GPS Attitude Determination for a JPALS Testbed: Integer Initialization and Testing

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***Abstract*- An attitude-based search algorithm was implemented for initial ambiguity resolution and integer determination, and then tested on a three-element equilateral array with baselines of 0.5m. Running the initialization algorithm at every epoch, it was found that the search algorithm gave efficient and reliable integer solutions for a static antenna array, but was sensitive to errors in a dynamic environment in the absence of multi-epoch filtering or solution checking procedures. Incorporating knowledge of allowable array orientation into the search greatly increased execution speed and reduced spurious integer estimates.**

This paper reviews the fundamentals of multi-antenna GPS-based attitude determination, develops in detail a simple and efficient 3-D search algorithm, discusses tradeoffs between execution speed (search spacing), signal phase noise, and estimate reliability, and presents results from static and in-motion automotive testing. Based on these findings, the utility of integration between GPS and inertial systems for robust attitude determination cannot be overstated.

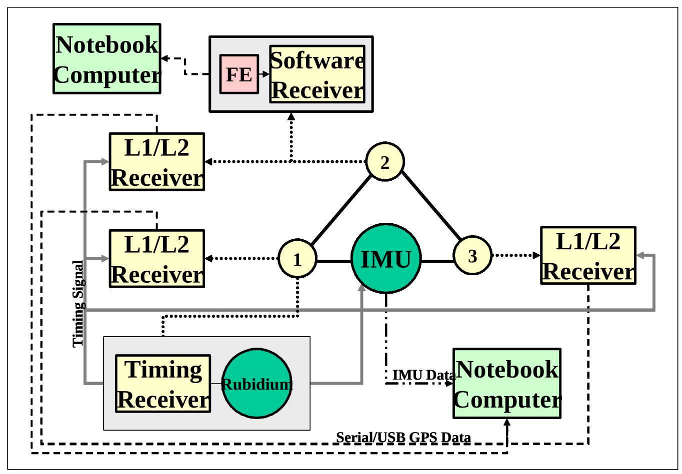

Fig. 1. GPS/INS Testbed.

I. Introduction

Stanford University is leading a multi-disciplinary, multi-university team (which includes The Illinois Institute of Technology and the University of Minnesota, Twin Cities) in support of Joint Precision and Approach Landing System (JPALS) system definition and trade studies. JPALS is a United States Navy and Air Force project to provide local-area augmentation to pilots for aircraft carrier, fixed base, and tactical airfields.

For JPALS, Stanford University is developing a research testbed (Fig. 1) to study GPS/INS integration methodologies for integrity monitoring, shipboard reference-station antenna motion compensation techniques, and Doppler aided tracking loop performance. One component of this testbed is a multi-element antenna array, suitable for GPS-based attitude determination, in conjunction with an inertial measurement unit. Precise attitude knowledge will be an important part of the performance evaluation.

II. Attitude Determination & Initialization

The fundamentals of GPS-based attitude determination are well covered in the literature, with [i] being the standard reference. Consequently, a rather brief overview is all that will be required in this paper.

In two dimensions, the determination of orientation by using measurements of the phase of incoming plane waves proceeds logically (Fig. 2 – the index i corresponds to baseline and j corresponds to satellite). Two antennas, by convention labeled "master" and "slave", define a baseline b coordinatized in a body-fixed basis. The carrier wave from a far-distant source, in this case a GPS satellite, is incident at each antenna; accurate measurement of the arrival phase φ is made simultaneously (or nearly so) at each antenna. The precise distance to the GPS satellite, and hence the exact whole number of carrier wavelengths, is not known without additional processing of the GPS signals (e.g., L1/L2 processing); this whole number ambiguity can be treated as a

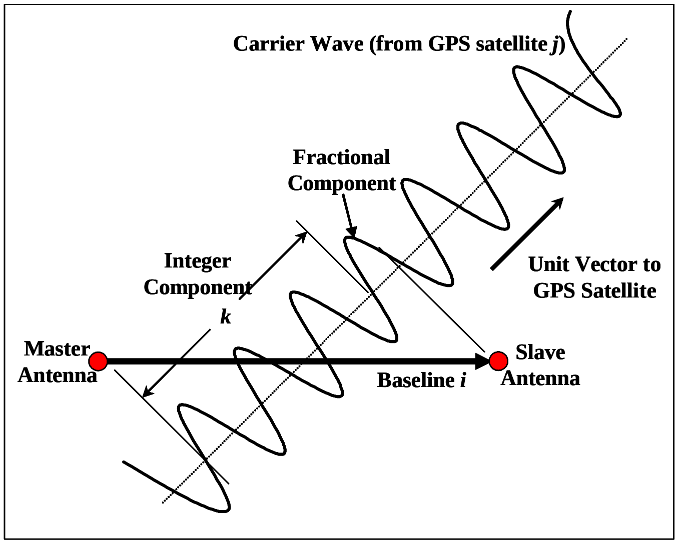

Fig. 2. Attitude determination – 2-D development.

random integer while lock is maintained. By taking the single-difference between the phase-plus-integer value at each antenna for several satellites ($\Delta\varphi+k$), the orientation A (a 3x3 transformation matrix) between the body-fixed basis and the external reference system can be found (Fig. 3). In addition, there may be some differential line bias, signal delay, or measurement asynchrony B between the signals measured at each receiver, as well as measurement noise v. The addition of a third antenna, defining a second baseline noncolinear with the first, allows a straightforward extension to three dimensions. (Hereafter, to simplify notation, all dimensions on the right hand side of (1) will be scaled by the L1 carrier wavelength.)

$$\Delta r_{ij} \equiv \left(\Delta\varphi_{ij} + k_{ij}\right)\lambda = \vec{b}_i^{\,T} A \hat{s}_j + B_i + v_{ij} \qquad (1)$$

Normal multi-antenna 3-D GPS attitude processing operates epoch-by-epoch according to (1) with the final product being an attitude transformation matrix between the external basis (e.g. ENU) and the body-fixed basis, as well as the antenna line biases (Fig. 4). The phases of the incoming carrier wave signals at each of the antennas are measured, satellite ephemerides are decoded from the navigation message, and baseline geometry is available from previous survey or calibration. Given knowledge of the number of integer wavelengths along each baseline for each satellite in view, (1) is solved by, for example, least-squares minimization of an appropriate cost-function [i] or deterministic (closed-form) attitude and bias updates [ii]. In practice, a deterministic attitude solution, such as that shown below based on a Singular Value Decomposition, may display better convergence properties and faster execution speed than least-squares techniques [ii].

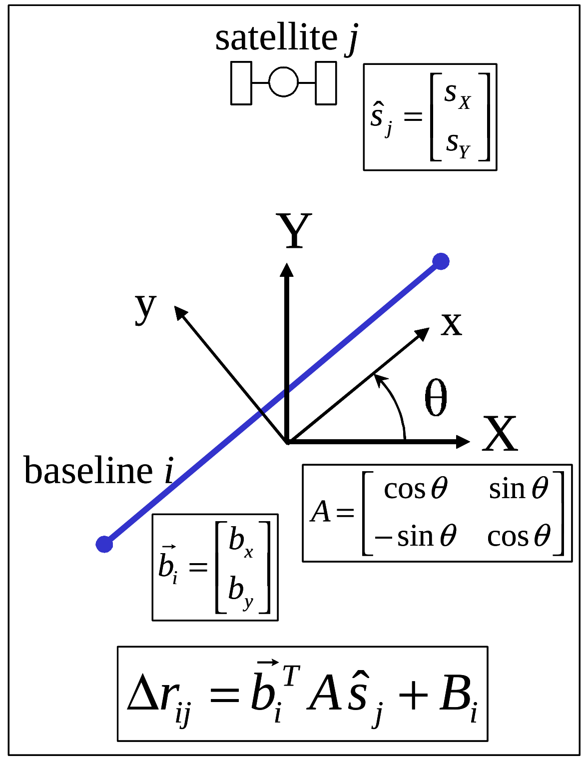

Fig. 3. Attitude determination and 2-D equation model.

$$\begin{aligned} G &= \mathbf{b}\left(\Delta\varphi + \mathbf{k} - \mathbf{B}\right)\mathbf{s}^{-1} \\ U\Sigma V^{T} &= SVD(G) \\ A &= U^{+} V^{+T} \end{aligned} \qquad (2)$$

$$\textit{where } X^{+} \equiv X \begin{bmatrix} 1 & & \\ & 1 & \\ & & |X| \end{bmatrix}$$

$$\textit{and } \mathbf{s}^{-1} \textit{ is a pseudoinverse}$$

Of course, if poor bias estimates B are used in initial attitude determination, it may be desirable to re-estimate biases after least-squares or closed-form calculation of A. There is also the possibility of performing integer re-initialization for occasional cycle-slips or for newly-risen or acquired satellites, although this occurs infrequently and leverages an attitude solution that has already converged.

The attitude initialization problem, where the external-to-platform orientation, the various integer ambiguities, and the antenna line biases are unknown, is a key challenge to GPS-based attitude determination. Because the measurements of phase are modulo one wavelength, it is not possible to solve analytically for the integer ambiguities. There are several general classes of methods presented in the literature to accomplish this ambiguity resolution, each with tradeoffs between complexity, convergence time, and robustness.

One set of methods relies on platform or satellite constellation motion to achieve sufficient observability of the cycle ambiguities; after constraining the cycle ambiguities to integer values an attitude solution is reached [i, iii, iv]. While generally straight-forward to implement, these algorithms face a number of disadvantages. First, an initial attitude solution must wait ten or more minutes for the satellite constellation to change sufficiently to reach a solution, or platform motion must be prescribed to achieve the same observability. Neither of these options is desirable for a test environment. Further, it may be operationally prohibitive to execute large heading changes solely for the purposes of integer acquisition.

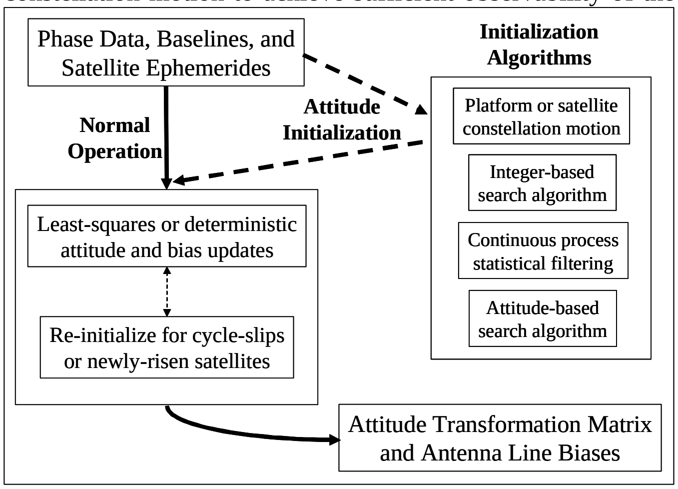

Fig. 4. Attitude processing and ambiguity resolution.

Another set of methods rely on exhaustive searches through possible integer combinations; when a minima of the attitude determination cost function is reached, the corresponding integer solution is used to solve for the platform attitude [v, vi, vii]. Of course, knowledge of the baseline geometry allows exclusion of a vast number of potential integer candidates. Also, a search tree may be traversed, with branches pruned when the attitude determination cost function exceeds a predetermined limit [vi], greatly improving search speed. Integer-based search methods can either become intractable for longer baselines, or the exclusion and pruning logic may grow quite complex.

There are also methods wherein the ambiguities are treated as continuous random variables, and then statistical filtering allows convergence to the desired integer values [viii, ix]. These methods tend to be complicated, and require many epochs of data for filter convergence. Neither of these properties was considered desirable for a test environment.

Finally, there are attitude-based search algorithms, where the attitude transformation space of possible orientations between the external and body-fixed bases is used to calculate integers and determine a cost function [x, xi, xii]. These methods operate at each epoch, allowing true single-epoch integer determination. Also, there are no complicated validity checks on baseline geometry, since true baseline geometry is always exploited in the generation of candidate solutions. All satellite data is used at each epoch, allowing for easy scaling with greater numbers of satellites or baselines. Most importantly for a test-and-development environment, attitude-based search methods are reasonably fast and noise resistant, are not too involved or complicated to develop/implement, are easy to understand and debug, and have excellent prospects of correct integer determination with multi-epoch processing. Finally, perhaps the greatest benefit to be realized from an attitude-based initialization algorithm is during cycle-slips or other short-term GPS attitude outages. In these cases, a reasonably good estimate of current platform attitude will be available, perhaps a very accurate estimate in the case of integration between GPS and INS subsystems; an attitude-based search is naturally suited to utilize this information to speed the integer determination process.

III. 3-D ATTITUDE SEARCH AND AMBIGUITY RESOLUTION

A search-based ambiguity resolution algorithm tests a series of candidate solutions (whether integer combinations or body orientations), constructs an appropriate cost function, and searches for a global minimum. But what does the "landscape" of a representative cost function look like? For example, consider a simple rearrangement of the measurement equation (1), giving the following cost function form:

$$J(A) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left(\Delta\varphi_{ij} + k_{ij} \right) - \left(\vec{b}_i^{\,T} A \hat{s}_j + B_i \right) \right)^2 \qquad (3)$$

The cost function value versus orientation of a single baseline can be plotted. For a single 2-wavelength baseline with true orientation at 0° azimuth and 0° elevation, 8 satellites randomly placed in the sky, and a search conducted over all azimuth and ±30° elevation, the cost function above yields the surface plot shown in Fig. 5. It is this landscape over which a search for the global minimum occurs – and it must be emphasized that it truly is a search, since gradient-based or similar methods will simply fall into the deepest local minima as shown by the inset contour plot. The other inset plot is a cut along the "equator" at 0° elevation, showing the existence of a global minimum at 0°/0°, as well as the relative depths of local minima for this example configuration. The addition of phase noise on the measurements causes a reduction in the depth of the global minimum, and an increase in the depths of the local minima, making solution identification problematic in the presence of significant phase noise. It is hoped that the cost function plot in Fig. 5 sufficiently motivates the challenges of integer identification and ambiguity resolution for 3-D GPS-based attitude determination.

The simplest way to visualize an attitude-based ambiguity search algorithm is to imagine all possible orientations of a body-fixed basis with respect to an external basis. Possible transformations may be created by, for example, considering a series of successive rotations (Euler angles), a rotation about

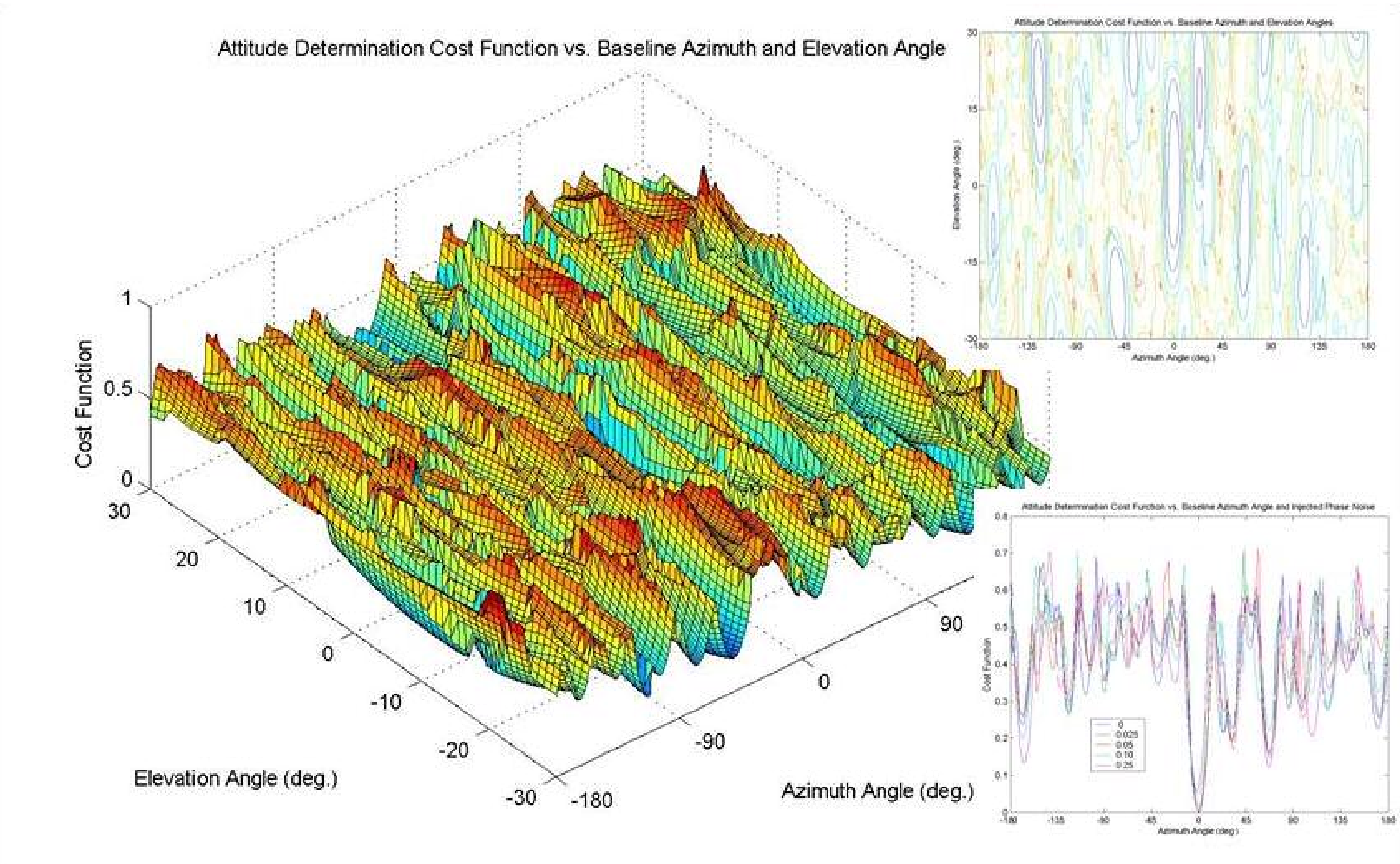

Fig. 5. Cost function plot vs. single baseline orientation, 8 SVs visible and true orientation at 0°/0° (upper insert shows contour plot of main graph, lower insert shows sectional cut at 0° elevation with successively greater injected measurement noise).

an axis fixed in either basis (Euler axis/angle), a quaternion representation of attitude (Euler symmetric parameters), or a direction cosine matrix [xiii].

However, conducting a search basis-to-basis is needlessly inefficient, as any attitude representation requires at least three parameters over which a search must traverse. Rather, it is possible to first determine candidate orientations of a single baseline, parameterized by azimuth and elevation only. A cost function is calculated for each candidate orientation, and the smallest values of the cost function represent orientations of the first baseline that are most likely to agree with reality. Then, it is possible to test only those orientations of the second baseline that match the pre-surveyed baseline geometry, by using the first baseline, now fixed, as an Euler axis about which the second baseline is rotated (Fig. 6). The process proceeds as follows:

1. Determine cost function values for each orientation of the first baseline
2. Select that orientation with the lowest cost function value
3. Determine cost function values for each geometrically permissible orientation of the second baseline
4. Repeat 1-3 for the second baseline
5. Repeat 2-4 for successively higher values of the cost function found in #1 on each baseline, until some predetermined search depth is reached

Note that the process described here can be terminated

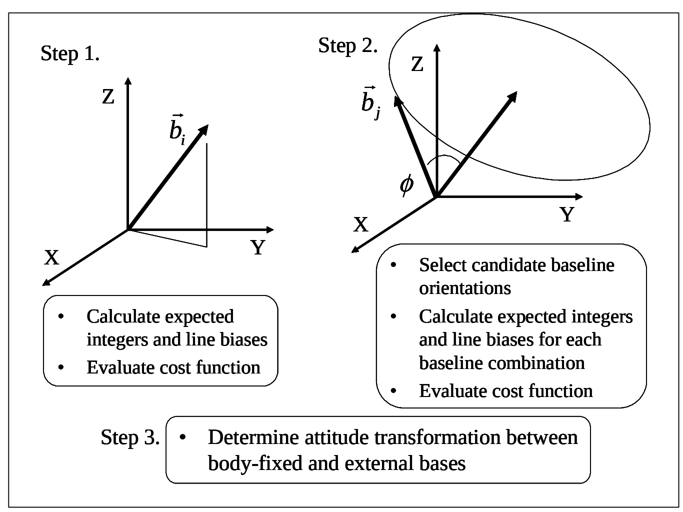

Fig. 6. Attitude-based search algorithm.

whenever a sufficiently small value of the cost function is reached. As indicated, this method has several tuning parameters, including the termination value of the cost function, the number of baseline orientations d evaluated in steps 2-3, and the search grid angular spacing a. Consequently, this search has a computation time that goes as (a^2+a) and is bounded by $2(a^2+ad)$ (where a is a measure of search grid angular spacing and d is a measure of search depth) when both baselines are used to seed the search process, rather than as a^3 or even a^4 when se eking the complete transformation in a single step.

The search algorithm model presented here adapts the standard measurement equation (1) to compute integers and line biases in a single step. Fundamentally, this attitude-based search knows or generates guesses for all terms on the right hand side of (4), and then evaluates the variance on the bias estimates required to fix the k_{ij} to integer values. The bias estimates B_i are those values that, when added to each element on the right-hand-side of (4), leave numbers that are as close as possible, on average, to integers. It is the variance on the bias estimates that forms the cost function in the method just described – the more successful the candidate A matrix is at producing phase values close to the $\Delta\varphi_{ij}$, the easier it will be to make integers k_{ij}, and the lower the variance on the residual of B_i. This variance is analogous to the noise term v_{ij} from (1).

$$k_{ij} - B_i = -\Delta\varphi_{ij} + \vec{b}_i^T A \hat{s}_j \tag{4}$$

In essence, an attitude-based search places a grid over the landscape displayed in Fig. 5 and evaluates a cost-function at each grid point. Ideally, the grid is constructed at constant solid-angle, so that there is not an increased density of grid points at higher elevations in the search domain. Also, it proves helpful to offset each search point slightly from a deterministic grid (i.e., pseudo-randomize the search), so that a correct solution missed during one epoch of the search may show up in a successive epoch, without having to make the search grid unnecessarily fine. The point here is that it is more efficient to operate with a slightly coarser grid, which executes substantially faster than a fine grid, and suffer a higher error rate on integer solutions, yet be able to process many more epochs of data even in real time, than to have the grid resolution exceedingly fine, spend precious time executing the $2(a^2+ad)$ search, and still have incorrect solutions due to phase noise, multipath, or poor satellite geometry.

An evaluation of the trade-offs between single-epoch integer determination error rate and median time to reach a solution, as a function of search grid spacing, makes this point more clearly (Fig. 7). For search grids of 0.2λ to 0.8λ, a simulation was run wherein the integers and initial attitude were estimated for a 3-antenna array, configured as two 2λ baselines at right-angles and with 8 GPS satellites in view. For random starting attitudes and line biases, noise was injected on the measured carrier phase; for the results shown here, this noise was zero-mean with a standard deviation of 0.025λ (although the findings are consistent with noise standard deviations of up to 0.1λ). The minimum median time to solution was achieved for a 0.4λ search grid, with a 0.3% error rate on the integers and attitude estimates. Making the search grid larger actually causes an increase in the time to reach a solution, since the first likely candidate orientations do not lead to a sufficiently low value of the cost function. In other words, the search is not likely to land sufficiently close to the correct solution to reach the termination value. It is tempting to seek a converged solution on the initial rough attitude guess produced by a coarse pseudo-random search, either by least-squares or deterministic means, essentially seeking the local minima over the search grid; however, the time taken to converge is actually longer than simply increasing the granularity of the search process. Conversely, increasing the fidelity of the initial search causes a rapid growth in solution time, which is not matched by corresponding gains in solution reliability. Therefore, desirable search grid spacing for attitude initialization will be strongly dictated by practical considerations such as antenna baseline length, measurement noise, multipath, and the number of visible satellites.

IV. Experimental results

A. GPS-Attitude Testbed

Evaluation of the attitude determination algorithms proceeded on static and in-motion data from a 3-element antenna array, with equilateral 0.5m baselines and mounted on the roof of a small sport utility vehicle. The equipment was arranged functionally according to Fig. 1, although data from the IMU were not required for the evaluation discussed below. Each antenna fed its signal to a NovAtel GPS receiver, with data collection synchronized at a 1Hz rate from an external timing source. Data were passed via USB interface to a laptop computer for storage and post-processing analysis. Data packets written to disk included pseudorange, carrier phase, and satellite ephemerides for each satellite/antenna combination. In all, approximately 24 minutes of data were collected; the vehicle was stationary for the first portion of the test, and started driving after 12 minutes. The static portion of the tests correspond to the vehicle parked outside of a large

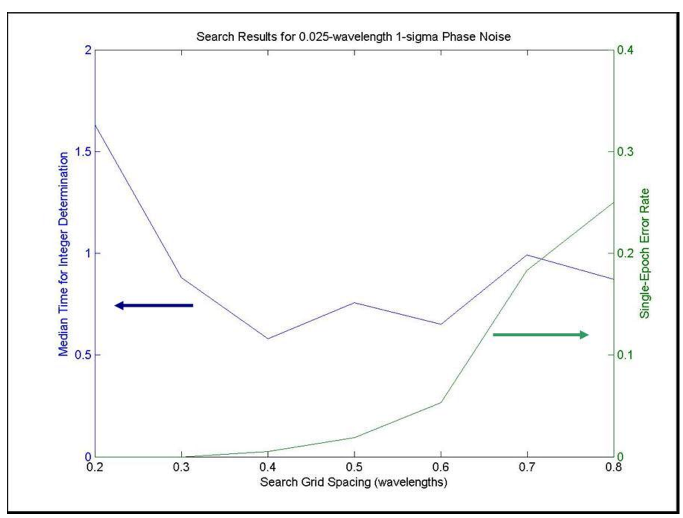

Fig. 7. Ambiguity search trade-offs.

parking structure at Stanford University, while in-motion data were collected navigating a counter-clockwise loop about campus (Fig. 8), an environment with 1-3 story buildings and many tall trees.

B. Data Collection

The integer determination and ambiguity resolution algorithms operated at every epoch that provided sufficient satellite visibility to reach a position solution. It should be emphasized that the goal was evaluation of the ambiguity resolution algorithm *per se*, not navigation, attitude determination, or the ability of discrimination, sanity checking, or multi-epoch filtering to distinguish incorrect solutions. Various combinations of the previously mentioned tuning parameters were tested, with a final search grid of 0.3λ, search depth of three (meaning that 3 candidate orientations of each baseline were utilized as Euler axes to generate potential transformation matrices), and termination value of the cost function of 0.010. This set of parameters yielded a reasonable compromise between search speed and attitude determination success.

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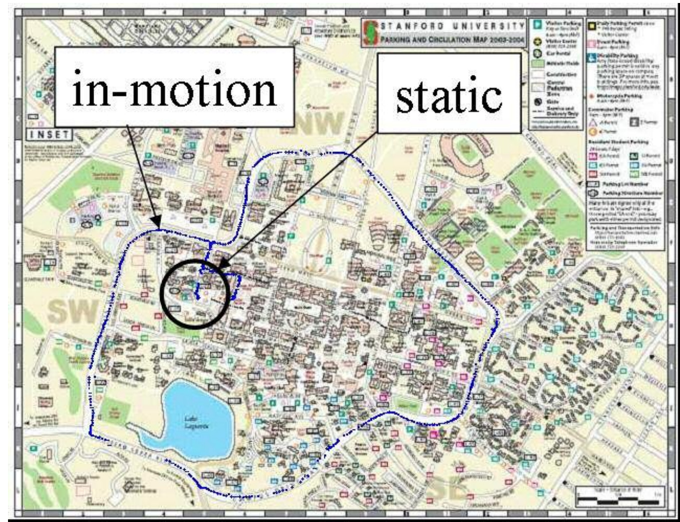

Fig. 8. Vehicle tests – 10/26/2003.

C. Analysis and Results

The roll/pitch/yaw time series as determined by the initialization algorithm are displayed in Fig. 9, along with the number of satellites visible and available in reaching that

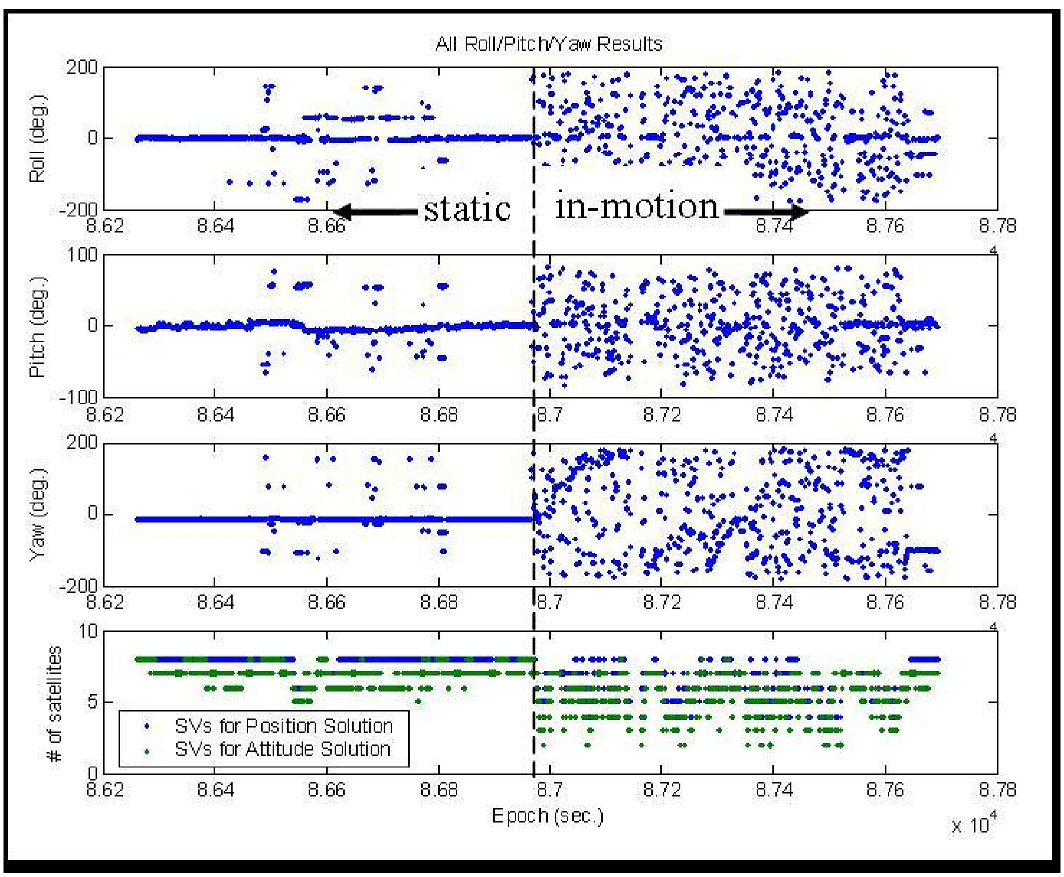

Fig. 9. Attitude initialization for static and in-motion data.

solution. The initial part of each chart shows solid prospects of correct integer determination while stationary; approximately 90% correct for this dataset and choice of tuning parameters. However, ambiguity resolution running at every epoch shows extreme vulnerability to incorrect solutions while the vehicle is in motion, as seen by roll and pitch predictions that span the available attitude space; of course, this is prior to any filtering, multi-epoch averaging, or applying any solution quality/sanity checks. Disallowing any integer combination that requires bias estimates greater than, for example, 3σ from the mean value while the vehicle was stationary still lets many poor integer estimates through. Applying a sanity check on vehicle pitch/roll angles also cleans up the ambiguity estimates, but by this time there is hardly any in-motion data left.

A more effective approach is using knowledge of platform attitude to constrain the initial search. For example, in an automotive application it is reasonable to assume that the vehicle is nominally level; starting with this requirement on baseline orientation greatly improves integer search performance. For example, constraining roll and pitch values to zero in the integer search and decreasing the search grid spacing from 0.3λ to 0.2λ (now in azimuth only) improves the correct solution rate above 98% for the static case and also allows a >10X speed improvement (Fig. 10a). In-motion data still shows the previous sensitivity to incorrect solutions, although this has been improved substantially and can be mitigated further by disallowing estimates that require roll or pitch values greater than some threshold, for example excluding epochs that require roll or pitch greater than 6σ from the mean (Fig. 10b).

At this point, it is instructive to investigate the underlying mechanisms that cause incorrect integer solutions. As mentioned previously, this occurs predominantly while the vehicle is in-motion and is associated with a reduction in the number of satellites visible by all antennas. What happens is that the cost function formed on the data at a particular epoch becomes a poor tool to estimate platform attitude – in fact, due to reduced satellite visibility and noise/multipath it will no longer be possible to detect a correct attitude and integer solution for that epoch. For example, at epoch 86,263 the vehicle was stationary and there were 8 satellites visible; making a cost function plot similar to Fig. 5 as a function of azimuth and elevation, and rotated so that there is again a correct solution at orientation 0°/0°, shows that there is indeed a well-pronounced global minimum (Fig. 11a). The cost function value calculated according to (3) reaches down to 0.023 in this minimum. However, while in-motion at epoch 87,146, satellite visibility drops to 4 (due to obstructions), and the cost function appears as in Fig. 11b. Now the global minimum, which reaches even lower to 0.002, does not correspond to the correct solution at 0°/0°. In fact, there is generally less structure (richness) in this plot due to fewer satellites available, and consequently less ability to withstand noise/multipath on the phase measurements caused by an obstruction-rich environment. There is no way to converge to a correct solution for this epoch, even by reducing the size of the search grid, since a cost function formed on the available data does not display a global minimum in the correct orientation; this is indeed the case for virtually all epochs that return an incorrect integer estimate. Multi-epoch filtering, solution-checking, or external inertial aiding would be a critical requirement if attitude initialization were to commence at this point.

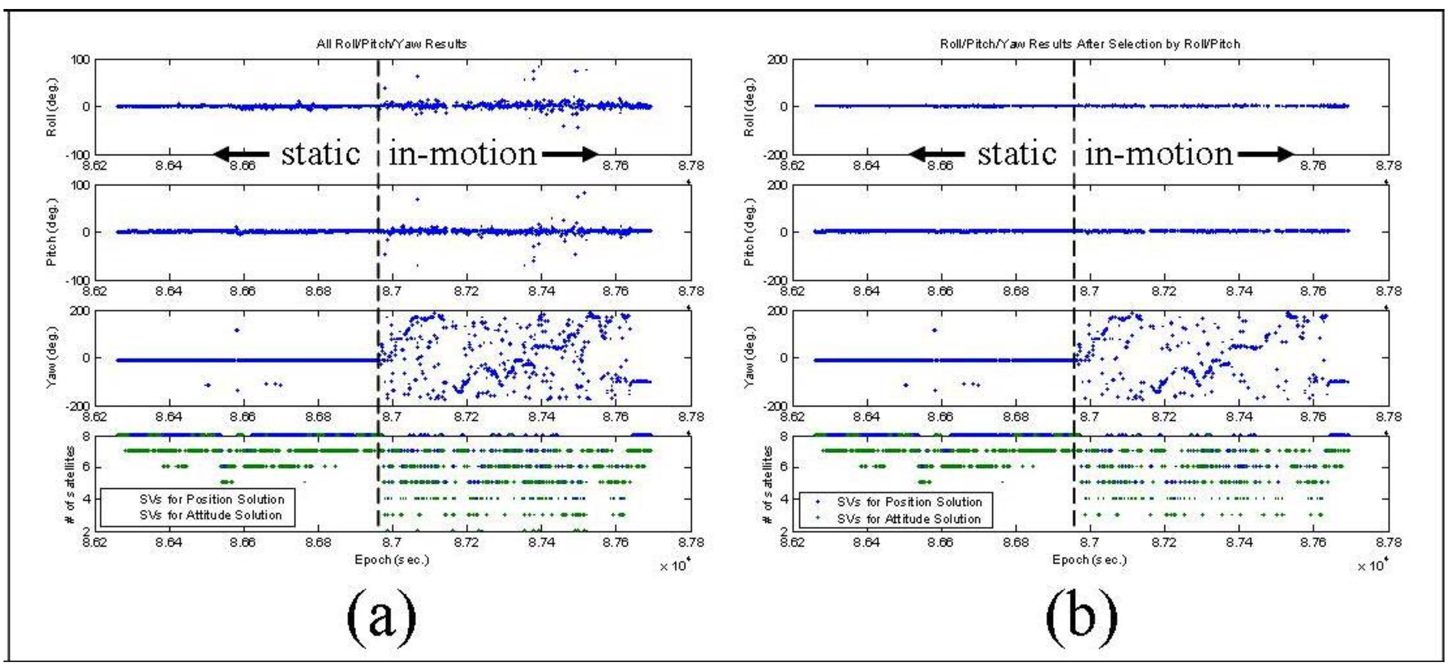

Fig. 10. Data for level-platform initialization assumption and roll/pitch data selection.

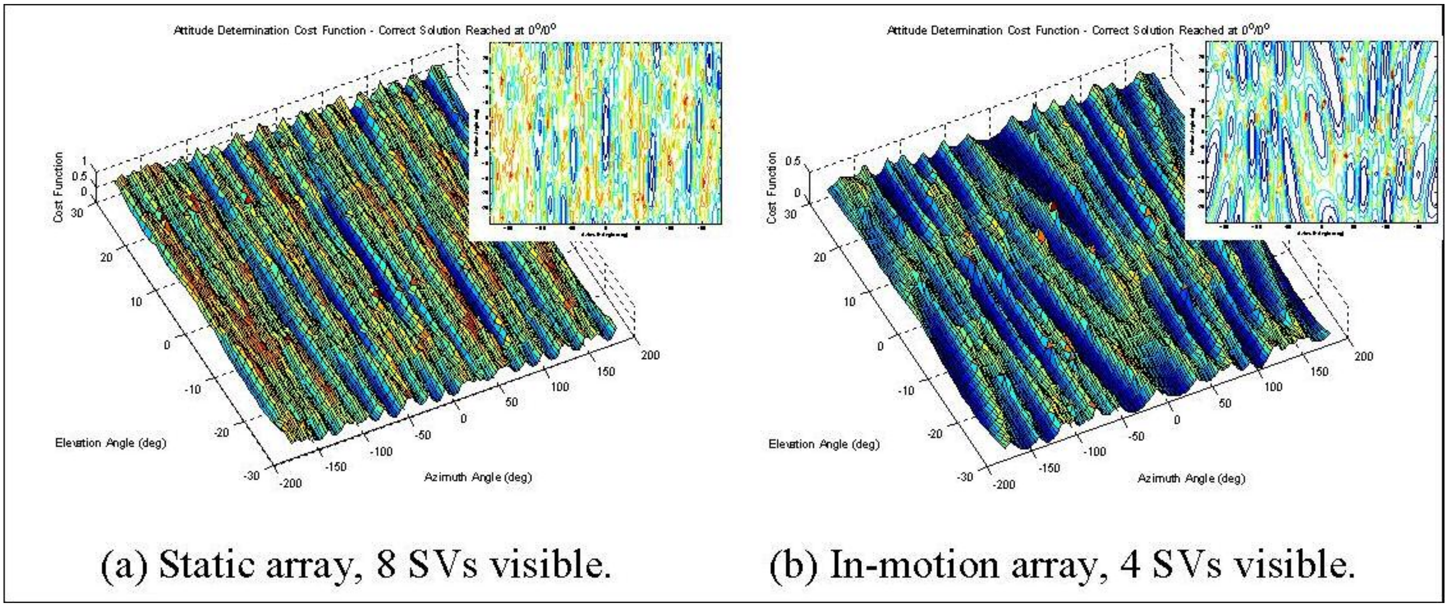

Fig. 11. True cost functions for static and in-motion data.

V. Conclusions

The goal of this research was to evaluate *attitude initialization* through single-epoch integer determination. It has been shown that an attitude-based ambiguity resolution algorithm is an efficient means of initialization in GPS-based attitude determination, particularly for static platforms. There is, however, a great sensitivity to noise, multipath, and reduced satellite visibility for in-motion platforms in an urban environment; multi-epoch averaging or sanity checks have the potential to greatly improve the integer determination robustness.

Another method of increasing solution reliability is to constrain the initial search to those attitudes most likely to be true – such as searching horizontal baseline orientations for an automobile application [x] or using inertial aiding to estimate platform attitude [x, xi].

There also are ways to use single-antenna GPS velocity and filtered acceleration information to infer platform attitude [xiv, xv]. For aircraft, if one assumes coordinated flight (no side-slip), then the velocity vector is aligned with the longitudinal axis and the acceleration vector (including the effect of gravity) is aligned out the aircraft belly; both estimates different from truth by the angle of attack. Conversely, for a land-based platform, it may be sufficient to assume zero roll angle and that the longitudinal vehicle axis is aligned with the velocity vector. In either case, it should then be possible to use the single-antenna attitude estimate to seed the integer ambiguity search for the multi-antenna attitude solution. This method of initialization is an area for future research.

Once initialization has been accomplished, a GPS-based attitude system is still susceptible to cycle-slips or loss of satellite coverage. In addition, the attitude update rate from a GPS navigation system is usually on the order of 1Hz, usually insufficient for closed-loop vehicle control. For these reasons, there is a great advantage to be realized by integrating GPS and inertial systems for robust attitude determination.

A further application of the hardware and algorithms described above is for antenna phase-center calibration. With a simple modification to the measurement equation (1), it is possible to use natural satellite constellation motion and planned reorientation of the antenna array to determine the azimuth- and elevation-dependent differential phase delay along each antenna baseline. For this application, accurate knowledge of the body-to-ENU transformation matrix is required, based either on *a priori* survey data or on a converged attitude solution. Phase-center calibration is an important component of Controlled Reception Pattern Antenna (CRPA) array development and evaluation [xvi].

$$\left(\Delta\varphi_{ij}+k_{ij}\right)\lambda=\vec{b}_i^{T}A\hat{s}_j+B_i+\underline{\delta B_{ij}\left(\alpha_j,\zeta_j\right)}+\nu_{ij} \quad (5)$$

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