Linear Time-of-Arrival Estimation in a Multipath Environment by Inverse Correlation Method

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BIOGRAPHY

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ABSTRACT

The Maximum Likelihood Estimator (MLE) is most often used to estimate the signal time delay in a multipath environment. There are various implementations of the MLE which can be divided into the time domain algorithms and the frequency domain algorithms. In this paper, the current time and frequency domain schemes are compared and analyzed in least square sense. Also novel solutions in both domains are developed to resolve the close-in multipath signals. They are tested by the Monte Carlo simulation and their performances are compared to the Cramer-Rao bound on the Direct Sequence Spread Spectrum (DSSS) signal.

I. INTRODUCTION

The accurate estimation of a multipath channel is the essential part of the positioning process in urban canyons and indoor areas where severe multipath is experienced and generates a high positioning error. While most of other types of errors in the positioning systems are now technically tracktable such as the ionospheric error in the Global Positioning System (GPS), the multipath error remains as a single dominant source of the positioning error. Therefore the mitigation of the multipath error is pursued extensively and various efforts have been made to estimate a multipath channel but they are either too complex to implement or not providing enough improvement.

The complexity of the problem comes from the fact that there are multiple parameters to be estimated such as time delay, amplitude and phase of each multipath signal. Thus the overall operation requires a multiple dimensional search where the dimension increases as the number of multipath signals increases. Especially the time delay is in a nonlinear form to the cost function and makes the estimation as a nonlinear optimization problem. Furthermore when a Line-of-Sight (LOS) signal has an enough separation from multipath signals, it can be well resolved but when close-in multipath signals exist, it becomes difficult to estimate because of the ambiguity between the LOS signal and the multipath signal. Hence the implementation of the Maximum Likelihood Estimator (MLE) in a multipath environment is proven to be a challenging task especially in small hand-held GPS devices or cell phones where power consumption due to high computation is least favored. Thus the focus of this paper is on the development of an algorithm with low complexity but still providing effective multipath mitigation for hand-held devices.

There are various approaches to implement the MLE which can be classified into two groups, the time domain estimators and the frequency domain estimators. The time domain methods investigate the correlation function of a received signal. The Waveform Shape Tracking focuses on the shape of small sections of the correlation function and matches it with a stored reference waveform measuring distortion and balancing the early and late parts of the waveform to find the true peak. It is simple but accompanies a loss of SNR because of giving up the information embedded in the rest of the correlation function. The Successive Cancellation is more sophiscated method which iteratively tracks dominant multipath components and removes them until the residue becomes lower than a threshold value. It more closely implementes the MLE but requires intensive computation. The Multipath Mitigation Technique (MMT) by Weill [1] formulates the multipath problem as a least square problem in time domain and reduces the complexity by breaking it into smaller problems in a reduced search space.

The frequency domain methods are based on a least square estimation problem trying to fit the weighted complex exponential functions which are a function of time delay, amplitude and phase of multipath components to a received signal spectrum. They are more convenient in estimating the time delays than the time domain methods because the time delays are expressed as exponents. It is initially proposed by Kirsteins [2] using the Tufts and Kumaresan algorithm [3] to solve the least square problem and a more robust scheme is given by Vaccaro [4] using the coarse search by the Kirsteins method and the fine search by the Gauss-Newton algorithm. The frequency domain approaches also generally accompany iterative searches.

The structure of the time domain and the frequency domain approaches are discussed as least square problems in Section II and novel approaches to the close-in multipath are proposed in Section III. The proposed schemes are compared with the Cramer-Rao Lower Bound (CRLB) by simulation in Section IV and the conclusion is given in Section V.

II. Least Square Estimation of Time Delay in Time and Frequency

In this section, the time delay estimation problem is formulated into least square problems in time domain and frequency domain fitting an estimated signal into a received signal which is equivalent to the MLE in the AWGN channel. The estimation of a received signal or a channel requires the reconstruction of the time delay, the amplitude and the phase of the channel impulse response but the ultimate goal is to find the time delay of the LOS signal τ_{LOS} only which is essential to the positioning. The channel model is defined as follows.

$$x(t) = s(t) * h(t) + n(t) \tag{1}$$

where $h(t) = \sum_{m=1}^{M} a_m \delta(t-\tau_m)$ is the channel impulse response function with the time delays $\boldsymbol{\tau} = (\tau_1, ..., \tau_M)$ and the complex amplitudes $\mathbf{a} = (a_1, ..., a_M)$ combining the amplitudes and the phases corresponding to the M multipath components in the ascending order of time delay. $n(t)$ is the additive white gaussian noise with zero mean and variance σ_n^2 and $s(t)$ is the transmitted signal and $x(t)$ is the received signal to be sampled at the sampling rate of $1/T_s$ to be total L samples. T is the signal time duration and W is the single-sided signal bandwidth. The cost function Γ of the least square problem is a function of the complex amplitudes $\mathbf{a}$ and the time delays $\boldsymbol{\tau}$. To find $\tau_1 = \tau_{LOS}$, the time delay of the LOS signal, first $\mathbf{a}$ minimizing Γ is derived analytically and then $\boldsymbol{\tau}$ minimizing Γ for given $\mathbf{a}$ is investigated.

A. Time Domain Least Square Solution

The time domain least square problem is given (2) and approximated by discrete samples and transformed into a matrix form (3).

$$\Gamma = \int_0^T \left| x(t) - \sum_{m=1}^{M} \hat{a}_m s(t-\hat{\tau}_m) \right|^2 dt \tag{2}$$

$$\begin{aligned} \Gamma &\approx \sum_{n=1}^{L} \left| x(t_n) - \sum_{m=1}^{M} \hat{a}_m s(t_n - \hat{\tau}_m) \right|^2 \\ &= \|\mathbf{x} - \mathbf{C}\hat{\mathbf{a}}\|^2 \\ &= R_X(0) - 2\mathbf{R}_{\mathbf{XS}}^{\mathbf{H}}\hat{\mathbf{a}} + \hat{\mathbf{a}}^{\mathbf{H}}\mathbf{R}_{\mathbf{S}}\hat{\mathbf{a}} \end{aligned} \tag{3}$$

where $\hat{\mathbf{a}} = (\hat{a}_1, ..., \hat{a}_M)$ is the estimation of $\mathbf{a}$ and $\hat{\boldsymbol{\tau}} = (\hat{\tau}_1, ..., \hat{\tau}_M)$ is the estimation of $\boldsymbol{\tau}$ and $\mathbf{x} = (x(t_1), ..., x(t_L))$ and

$$\mathbf{C} = \begin{bmatrix} s(t_1-\hat{\tau}_1) & \dots & s(t_1-\hat{\tau}_M) \\ s(t_2-\hat{\tau}_1) & \dots & s(t_2-\hat{\tau}_M) \\ \vdots & \ddots & \vdots \\ s(t_L-\hat{\tau}_1) & \dots & s(t_L-\hat{\tau}_M) \end{bmatrix}$$

and $R_X(t)$ and $R_S(t)$ are the autocorrelation functions of $x(t)$ and $s(t)$ respectively and $R_{XS}(t)$ is the cross correlation function of $x(t)$ with $s(t)$ and $\mathbf{R}_{\mathbf{XS}} = (R_{XS}(\hat{\tau}_1), ..., R_{XS}(\hat{\tau}_M))$ and

$$\mathbf{R}_{\mathbf{S}} = \begin{bmatrix} R_S(0) & R_S(\hat{\tau}_1-\hat{\tau}_2) & \dots & R_S(\hat{\tau}_1-\hat{\tau}_M) \\ \vdots & \vdots & \ddots & \vdots \\ R_S(\hat{\tau}_M-\hat{\tau}_1) & R_S(\hat{\tau}_M-\hat{\tau}_2) & \dots & R_S(0) \end{bmatrix}$$

$\hat{\mathbf{a}}$ minimizing Γ can be calculated by the product of $\mathbf{x}$ and $\mathbf{C}^{\dagger} = (\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-\mathbf{1}}\mathbf{C}^{\mathbf{H}}$ the pseudo inverse of $\mathbf{C}$ or by $\mathbf{R}_{\mathbf{S}}$ and $\mathbf{R}_{\mathbf{XS}}$.

$$\hat{\mathbf{a}}^* = \mathbf{C}^{\dagger}\mathbf{x} = \mathbf{R}_{\mathbf{S}}^{-\mathbf{1}}\mathbf{R}_{\mathbf{XS}} \tag{4}$$

Given $\hat{\mathbf{a}}^*$, Γ is now a function of only the time delay $\boldsymbol{\tau}$ and can be expressed as the maximization problem of another quadratic form excluding a constant term.

$$\begin{aligned} \min \Gamma(\hat{\mathbf{a}}^*, \hat{\boldsymbol{\tau}}) &= \min \left\| \left(\mathbf{I} - \mathbf{C}\mathbf{C}^{\dagger}\right)\mathbf{x} \right\|^2 \\ &= \mathbf{x}^{\mathbf{H}}\mathbf{x} - \max \mathbf{x}^{\mathbf{H}}\mathbf{C}\mathbf{C}^{\dagger}\mathbf{x} \\ &= R_x(0) - \max \mathbf{R}_{\mathbf{XS}}^{\mathbf{H}}\mathbf{R}_{\mathbf{S}}^{-\mathbf{1}}\mathbf{R}_{\mathbf{XS}} \end{aligned} \tag{5}$$

However, because both $\mathbf{R}_{\mathbf{XS}}$ and $\mathbf{R}_{\mathbf{S}}$ are nonlinear functions of $\hat{\boldsymbol{\tau}}$, it can not be solved linearly and the authors are not aware of an analytic solution to this problem except the iterative search. Weill uses a step-wise iterative search coming back and forth between (4) and (5) updating $\hat{\boldsymbol{\tau}}$ and $\hat{\mathbf{a}}$ each time [1].

B. Frequency Domain Least Square Solution

The frequency domain least square problem can be formulated similarily (6). The cost function Γ (2) can also be expressed in frequency domain through the Parseval's theorem and transformed into a matrix form (7).

$$\Gamma = \int_{-W}^{W} \left| X(f) - \sum_{m=1}^{M} \hat{a}_m S(f) e^{-j2\pi f \hat{\tau}_m} \right|^2 df \tag{6}$$

$$\begin{aligned} &\approx \sum_{k=1}^{L} \left| X(f_k) - S(f_k) \sum_{m=1}^{M} \hat{a}_m e^{-j2\pi f_k \hat{\tau}_m} \right|^2 \\ &= \|\mathbf{X} - \mathbf{D}\hat{\mathbf{a}}\|^2 \end{aligned} \tag{7}$$

where $X(f)$ and $S(f)$ are the Fourier transform of $x(t)$ and $s(t)$ each and $X(f_k)$ and $S(f_k)$ are their discrete correspo-nents and $f_k = (k-1)/T$ and $\mathbf{X} = (X(f_1), ..., X(f_L))$ and

$$\mathbf{D} = \begin{bmatrix} S(f_1)e^{-j2\pi f_1 \hat{\tau}_1} & \dots & S(f_1)e^{-j2\pi f_1 \hat{\tau}_M} \\ S(f_2)e^{-j2\pi f_2 \hat{\tau}_1} & \dots & S(f_2)e^{-j2\pi f_2 \hat{\tau}_M} \\ \vdots & \ddots & \vdots \\ S(f_L)e^{-j2\pi f_L \hat{\tau}_1} & \dots & S(f_L)e^{-j2\pi f_L \hat{\tau}_M} \end{bmatrix}$$

$\mathbf{X}$ and $\mathbf{D}$ each represent the received signal spectrum and the estimated spectrum composed of the weighted exponential functions.

$\hat{\mathbf{a}}$ minimizing Γ is given as the product of $\mathbf{X}$ and $\mathbf{D}^\dagger = (\mathbf{D^H D})^{-1}\mathbf{D^H}$ the pseudo inverse of $\mathbf{D}$.

$$\hat{\mathbf{a}}^* = \mathbf{D}^\dagger \mathbf{X} \tag{8}$$

Given $\hat{\mathbf{a}}_{\mathbf{min}}$, Γ is now a function of only time delay $\hat{\boldsymbol{\tau}}$ and can be expressed as a maximization problem.

$$\begin{aligned} \min \Gamma(\hat{\mathbf{a}}^*, \hat{\boldsymbol{\tau}}) &= \min \left\| \left(\mathbf{I} - \mathbf{D}\mathbf{D}^\dagger\right) \mathbf{X} \right\|^2 \\ &= \mathbf{X^H X} - \max \mathbf{X^H D D^\dagger X} \end{aligned} \tag{9}$$

$\hat{\boldsymbol{\tau}}$ minimizing Γ can be found by either an iterative search or a linear prediction technique because (6) is in a form of an exponential function fitting problem. However the linear prediction method is generally limited to the signals with a flat spectrum [2] [3] [4].

C. Equivalence of Time and Frequency Domain Solutions

There is high similarity between the time domain least square solution and the frequency domain least square solution in their derivation and structure. And it turns out that their results are equivalent to each other. The correlation of two function can be calculated either in time or frequency domain.

$$x(t) \star \overline{s(t)} \leftrightarrow X(f)\overline{S(f)} \tag{10}$$

In discrete form, the correlation function can be expressed as follows after the Inverse Fourier Transform on the frequency domain result.

$$\begin{aligned} R_{XS}(\tau) &= \sum_{n=1}^{L} x(t_n)\overline{s(t_n - \tau)} \\ &= \sum_{k=1}^{L} X(f_k)\overline{S(f_k)e^{-j2\pi f_k \tau}} \end{aligned} \tag{11}$$

Based on (11), $\mathbf{R_{XS}}$ and $\mathbf{R_S}$ can be obtained from either $(\mathbf{C}, \mathbf{x})$ or $(\mathbf{D}, \mathbf{X})$ because $s(t_n - \tau)$ and $S(f_k)e^{-j2\pi f_k \tau}$ are the elements of $\mathbf{C}$ and $\mathbf{D}$.

$$\begin{aligned} \mathbf{R_{XS}} &= \mathbf{C^H x} = \mathbf{D^H X} \\ \mathbf{R_S} &= \mathbf{C^H C} = \mathbf{D^H D} \end{aligned} \tag{12}$$

Therefore the time domain solution and the frequency domain solution are equivalent to each other.

$$\hat{\mathbf{a}}^* = \mathbf{R_S}^{-1}\mathbf{R_{XS}} = \mathbf{C}^\dagger \mathbf{x} = \mathbf{D}^\dagger \mathbf{X} \tag{13}$$

III. Inverse Correlation Method and Inverse Spectrum Method

In the previous section, the least square estimation problems are described in time and frequency domains where the optimization problems are derived to be functions of only the time delay parameters removing other variables (5)(9). However, they are hard to be solved straightforward without iterative searches due to the nonlinearity. In this section, the Inverse Correlation Method (ICM) and the Inverse Spectrum Method (ISM), are proposed which provide an effective multipath estimation without iterative searches under certain approximations. For them the cost function is re-defined to fit the received signal with a set of uniformly delayed signals with $\hat{\mathbf{a}} = (\hat{a}_1, ..., \hat{a}_L)$ at $(t_1, ..., t_L)$.

$$\begin{aligned} \Gamma &= \sum_{n=1}^{L} \left| x(t_n) - \sum_{l=1}^{L} \hat{a}_l s(t_n - t_l) \right|^2 \\ &= \sum_{k=1}^{L} \left| X(f_k) - \sum_{l=1}^{L} \hat{a}_l S(f_k) e^{-j2\pi f_k t_l} \right|^2 \end{aligned} \tag{14}$$

We assume that the sampling rate $1/T_s$ is high enough and the number of signal samples is sufficiently larger than the number of multipath components i.e. $L \gg M$ and only a single multipath component exists at each time sample. Then the complex amplitudes $\hat{\mathbf{a}} = (\hat{a}_1, ..., \hat{a}_L)$ at $(t_1, ..., t_L)$ become indicators of the existence of multipath components at given time bins. In other words, instead of estimating both $(\mathbf{a}, \boldsymbol{\tau})$ which requires iterative searches, only the amplitudes at the fixed time bins are to be estimated by non-iterative methods. They would be comparable to the iterative schemes for the multipath signals close to sample times but could be less accurate otherwise. However their simpler structure makes them more suitable solutions for the hand-held devices.

A. Inverse Correlation Method

A new approach can be found in (4) where $\hat{\mathbf{a}}$ the estimate of $\mathbf{a}$ is shown to be achievable by the simple matrix multiplications. After re-defining $\mathbf{R_S}, \mathbf{R_{XS}}$ and $\mathbf{C}$ for $(\hat{\tau}_1, ..., \hat{\tau}_L) = (t_1, ..., t_L)$, the solution to the new cost function (14) is given as $\hat{\mathbf{a}}_{\text{ICM}}$.

$$\boxed{\hat{\mathbf{a}}_{\text{ICM}} = \mathbf{C}^\dagger \mathbf{x} = \mathbf{R_S^{-1} R_{XS}}} \tag{15}$$

where $\mathbf{x} = (x(t_1), ..., x(t_{2L}))$ collected for $[0, 2T)$ assuming $\tau_{LOS} \in [0, T)$ and $t_l = T(l-1)/L$ and

$$\mathbf{C} = \begin{bmatrix} s(t_1) & 0 & \dots & 0 \\ s(t_2) & s(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s(t_L) & s(t_{L-1}) & \dots & s(t_1) \\ 0 & s(t_L) & \dots & s(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s(t_L) \end{bmatrix}$$

Because only a single multipath component exists at each time sample $\hat{\mathbf{a}}_{\text{ICM}}$ becomes a delay profile of the received signal

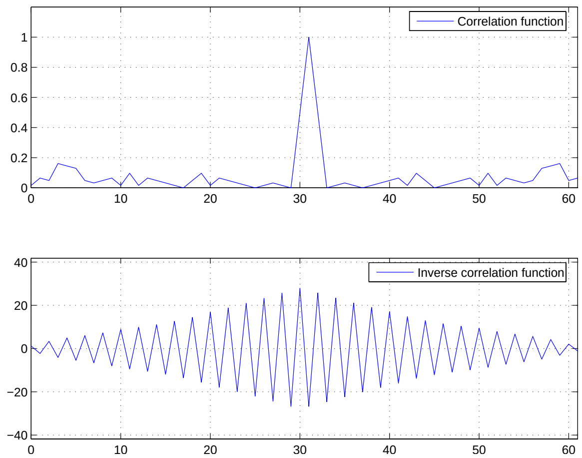

Fig. 1. Correlation function $\mathbf{R_{XS}}$ and inverse correlation function from $\mathbf{R_S^{-1}}$

and the amplitude $\hat{a}_{\text{ICM},l}$ indicates the existence of a multipath signal at each bin.

$\mathbf{R_S^{-1}}$ represents the inverse process of the correlation function and thus it is called as the Inverse Correlation Matrix. Fig. 1 displays an example of the auto-correlation function and its inverse correlation function based on the 31 chip Gold sequence. The rows of $\mathbf{R_S^{-1}}$ are the inverse correlation functions corresponding to the correlation functions with different delays. The ICM process is basically differentiating the correlation output to generate a sharper peak. In (15) the calculation of the inverse matrix $\mathbf{C}^\dagger$ requires large computations but can be calculated in advance and stored in memory using the prefixed time instances and the known autocorrelation function of the reference signal.

B. Inverse Spectrum Method

A similar approach can be found in (8) where $\mathbf{a}$ is again shown to be achievable by a simple matrix multiplication and can represent a delay profile of the received signal. $\mathbf{D}$ is redefined for $(\hat{\tau}_1, ..., \hat{\tau}_L) = (t_1, ..., t_L)$, the solution to the new cost function (14) is given as $\hat{\mathbf{a}}_{\text{ISM}}$.

$$\boxed{\hat{\mathbf{a}}_{\text{ISM}} = \mathbf{D}^\dagger \mathbf{X} = \mathbf{R_S^{-1}} \mathbf{R_{XS}}} \tag{16}$$

where $f_k = (k-1)/2T$ and $\mathbf{X} = (X(f_1), ..., X(f_{2L}))$ and

$$\mathbf{D} = \begin{bmatrix} S(f_1)e^{-j2\pi f_1 t_1} & \dots & S(f_1)e^{-j2\pi f_1 t_L} \\ S(f_2)e^{-j2\pi f_2 t_1} & \dots & S(f_2)e^{-j2\pi f_2 t_L} \\ \vdots & \ddots & \vdots \\ S(f_{2L})e^{-j2\pi f_{2L} t_1} & \dots & S(f_{2L})e^{-j2\pi f_{2L} t_L} \end{bmatrix}$$

$\mathbf{D}^\dagger$ represents the inverse process of the signal spectrum and thus it is called as the Inverse Spectrum Matrix. The pseudo inverse matrix $\mathbf{D}^\dagger$ can be calculated in advance and stored in memory and only the signal spectrum $\mathbf{X}$ needs to be calculated based on the received signal. Fig. 2 displays the signal spectrum of the 31 chip Gold sequence and its inverse spectrum. The rows of $\mathbf{D}^\dagger$ are the inverse signal spectrums corresponding to the spectrum of the signals with different delays. As shown in the figure, the ISM is multiplying an inverted spectrum to the incoming spectrum to make it a flat spectrum. Therefore the higher frequency components of the signal is enhanced and generates a sharper peak in time domain at the expense of the SNR because the high frequency noise is also boosted.

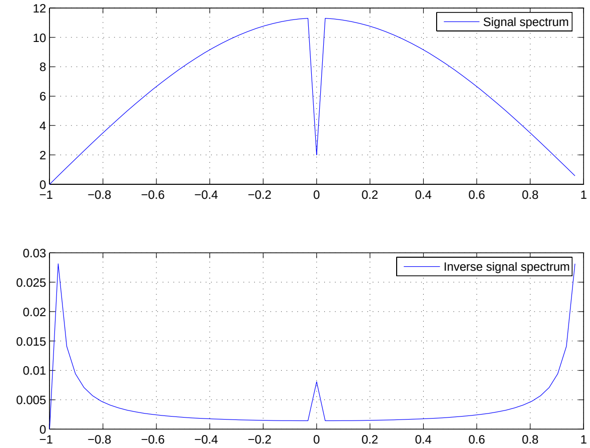

Fig. 2. Signal spectrum $\mathbf{X}$ and inverse signal spectrum $\mathbf{D}^\dagger$

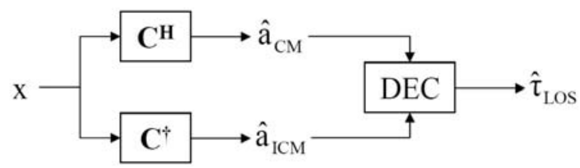

Fig. 3. Complementary implementation of correlation method and Inverse correlation method based on $\mathbf{C}$ and $\mathbf{C}^\dagger$

C. Implementation

Because the outputs of the ICM and the ISM are mathematically equivalent to each other but the ISM requires the Fourier Transform of the received signal, the ICM is easier to be adopted in the conventional receivers than ISM.

Fig. 3 displays an implementation of the ICM along with the correlation method (CM) both taking a common input and using matrix multiplications. $\hat{\mathbf{a}}_{\mathbf{CM}} = \mathbf{C^H x}$ is the output of the conventional correlator and is optimal in terms of noise suppression but less effective to mitigate multipath because of the smooth shape. Contrarily the ICM output $\hat{\mathbf{a}}_{\mathbf{ICM}} = \mathbf{C}^\dagger \mathbf{x}$ has a sharper waveform which is critical to resolve the close-in multipath. However it could suffer in low SNR cases. The conventional correlation output and the ICM output have complementary characteristics and hence the addition of the ICM to the existing structure can improve the receiver performance especially in the multipath environment.

In this paper, the correlation output and the ICM output are used separately for time delay estimation and for simplicity, the LOS is declared based on a specified threshold value. Further study is required for development of the optimal combination scheme of the two outputs.

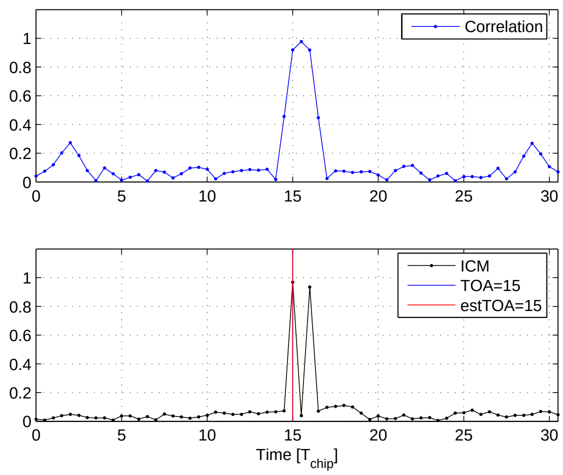

Fig. 4. Resolving two closely located multipth signals at SNR = 20 dB

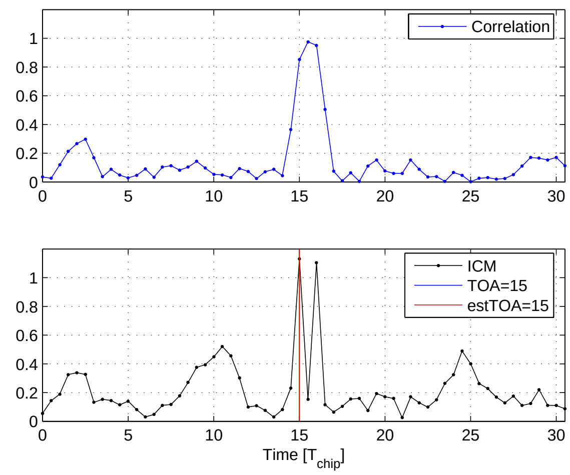

Fig. 5. Resolving two closely located multipth signals at SNR = 5 dB

IV. Results

In this section, the proposed linear estimation schemes, the ICM and the ISM are compared with the CRLB based on the Monte Carlo simulation but because of equivalency of the ICM and the ISM, only ICM result is discussed. The 31 chip Gold sequence is used as a signal and a complex baseband channel is assumed. The single sided signal bandwidth W is assumed to be same as the signal chip rate $1/T_{chip}$ and the sampling time is a half of the chip time $T_s = T_{chip}/2$ and the signal is lowpass filtered at the signal bandwidth at the transmitter and at the receiver.

Fig. 4 shows the example of the ICM processing at SNR = 20 dB where a LOS signal and a multipath are located closely ($\Delta T = 1\,T_{chip}$). The correlation functions of the LOS signal and the multipath signal are overlapped and thus the resulting correlation function becomes wider and contains uncertainty about τ_{LOS}. Its peak is actually 1 sample apart from τ_{LOS}. However the ICM output shows two distinct peaks and thus τ_{LOS} can be easily resolved without ambiguity. Fig. 5 displays the low SNR case (SNR = 5 dB) where noise components are forming many false peaks in the ICM output but the correlation output is relatively safe from them. Fig. 4 and Fig. 5 demonstrate the strength of the correlation output and the ICM to the high noise case and to the close-in multipath case respectively and their the weakness vice versa. In Fig. 6, the root-mean-square (rms) error of the time delay estimation is plotted against the time separation between the LOS and a multipath signal ΔT at SNR = 20 dB. The ICM generally outperforms the correlation output in this close-in multipath case as expected. Especially for $\Delta T = 0.4 \sim 0.65\,[T_{chip}]$, the ICM is superior to the correlation output.

The ICM is tested along with the correlation method in the AWGN channel without multipath signals and the result is shown in Fig. 7. The estimation error is plotted over the SNR from -10 dB to 20 dB. The curves are their rms errors and the straight line is the CRLB. At low SNR, they deviate from the

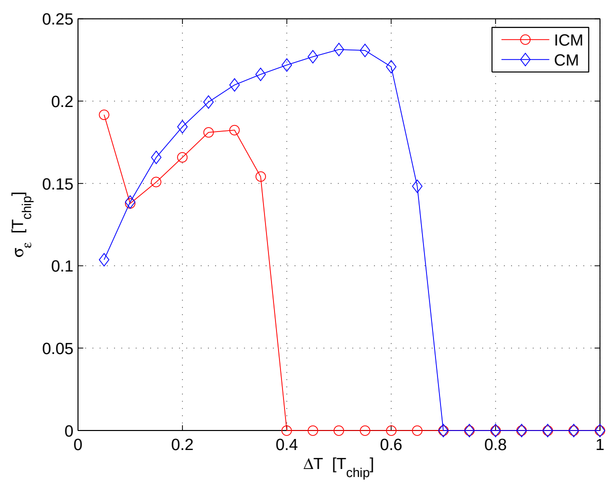

Fig. 6. RMS time delay estimation error versus time separation between LOS signal and a multipath signal

CRLB but become closer to it as the SNR increases which is predicted by the detection theory bound [8][9]. The correlation method outperforms in this AWGN case. Fig. 8 shows the case where a single multipath signal is located apart from the LOS signal with time separation $\Delta T = 0.05 \sim 1\,[T_{chip}]$ equally likely. Within this study, we focus on the close-in multipath and thus the multipath signal with ΔT less than $1\,T_{chip}$ is considered and beyond $1\,T_{chip}$, the multipath is relatively easier to resolve than the close-in multipath case. Clearly the error due to the multipath prevents the convergence to the CRLB and keeps both curves at a constant distance. It is because the CRLB only considers the AWGN and not the multipath and thus the error due to the multipath creates the gap between the CRLB and the performance of the estimators. Although the correlation method is generally better than the ICM, at high SNR the performance curves are reversed and the ICM generates less error which confirms the advantage of

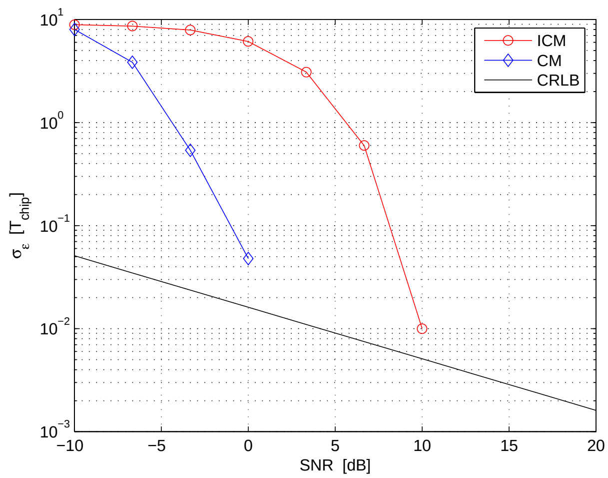

Fig. 7. RMS time delay estimation error versus SNR on AWGN channel without a multipath

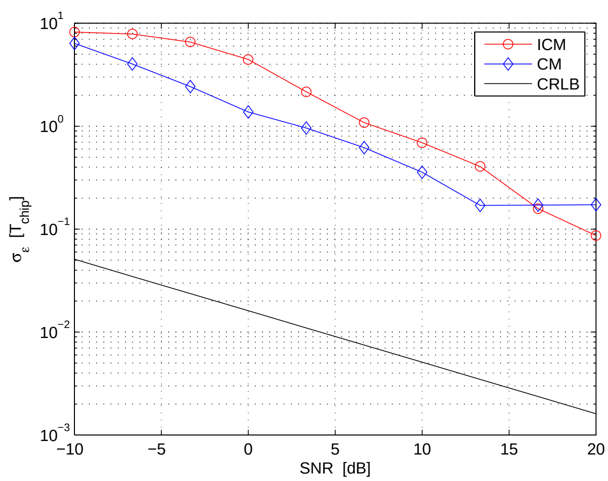

Fig. 8. RMS time delay estimation error versus SNR on AWGN channel with a single multipath ($\Delta T = 0.05 \sim 1\ [T_{chip}]$)

the ICM in high multipath and high SNR cases.

In the AWGN channel the results show that the conventional correlation output is better than the ICM and quickly converges to the CRLB. However when the SNR is high and the close-in multipath is introduced, the estimation error is dominated by the multipath error and the strong multipath resolving capability of the ICM starts to have an effect. It clearly supports that the introduction of the ICM to the existing correlator can provide an alternative interpretation of the received signal which can complement the correlator output especially in the severe multipath environment.

V. Conclusion

The accurate estimation of multipath channels is strongly desired for the precise positioning. However, it still remains as a hard problem to be solved due to the high computational burden of the estimators requiring multi-dimensional searches and the ambiguity in the close-in multipath cases. In this paper, multipath estimation problem is analyzed as the least square problems in time domain and frequency domain and the novel estimation methods, the Inverse Correlation Method (ICM) and the Inverse Spectrum Method (ISM), are proposed which can be implemented by the linear matrix multiplications without iterative searches. The channel is approximated by the discrete channel model and the ICM and the ISM are the least square fits to the received signal in time and frequency respectively. Even though there is weakness to low SNR cases, they are shown to perform effectively in the very close multipath cases and could improve the performance of the conventional correlator in the multipath environment.

The proposed methods can be easily implemented within the existing receiver structure with minimal changes and computation. Further study is in need to investigate their optimal integration with the existing correlation output.

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