Design and Performance of a Minimum-Variance Hybrid Location Algorithm Utilizing GPS and Cellular Received Signal Strength for Positioning in Dense Urban Environments

David S. De Lorenzo, *Stanford University*
Sherman C. Lo, *Stanford University*
Per K. Enge, *Stanford University*
Marty Feuerstein, *Polaris Wireless*
Tarun K. Bhattacharya, *Polaris Wireless*
Steve Spain, *Polaris Wireless*
Zhengjiu Kang, *Polaris Wireless*

BIOGRAPHY

Dr. David De Lorenzo received the Ph.D. degree in Aeronautics and Astronautics from Stanford University, where he works as a researcher in the GPS Laboratory. His current research is in GNSS software receivers, adaptive beamsteering antenna arrays, indoor and urban reception of radio signals, and navigation system security and integrity.

Dr. Sherman Lo is a research associate at the Stanford University GPS Laboratory, where he is the Associate Investigator for Stanford University efforts on the technical evaluation of Loran.

Dr. Per Enge is a Professor of Aeronautics and Astronautics at Stanford University, where he is the Kleiner-Perkins, Mayfield, Sequoia Capital Professor in the School of Engineering. He directs the Stanford University GPS Research Laboratory.

Dr. Marty Feuerstein has more than 20 years experience in research, development, and deployment of wireless products, including work at Northern Telecom, AirTouch, Lucent Technologies, Bell Laboratories, and Metawave Communications. Dr. Feuerstein earned the Ph.D. degree in Electrical Engineering from Virginia Tech.

Dr. Tarun K. Bhattacharya has over 15 years experience in the design and development of advanced signal processing systems for commercial and military applications. Dr. Bhattacharya's areas of expertise include spatio-temporal filtering, neural networks, pattern recognition, time-frequency distributions, geolocation, and multi-target tracking with data fusion.

Dr. Steve Spain serves as Chief Technology Architect of Polaris Wireless. Dr. Spain has more than 25 years of experience in the development of estimation, control, and signal processing applications for defense and commercial customers, including work at Tera Research, Integrated Systems, Inc., Advanced Decision Systems, ESL Inc., and Systems Control Inc. Dr. Spain earned the Ph.D. degree in Electrical Engineering from Stanford University.

Dr. Zhengjiu Kang has been a principal algorithm engineer at Polaris Wireless since 2005, where his major responsibility has been focusing on Location Based Services (LBS) algorithm and system R&D. Before he joined Polaris Wireless, he worked at UtopiaCompression Corp. as a senior R&D scientist. Dr. Kang received his Ph.D. degree in Electrical Engineering from the University of California at Los Angeles in 2004.

ABSTRACT

This paper shows how the complementary advantages of GPS and cellular received signal strength (RSS) positioning methods improves hybrid location estimation performance in dense urban environments. In general, GPS techniques work best in rural or suburban environments where there is only moderate building or

landscape clutter to interrupt sky visibility or to introduce multipath errors from reflected satellite signals. In contrast, cellular techniques work best in urban environments where the density and geometry of cell towers is favorable.

Test results show that cellular-RSS effectively can be used to eliminate GPS outliers (location errors > 1 km, such as those associated with cell-ID fallback mode), while GPS can improve the accuracy of the combined solution. Based on field trial data, the accuracy of the Minimum-Variance Hybrid Algorithm (in dense urban environments) is approximately 45m for 67% of E911 calls and 110m for 95% of E911 calls.

INTRODUCTION

Ubiquitous positioning – locating a user or a device anywhere and at any time – is the ultimate goal for navigation engineers. This effort is driven not only by consumer acceptance and market uptake of Location Based Services (LBS), but also by government directives such as the FCC's E911 mandate. The FCC wireless Enhanced 911 standard requires cellular network carriers to estimate the location of a mobile handset making a 911 emergency call, and to provide this information to the dispatcher in a timely manner [1]. Phase II of the standard specifies accuracy targets generally to within 50 to 300 meters over a selected geographic region.

The Global Positioning System (GPS) is the preferred solution for low-cost commercial positioning, as seen in personal navigation devices and mobile handsets. However, the performance of GPS in mobile handsets is problematic in meeting the guidelines of the E911 standard for several reasons. First, mobile handset operation during 911 emergency calls frequently occurs in urban, dense urban, or indoor environments [2]. Second, as navigation is not the primary purpose of a mobile handset, the design and placement of the L-band antenna is non-ideal leading to further performance degradation [3, 4].

Many technologies have been developed to meet the challenges of urban navigation, particularly as applied to E911 mobile handset location. Assisted-GPS (A-GPS) increases acquisition sensitivity and decreases time-to-first-fix (TTFF) [5]. Other techniques leverage information measured by the handset and reported to the network during the course of active and/or standby modes of operation. These techniques include estimating the location of the handset based on the coordinates of the serving cell tower [6], using timing information to trilaterate handset location [7], and pattern-matching to the received signal strength (RSS) measurements of the serving cell tower and of neighbor cell towers [8, 9].

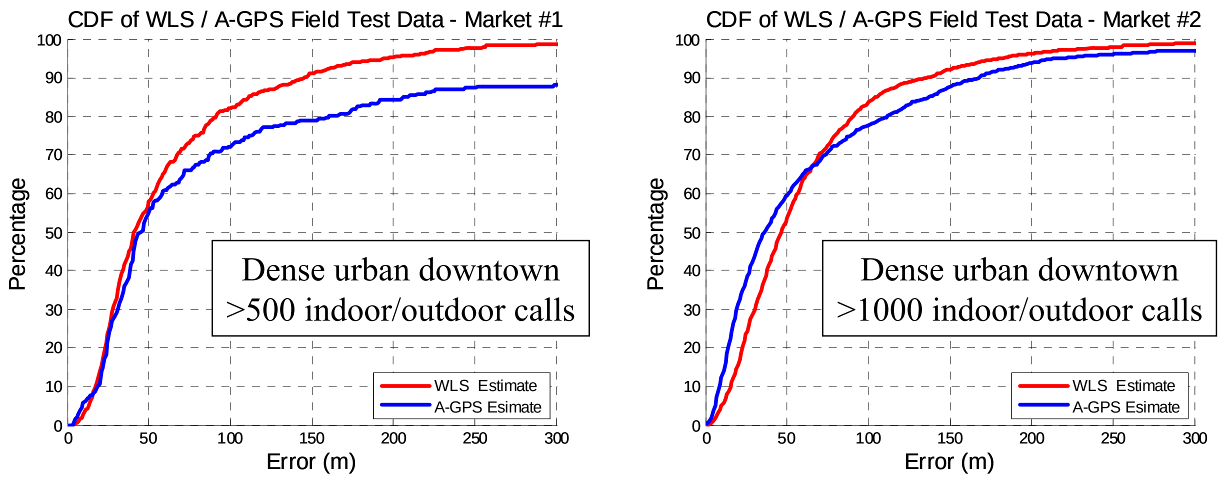
This paper shows how the complementary advantages of GPS-based and cellular-RSS-based positioning methods improves hybrid location estimation performance in dense urban environments. In general, GPS techniques work best in rural or suburban environments where there is only moderate building or landscape clutter to interrupt sky visibility or to introduce multipath errors from reflected satellite signals. In contrast, cellular techniques work best in urban environments where the density and geometry of cell towers is favorable.

First, this paper derives a method of combining GPS and cellular-RSS location estimates. This method utilizes quality-of-fix attributes reported by the GPS and RSS-based algorithms to minimize the error variance of the hybrid location estimate, and incorporates cross-checking between GPS and cellular-RSS estimates for outlier exclusion to avoid contaminating the position solution. The algorithm operates in the position domain, simplifying vendor- and hardware-specific implementation issues. This minimum-variance combiner represents an optimal use of the GPS and cellular-RSS location and uncertainty estimates to calculate handset location.

Second, this hybrid location technique is verified with field trial data from several dense urban markets in the U.S., Canada, and Japan. This field trial data allows demonstration of the procedures for determining algorithm parameters, and enables performance comparison between GPS, cellular-RSS, and hybrid GPS/RSS methods. The implementation procedure involves multiple steps to align various information inputs, to detect and to filter outliers, and to develop the optimal solution. One major step is to develop correspondence between the uncertainty values output by the GPS and by the cellular-RSS positioning engines; for example, as different GPS chipset vendors have different means and metrics to report uncertainties, the algorithm accounts for these differences and aligns them with those reported by the cellular-RSS location engine. Test results show that cellular-RSS effectively can be used to eliminate GPS outliers (location errors > 1 km), while GPS can improve the accuracy of the combined solution. Based on field trial data, the accuracy of the Minimum-Variance Hybrid Algorithm (in dense urban outdoor environments) is approximately 45m for 67% of E911 calls and 110m for 95% of E911 calls.

OVERVIEW OF WLS/GPS MINIMUM-VARIANCE HYBRID ALGORITHM

Wireless Location Signatures (WLS) uses received signal strength and network timing measurements from the serving cell tower and from neighboring cell towers to estimate handset location. GPS processing uses pseudorange measurements to several GPS satellites to



Estimation Errors	Market #1		Market #2	
	67%	95%	67%	95%
WLS Estimate	61 m	194 m	65 m	179 m
A-GPS Estimate	77 m	2122 m	65 m	219 m

Figure 1. WLS & GPS location estimation accuracy.

estimate handset location. In cases where GPS time-to-fix exceeds the limits established for the E911 call flow, a fallback position estimate derived from the location of the serving cell tower (and antenna sector) is presented instead (fallback to cell-ID positioning mode may or may not be reported back through the network). Figure 1 shows WLS and GPS location estimation accuracy from field trials of simulated E911 calls in two dense urban test markets. Note particularly that 1-σ accuracy in these field trials is roughly comparable between WLS and GPS, but also that the GPS error distributions have long tails (especially for test market #1) which is associated with severe urban multipath and cell-ID fallback inaccuracy.

The Minimum-Variance Hybrid Algorithm represents an optimal use of WLS and GPS location and uncertainty estimates to calculate handset location. Namely, if it were known *a priori* which estimate were better (WLS or GPS), then more weight could be applied to that estimate in the final positioning report. Furthermore, if there were a method to detect outliers, those estimates which are likely to deviate significantly from the core of the error distributions, then the contribution from those estimates also could be excluded from the final positioning report. Specifically, a minimum-variance estimator seeks an optimal method for combining estimates, in this case estimates taken from two error populations associated with WLS and with GPS, and then combining those estimates in a ratio determined from their respective population variances.

In other words, we seek an estimator which utilizes data from more than one source and combines that data using weighting coefficients which minimize the error variance of the final result (see Appendix A for a derivation). For WLS and GPS location estimates $\vec{p}_{WLS}$ and $\vec{p}_{GPS}$, of known or estimated error variance σ^2_{WLS} and σ^2_{GPS}, the hybrid location estimate $\vec{p}_{hybrid}$ is calculated as follows:

$$w_{WLS}\vec{p}_{WLS} + w_{GPS}\vec{p}_{GPS} = \vec{p}_{hybrid} \qquad (1)$$

To minimize location error variance under the constraint that $w_{WLS} + w_{GPS} = 1$, the weighting coefficients are calculated as follows (see Appendix A):

$$\begin{aligned} w_{WLS} &= \frac{\sigma^2_{GPS}}{\sigma^2_{WLS} + \sigma^2_{GPS}} \\ w_{GPS} &= 1 - w_{WLS} \end{aligned} \qquad (2)$$

To summarize, the minimum-variance hybrid algorithm blends WLS and GPS estimates to improve accuracy, by

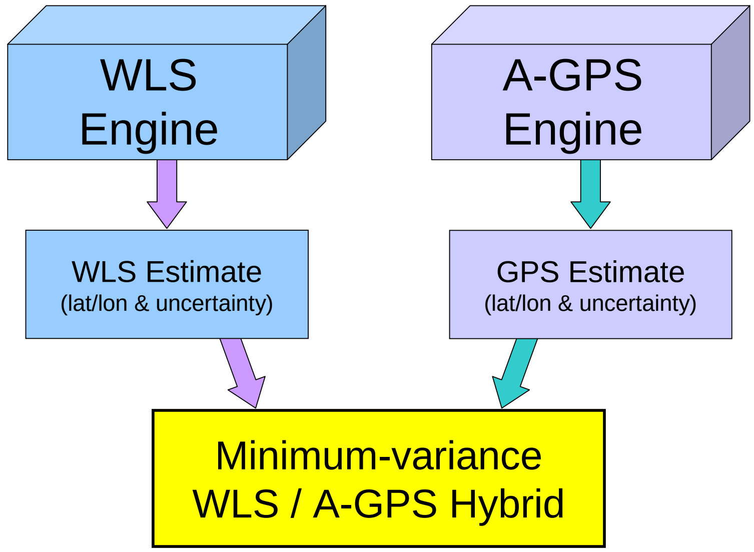


Figure 2. Minimum-variance WLS/GPS hybrid algorithm.

leveraging complementary features of WLS and GPS and by seeking to eliminate large errors and outliers. The weighting coefficients in the hybrid algorithm depend on WLS and GPS estimation uncertainties although, as will be discussed below, GPS uncertainty reporting can be problematic as it not standardized by chipset vendor or handset implementation. Finally, this is a position-domain hybrid algorithm, meaning that it does not need to access (nor can it modify) the intermediate location processing steps – in other words, it consider GPS and WLS to be location "black boxes" (see Figure 2).

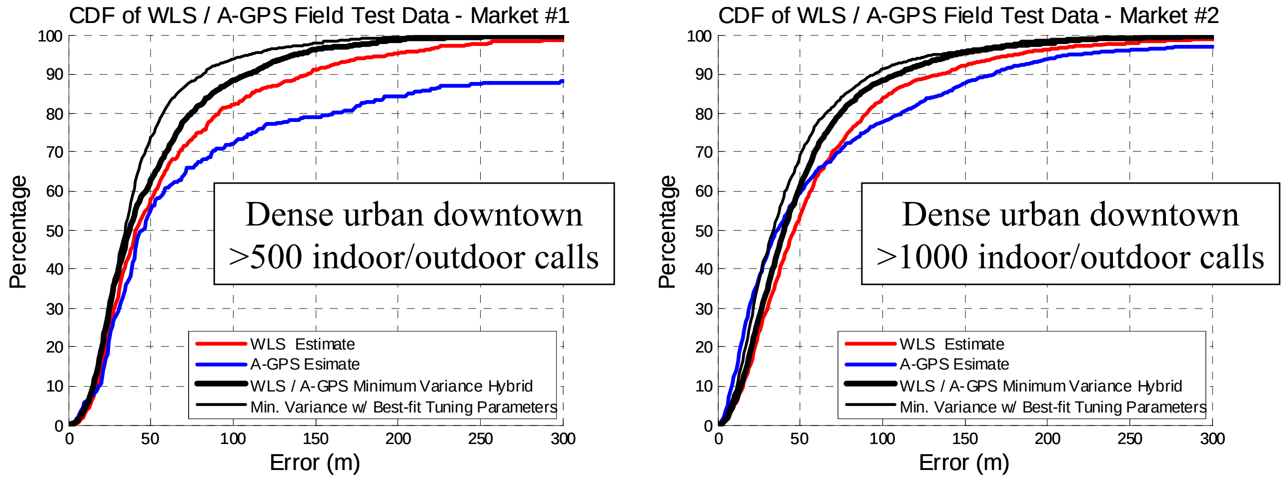
When the minimum-variance estimator is applied for the purpose of combining WLS and GPS location estimates, then there are several factors of which to be aware.

1. The estimation of 2-D location on the surface of the Earth often can be decomposed into independent and orthogonal estimates of North/South location (latitude) and East/West location (longitude) – this de-coupling characteristic applies (on average) to location estimates from WLS and from GPS. This means that the hybrid location estimation problem can be broken down into separate problems of determining latitude and longitude using WLS and GPS estimates. It follows from this that the total positioning error of the minimum-variance hybrid estimate can be calculated from the square root of the sum of the squares of the North/South error and of the East/West error.

2. The WLS and GPS location estimates are mutually independent (the error variances may or may not be mutually independent, but the minimum-variance algorithm does not require variance independence, so this distinction will not be further discussed). This means that the error magnitude for one of the estimates does not correlate with the error magnitude of the other estimate.

3. When expressed in units of meters, the latitude and longitude estimation errors for either method (WLS or GPS) have equal variance. (This may not be true either for WLS or for GPS. Currently the Polaris Location Engine reports only one value of "uncertainty". Some GPS applications report elliptical error estimates, but to date these neither have proven well-matched to the field trial data nor have been useful in characterizing the estimation errors. If this circumstance no longer holds for newly tested markets, either for WLS or for GPS, then this assertion can be re-visited.) This means that each calculation of hybrid location will rely on one measure of estimation variance for WLS and one measure of estimation variance for GPS; each of these variances will apply both to the North/South estimate as well as to the East/West estimate for their respective positioning methods.

4. The WLS and GPS location estimates are zero-mean (unbiased) and approximately normally distributed (Gaussian). (The normality requirement is weak, due to the Central Limit Theorem.) This means that the minimum-variance estimator derived in Appendix A applies to the problem of combining WLS and GPS location estimates, assuming some mechanism exists to



Estimation Errors	Market #1		Market #2	
	67%	95%	67%	95%
WLS Estimate	61 m	194 m	65 m	179 m
A-GPS Estimate	77 m	2122 m	65 m	219 m
Min. Variance Hybrid	44 m	110 m	48 m	133 m
Min. Variance Hybrid / Compromise	55 m	137 m	56 m	144 m

Figure 3. Minimum-variance WLS/GPS location estimation accuracy.

estimate the WLS and GPS location error variances.

5. In cases where either WLS-based or GPS-based estimation suffers a gross failure, then there should be a mechanism for detecting and then excluding these outliers. The heuristic tools to detect and to exclude outliers, as is to be expected, are market dependent and will be discussed further below.

6. There is a relationship between the "uncertainty" reported by the WLS and GPS positioning applications and the error variance of the North/South and East/West estimates. (It is not clear that this must or even should be the case *in theory* without delving into the core of the WLS and/or GPS applications; it only is necessary to show that this is the case *in practice*.) This means that there will be functions to map "uncertainty" (however it is derived and in whatever units it is reported) to error variance (in meters-squared) both for WLS and for GPS.

This final requirement is the most stringent to consider in developing the Minimum-Variance Hybrid Estimator, as it is likely that the relationship between reported "uncertainty" and error variance will be handset dependent – certainly so for GPS as the calculation of "uncertainty" depends on the implementation of the chipset vendor, and possibly so for WLS if there are dependencies on antenna gain pattern, analog hardware design, signal power estimation algorithms, etc. The possible requirement to develop a handset-specific Minimum-Variance Hybrid Algorithm is beyond the scope of this note, and is not discussed further here.

APPLYING THE WLS/GPS MINIMUM-VARIANCE HYBRID ALGORITHM

The following is an overview of the steps involved in executing the WLS/GPS Minimum-Variance Hybrid Algorithm (accuracy results summarized in Figure 3). The first steps check the validity of the assertions required to combine the WLS and the GPS location estimates in a minimum-variance fashion. The next steps are designed to fix the algorithm parameters (i.e., determine the functions that map "uncertainty" to error variance and develop the WLS and GPS outlier exclusion criteria). The final steps invoke the algorithm on WLS and GPS location estimates – these final steps are those that would

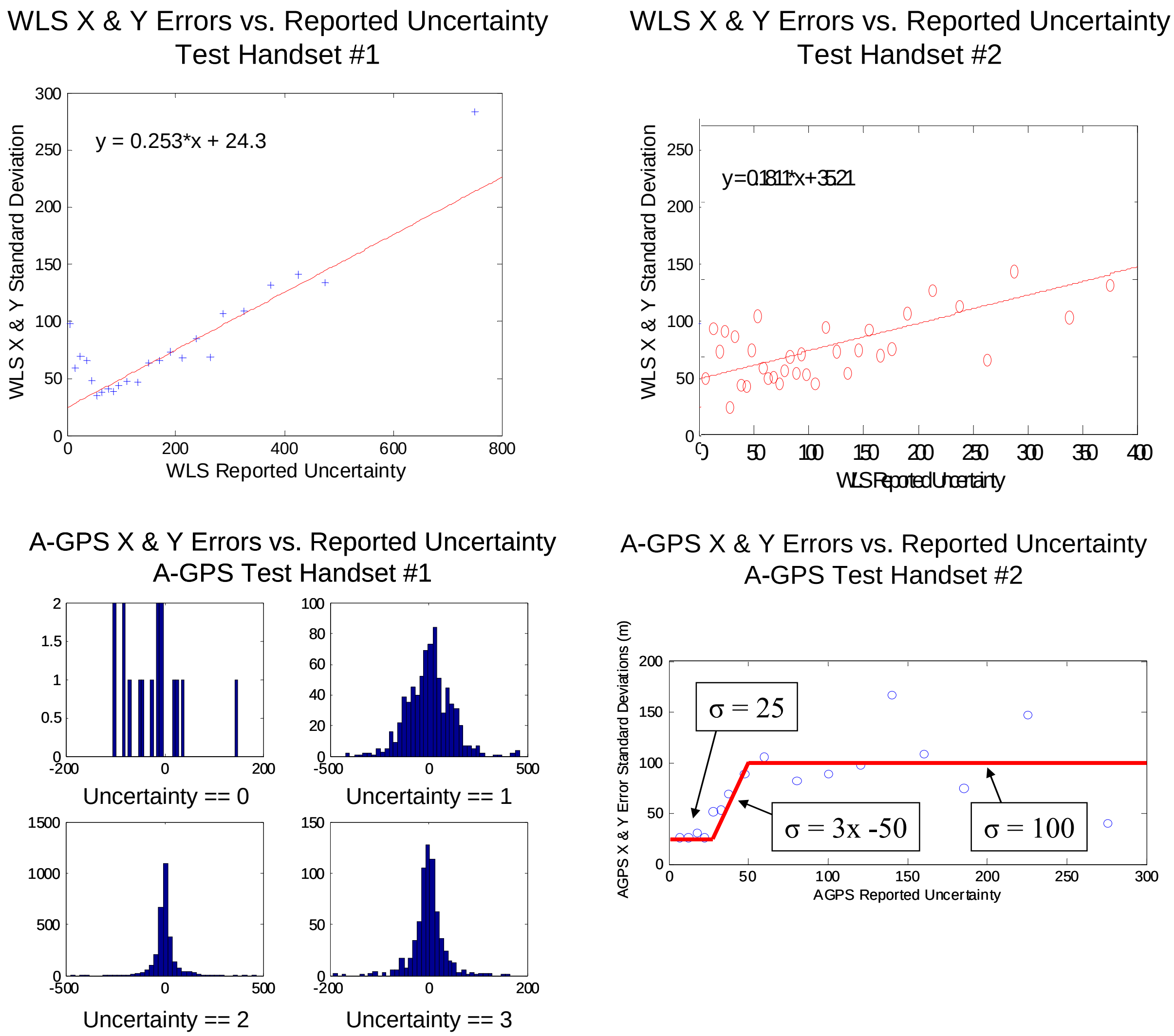


Figure 4. Plotting WLS and GPS "uncertainty" vs. standard deviation.

be deployed in an operational minimum-variance hybrid system.

The first steps evaluate error characteristics: independence of latitude vs. longitude, independence of WLS vs. GPS, normality of WLS and GPS.

1. Plot latitude vs. longitude errors for WLS and for GPS; plot WLS vs. GPS latitude errors; plot WLS vs. GPS longitude errors; calculate R^2 value (also called 'coefficient of determination') between errors (but only for total errors less than some threshold, e.g., 500m); verify that R^2 value is suitably low (i.e., << 0.25).

2. Plot histograms of latitude and longitude errors for WLS and for GPS; confirm zero-mean and nominally "Gaussian" in appearance (may have long tails due to outliers); calculate standard deviations of WLS and GPS latitude and longitude errors.

The next steps fix the algorithm parameters mapping "uncertainty" to standard deviation and determine outlier exclusion criteria.

1. Plot reported "uncertainty" vs. error standard deviation by binning data in suitable ranges (see Figure 4). Note that the proprietary WLS Location Engine enables consistent uncertainty reporting across several markets, while the GPS uncertainty reporting is not standardized across different handsets.

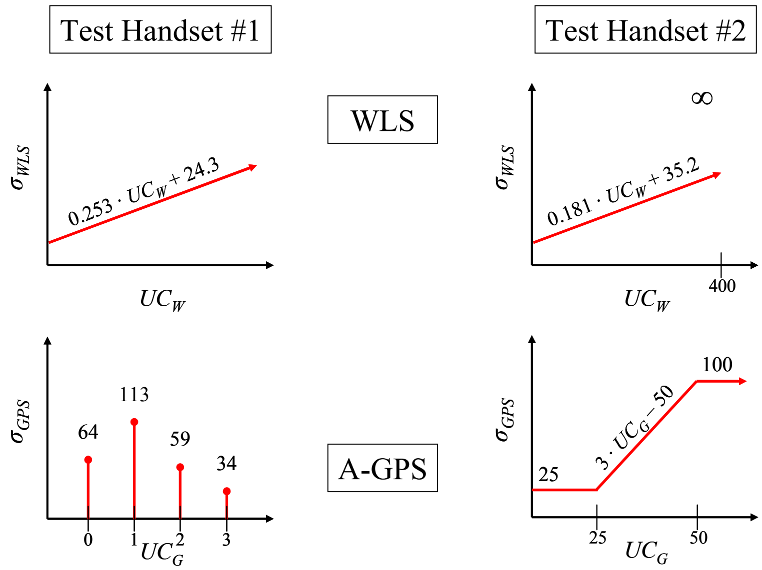


Figure 5. Mapping WLS and GPS "uncertainty" to standard deviation.

2. Determine mapping functions between "uncertainty" and latitude and longitude standard deviation (see Figure 5).

3. Establish outlier exclusion criteria, e.g.:
   a. scatter plot WLS and GPS error magnitudes vs. reported "uncertainty" to estimate reported "uncertainty" threshold above which WLS and GPS estimates may be considered outliers
   b. plot WLS and GPS errors vs. WLS-to-GPS separation to identify separation threshold above which GPS estimates may be considered outliers

   For the field trial data analyzed to date, if the WLS-to-GPS position estimates were separated by more than 500m, then hybrid positioning errors were reduced by defaulting to the WLS position estimate (i.e., considering the GPS estimate to be an outlier):

$$\begin{aligned} w_{WLS} &= 1 \\ w_{GPS} &= 0 \end{aligned} \tag{3}$$

To execute the minimum-variance algorithm on WLS and GPS positioning estimates $\{Lat_{WLS}, Lon_{WLS}\}$ and $\{Lat_{GPS}, Lon_{GPS}\}$ with estimated variances σ^2_{WLS} and σ^2_{GPS}, calculate the hybrid estimate $\{Lat_{hybrid}, Lon_{hybrid}\}$:

$$\begin{aligned} w_{WLS} Lat_{WLS} + w_{GPS} Lat_{GPS} &= Lat_{hybrid} \\ w_{WLS} Lon_{WLS} + w_{GPS} Lon_{GPS} &= Lon_{hybrid} \end{aligned} \tag{4}$$

and the hybrid standard deviation in latitude/longitude as:

$$\sigma_{hybrid} = \frac{\sigma_{WLS}\sigma_{GPS}}{\sqrt{\sigma^2_{WLS} + \sigma^2_{GPS}}} \tag{5}$$

Based on the field trial data summarized in Figure 1, minimum-variance WLS/GPS positioning accuracies improve by ~30-40% for the 1-σ value (see Figure 3 and the 3rd row of results shown in the table contained therein). Furthermore, the 95% accuracy improves dramatically, removing the long tails of the error distributions arising predominantly from GPS outliers and cell-ID fallback mode position estimates.

These results are based on market- and handset-specific calibration of the mapping functions between reported "uncertainty" and latitude and longitude standard deviation. A practical implementation considers a compromise that avoids this labor-intensive step. Parameter sensitivity studies determined that a reasonable compromise set the mapping functions to be:

$$\begin{aligned} \hat{\sigma}_{WLS} &= 0.2 \cdot UC_{WLS} + 30 \text{ m} \\ \hat{\sigma}_{GPS} &= 100 \text{ m} \end{aligned} \tag{6}$$

With these compromise modeling parameters applied to the field trial data from markets #1 and #2, the 1-σ accuracy improvement still exceeds ~10%, and the outlier exclusion benefits to the 95% positioning accuracy continue to persist.

WLS/GPS Minimum-Variance Hybrid Algorithm represents near-optimal use of WLS and GPS location and uncertainty estimates.

ACKNOWLEDGMENTS

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APPENDIX: MINIMUM-VARIANCE ESTIMATION

Given:

1. p_1 and p_2 are independent and unbiased estimates of an unknown quantity p
2. the error in estimate p_1 is Gaussian with standard deviation σ_1
3. the error in estimate p_2 is Gaussian with standard deviation σ_2

Find:

1. the minimum-variance estimate for p called p_{hybrid}
2. the standard deviation of the error in this estimate σ_{hybrid}

Method:

The weighted sum of two estimates p_1 and p_2:

$$w_1 p_1 + w_2 p_2 = p_{hybrid} \tag{A.1}$$

The constraint that the sum of the weights w_1 and w_2 equals unity:

$$w_1 + w_2 = 1 \tag{A.2}$$

The variance of the weighted sum of independent, zero-mean, normally distributed variables whose variances are σ_1^2 and σ_2^2:

$$(w_1\sigma_1)^2 + (w_2\sigma_2)^2 = \sigma_{hybrid}^2 \tag{A.3}$$

Substitute for w_2:

$$(w_1\sigma_1)^2 + ((1 - w_1)\sigma_2)^2 = \sigma_{hybrid}^2 \tag{A.4}$$

Expand:

$$\begin{aligned} w_1^2\sigma_1^2 + \sigma_2^2 - 2w_1\sigma_2^2 + w_1^2\sigma_2^2 \\ = \sigma_{hybrid}^2 \end{aligned} \tag{A.5}$$

Differentiate with respect to w_1, noting that $d\sigma_i / dw_1 = 0$ and $dw_1 / dw_1 = 1$:

$$2w_1\sigma_1^2 + 0 - 2\sigma_2^2 + 2w_1\sigma_2^2 = 2\sigma_{hybrid} \frac{d\sigma_{hybrid}}{dw_1} \tag{A.6}$$

Minimize σ_{hybrid} by setting $d\sigma_{hybrid} / dw_1 = 0$, and divide through by 2:

$$w_1\sigma_1^2 - \sigma_2^2 + w_1\sigma_2^2 = 0 \tag{A.7}$$

Solve for w_1:

$$w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \tag{A.8}$$

Substitution likewise can show that:

$$\sigma_{hybrid} = \frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \tag{A.9}$$