, illustrates that consistent improvements are available within wide ranges of integrity protection thresholds and SNR correction ratios. Rule of thumb, the integrity protection threshold can be set to about 3-σ of multipath error, with SNR correction ratio around $\sigma_{multipath}/\sigma_{SNR}$ (approximately 0.8 in above example).

4.2.2 SNR Integrity Monitoring: SNR measurements are generally not as robust as code measurements. An SNR quality control routine is necessary to guaranteed integrity. Typical SNR measurements outages include: abrupt mean changes, outliers or breaks. Due to the high stability of SNR nominal measurements, these anomalous points can be readily repaired or flagged autonomously based on prior nominal SNR pattern calibrations. For high integrity, isolated anomalous points are usually discarded and used as whole system protection alarms. This autonomous monitoring process could markedly improve both system and SNR calibration scheme accuracy and robustness.

4.3. Repeatability Statistical Analysis and Integrity

The spline functions used to model the repeatable multipath will face the biggest challenge from environment changes due to strong dependence on local geometry. On-line monitoring is difficult due to the unobservability of these changes. Integrity must stem from reference station environment maintenance and timely calibration of the repeatable multipath spline curves. Physical ground specular reflector range from reference receiver is given in Figure 10. Conceptually, low frequency multipath component most likely relies on near-field (within meters) environment geometry,

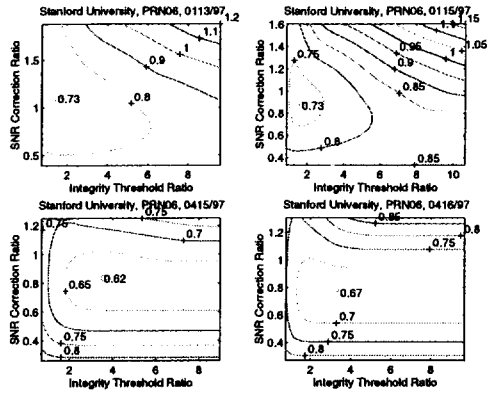

Figure 9: Sensitivity analysis of SNR correction ratio and integrity protection threshold ratio (relative to $\sigma_{multipath}$). Contour lines are the multipath residual RMS after calibration, normalized by multipath RMS before calibration. Consistent improvements are available within wide ranges of integrity protection thresholds and SNR correction ratios.

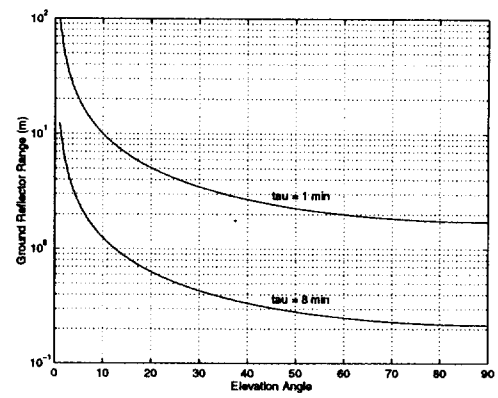

Figure 10: Ground specular reflector range (horizontal distance) from reference station receiver as a function of elevation angle of GPS satellite. Multipath is assumed to be reflected from ground reflection. Resulted multipath time constant is marked in the plot.

and is low amplitude (usually with maximum level less than 0.5 meters) for well-sited antennas. Thus, reference receiver environment maintenance is manageable and poses less of an integrity risk.

An example of the repeatability of low frequency multipath component and spline function calibration performance is shown in Figure 11. For the common case, 80-90% improvement is reasonable. Long-term temporal correlation of low frequency multipath error, shown in Figure 12, encourages timely recalibrations for this scheme.

It is noted that this strategy can only be used by WAAS, as LAAS single-frequency reference stations can not get the true ionospheric delay and full multipath observability, rendering the repeatable components unavailable.

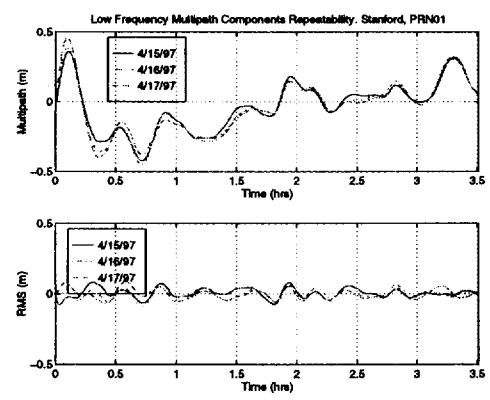

Figure 11: Repeatability of low frequency multipath error components and spline function calibration performance example with 4th order spherical harmonic fit. Data collected at Stanford University, PRN01 on April 15,16,17.

5. Applications for WAAS Reference Stations

The multipath mitigation strategy presented above was originally motivated for WAAS ground reference stations with dual-frequency receivers and well sited antennas. Both code and carrier phase measurements on $L1$ and $L2$ are available for data processing, which can accurately characterize the ionospheric delay and provide potential to reduce the multipath errors to centimeter level in asymptotic sense.

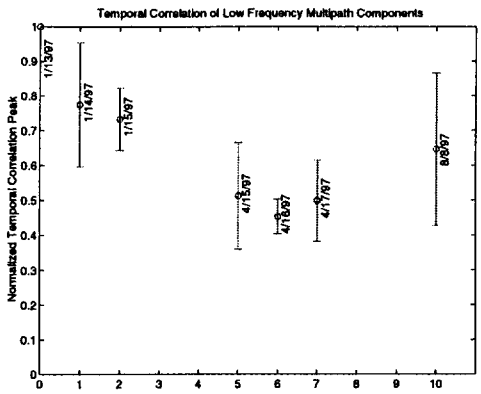

Figure 12: Temporal correlation of low frequency multipath error components. Data collected from Stanford University and Dayton reference stations, including PRN1, 4, 5, 6

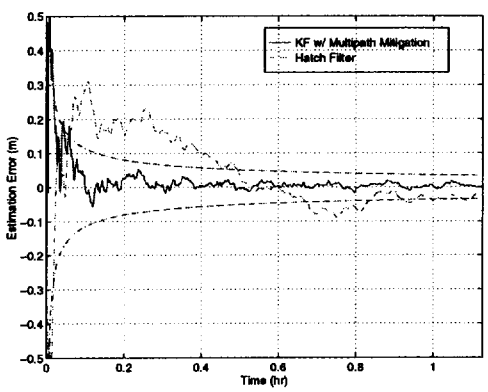

Figure 13: Performance of multipath mitigation scheme vs. Hatch Filter. Data from Stanford CA, PRN15, January 14, 1997

5.1. Implementation for WAAS application

SNR and repeatability calibration can be easily integrated into a Kalman filter scheme. Overall filter bandwidth can be two orders higher than Hatch filter with about 30% accuracy improvement. A typical filtering result is given in Figure 13. Faster convergence and improved accuracy are obvious.

Further analysis on asymptotic behavior of carrier smoothing and performance of above multipath mitigation techniques will give more insights of this synthesizing scheme.

5.2. Asymptotic Performance in WAAS Application

5.2.1 Carrier Smoothing: The mechanics of carrier smoothing stem from complementary filtering. Cutoff frequencies of code low pass and carrier

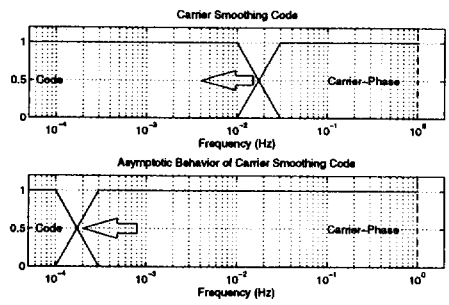

Figure 14: Frequency behavior of carrier smoothing and its asymptotic performance

high pass filters, determined by the width of the averaging window, are lower with time as more data is incoming. The frequency behavior of carrier smoothing and its asymptotic performance are conceptually shown in Figure 14. The carrier phase will dominate the strength of the combined measurements, except code DC component which is exactly the integer ambiguity of carrier. Hence, carrier smoothing are usually assumed to be unbiased to the code asymptotically.

5.2.2 Multipath Mitigation Scheme: The multipath mitigation technique decomposes the multipath error in the frequency domain. Figure 15 illustrates this filtering scheme with frequency domain behaviors. As shown, the filtered signal is a high performance combination of code, carrier, SNR measurements and daily-repeatable spline data. The carrier measurement strength still dominates the resulting high frequency response. Different from the pure carrier-smoothing method, medium and low frequency multipath error components of code measurements are notched out by SNR and spline curve. As a result, the code phase always dominates the very low frequency response, but with a very narrow bandwidth. The whole system bandwidth is dominated by the carrier smoothing parameter $k(= n)$, which defines the cutoff frequency of the equivalent carrier high pass filter. The resulting system provides high performance multipath mitigation with high bandwidth, which is greatly desirable to both integrity and accuracy.

The asymptotic performance of this multipath mitigation presents consistency with carrier smoothing. With data coming in, the carrier high pass filter cutoff frequency will decrease and finally dominate most frequency span, and perform equivalent to carrier smoothing case asymptotically.

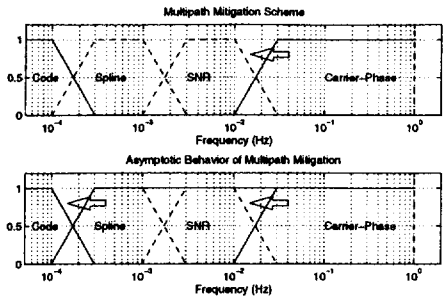

Figure 15: Frequency behavior of the multipath mitigation scheme and its asymptotic performance

6. Applications in LAAS Reference Stations

This spectral decomposition based multipath mitigation concept could also be applied readily at typical LAAS ground reference station.

6.1. LAAS Algorithm and Multipath Error

LAAS algorithm is a most exhaustive application of the differential concept. All the common errors between users and reference stations, including ionospheric, tropospheric, and satellite clocks are eliminated by single differencing. Quickly resolving the integer ambiguity is crucial for accurate carrier positioning of LAAS. The integer ambiguity is directly solved by the following difference:

$$\Delta\phi - \Delta\tau = \Delta N\lambda + \Delta M \varepsilon_{\phi,\tau}$$

where, the reference station and user multipath and receiver noise become the biggest error sources for LAAS. Physically, a static receiver, such as a reference station receiver, multipath decorrelation time constant is generally longer than that of a dynamic one. Generally speaking, in aviation applications, the user measurement errors are dominated by fast decorrelated multipath and white noise. As a result, the reference station multipath becomes the dominant limitation for LAAS accuracy. Effective mitigation of the multipath at the reference station could be one possible solution to speedup LAAS carrier positioning.

6.2. Multipath Mitigation Schemes

6.2.1 Carrier Smoothing Hatch Filter: Carrier smoothing Hatch filters are commonly used for the single frequency case. The basic equation is given by equation [3]. As mentioned before, the smoothing performance is limited by code and carrier ionospheric delay divergence, shown in Figure 2.

6.2.2 Ionospheric Dynamic Model: In above carrier smoothing case, no explicit assumption is made regarding ionospheric dynamics. Further improvement could result from deterministic knowledge of ionospheric dynamics. A dynamic model used to characterize the physical time-variant ionospheric effects has the form:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{t})\mathbf{x}(t) + \mathbf{w}(t)$$

where,

$$\mathbf{x} = \begin{bmatrix} x & \dot{x} & \ddot{x} \end{bmatrix}^T$$

$$\mathbf{w} = \begin{bmatrix} 0 & 0 & w \end{bmatrix}^T$$

$$\mathbf{F}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau(t)^{-1} \end{bmatrix}$$

Here, the acceleration of the ionospheric delay is modeled as a first-order Gaussian-Markov process. The correlation time constant $\tau(t)$ and driving noise w can be time-variant functions adjusting filter tracking ability to get better performance.

The above ionospheric dynamic model should be augmented with colored multipath error model, as described earlier, to get reasonable covariance information. The augmented system can be readily implemented as Kalman Filter.

Numerical Results Some numerical results of above algorithm present about 30% improvement over carrier smoothing. The true ionospheric delay is obtained from dual-frequency carrier phase difference, with subcentimeter level accuracy. Some numerical results are give in Table 1 for comparison. Typical ionospheric delay estimation is shown in Figure 16. Only relative ionospheric delay change is observable and matters here.

Table 1. Results of Ionospheric Delay Estimation

Filter Scheme	Error STD (m)	Peak (m)
Hatch	0.2064	0.6898
KF w/ Iono. Model	0.1327	0.5241
Hatch w/ SNR	0.1824	0.6527
KF w/ Iono. + SNR	0.1339	0.5059

Here, the Hatch filter time constant is 300 seconds. Obviously, the Kalman filter scheme with dynamic ionospheric model can improve the accuracy as expected.

6.2.3 SNR Aiding Multipath Mitigation: The decomposition based multipath mitigation concept can be readily applied here. Unfortunately, only

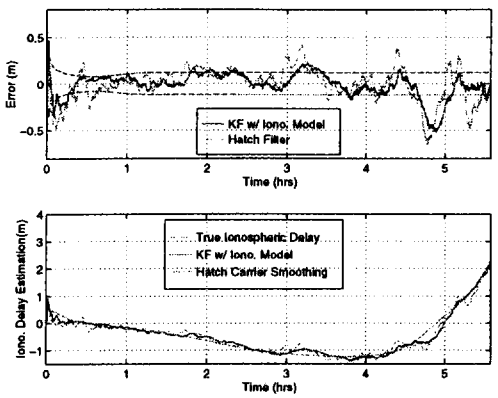

Figure 16: Typical results of comparison of ionospheric delay estimation between Hatch carrier smoothing and Kalman filter with ionospheric dynamic model and multipath model. Data from Stanford University, PRN01, January 15, 1997

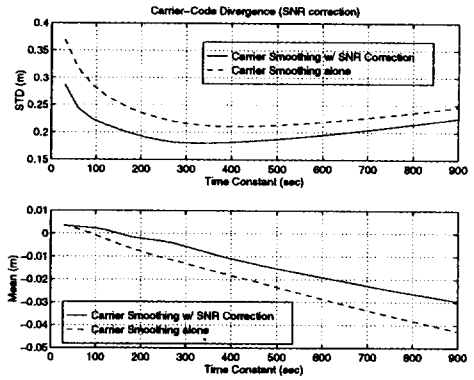

Figure 17: Typical results of performance comparison of carrier smoothing with and without SNR aiding multipath correction scheme. Data from Stanford University, PRN01, January 15, 1997

SNR aiding multipath calibration is available for the single-frequency user. Unlike the dual-frequency case, the true ionospheric delay and true multipath error can not be accurately known, nor can the repeatability. Here, SNR calibration performance is verified by dual-frequency measurements.

Numerical Results Numerical results demonstrate the improvement by SNR correction method for single-frequency receiver. Figure 17 shows typical improvement over simple carrier smoothing code by using the SNR correction. Consistent improvements, tighter variance bounds and less mean drifts, are achieved with different time constant. SNR correction profiles are generated using the simplified scheme describe before. The SNR corrections are also applied to the Kalman filter with dynamic ionospheric model. Results are also shown in Table 1. Improvement from SNR calibration for Kalman filter case is not obvious. Ionospheric dynamic model scheme can reach a much longer time constant than the Hatch filter, which effectively smoothes out medium frequency multipath, causing SNR calibration effects to fade out.

6.3. Dual-frequency LAAS Reference Stations

Compared with the single-frequency and dual-frequency receivers described above, obvious advantages of the dual-frequency receiver over its single-frequency counterpart are not only the observability of ionospheric delay, but also the potential to mitigate the multipath errors down to centimeter level. If the dual-frequency receiver could be equiped at LAAS reference station, so-called multipath-free pseudorange correction could be available to LAAS user with enhanced performance and integrity.

In recursive form, in carrier smoothing sense, multipath-free pseudorange is given by

$$\begin{aligned}\hat{\tau}_{L1}(n) &= \frac{1}{n}\tau_{L1}(n) + \frac{n-1}{n}[\hat{\tau}_{L1}(n-1) + \Delta\phi_{L1}(n) + \\ &\quad \frac{2}{\gamma-1}(\Delta\phi_{L1}(n) - \Delta\phi_{L2}(n))]\end{aligned}$$

The above mechanics are the same as those in WAAS algorithms, and spectral decomposition based multipath mitigation techniques can be further applied here.

Furthermore, the ionospheric effects could be accurately estimated within dual-frequency LAAS senarios. An independent ionospheric integrity monitoring for WAAS could be readily available. In future implementations, dual-frequency LAAS stations could also be augmented as a reference station into WAAS network. And LAAS and WAAS system could be integrated naturally to provide high performance and integrity, serving as a primary means navigation system.

7. Conclusion

This paper presents a spectral decomposition based multipath mitigation technique, which combines carrier smoothing, SNR and repeatability. Multipath stochastic modeling, SNR and repeatability aiding schemes are investigated from the integrity viewpoint in detail. Applications in WAAS and LAAS reference station receivers are demonstrated.

The conclusions based on this research include:

- Optimal combination of carrier and code measurements should be augmented with correct stochastic model of colored multipath error for reasonable covariance bounds with integrity. Investigations of different filtering schemes, including RLS, KF, EKF and H_∞ filter, provide a good reference. Multipath AR model with KF implementation gives a reasonable trade off between performance and integrity.
- Multipath Spectral Analysis characterizes the multipath error by decomposing it into three parts: high, medium and low frequency components. Spectral decomposition based multipath mitigation technique introduces the combination of carrier smoothing, SNR and repeatability.
- SNR and repeatability statistical analysis are detailed from the integrity viewpoint. About 30-40% of medium frequency and 80-90% low frequency multipath components can be removed by SNR and repeatability calibration schemes with high integrity.
- In WAAS applications, this multipath mitigation technique can improve both the filtering bandwidth by an order of two and 30-40% on accuracy over Hatch filter. Asymptotic behavior analysis verifies the consistent performance with carrier smoothing. Practical implementation is detailed and demonstrated by numerical results from NSTB TRS data.
- Multipath is the biggest error source at LAAS reference receivers. Dynamic ionospheric model and SNR aiding multipath correction schemes are proposed and verified by numerical results. About 30% accuracy improvement over the Hatch filter is demonstrated.
- Dual-frequency LAAS reference station design can most effectively mitigate code multipath corruptions. High integrity multipath-free pseudorange measurements/corrections could be available on such a platform. Potential integrity check for WAAS corrections, serving as LAAS/WAAS dual-functional station, or future integration with WAAS networks would be natural follow-on.

8. Acknowledgments

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To access this paper via internet, use `http://www-leland.Stanford.edu/~ dhdy/ion97`. Comments or questions may be directed to *dhdy@leland.Stanford.edu*.

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