# Multi-Fault Tolerant RAIM Algorithm for Hybrid GPS/TV Positioning

Ju-Yong Do, Matthew Rabinowitz, and Per Enge, Stanford University

### **BIOGRAPHY**

Ju-Yong Do is a Ph.D. candidate in Electrical Engineering at Stanford University. He received his B.S. degree from Yonsei University in Korea in 1998 and his M.S. degree from Stanford University in 2003 in Electrical Engineering. He worked for Motorola between 1998 and 2000 as a communication hardware engineer in a CDMA division. Major interests are positioning technology, wireless communication theory and estimation theory.

Dr. Matthew Rabinowitz is a consulting Professor of Aeronautics and Astronautics at Stanford University. He received his B.S. in Physics and his M.S. and Ph.D., both in Electrical Engineering, from the Stanford University in 1996 and 2000. He is the co-founder and the CTO of Rosum corp. Professor Rabinowitz's research focuses on the design of the terrestrial signal-based positioning system and the urban/indoor positioning technology.

Dr. Per Enge is a Professor of Aeronautics and Astronautics at Stanford University. He received his B.S. in Electrical Engineering from the University of Massachusetts at Amherst in 1975, and his M.S. and Ph.D., both in Electrical Engineering, from the University of Illinois at Urbana-Champaign in 1979 and 1983. Professor Enge's research focuses on the design of navigation systems which satisfy stringent requirements with respect to accuracy, integrity (truthfulness), time availability, and continuity.

# **ABSTRACT**

Terrestrial communication signals, such as television, cellular and wireless LAN signals, have been pursued as ranging sources for deeper penetration into urban and indoor areas. However, because these signals are not designed for positioning and travel in severer multipath environments, there tend to be more outlying pseudorange measurements in terrestrial signal-based positioning sys-

tems than in space (satellite) signal-based positioning systems. The resulting multiplicity of outliers makes it more challenging for receiver autonomous integrity monitoring (RAIM) algorithms to provide reliable position estimates because conventional RAIM algorithms for satellite systems assume usually a single satellite failure. To handle this multi-fault case, we modify the three conventional RAIM algorithms—the chi-square test, the horizontal protection level test (HPL) and the multi-hypothesis solution separation test (MHSS)—with iterative fault detection and exclusion steps as well as the clusterization of transmitters, and compare their performance based on hybrid GPS and TV pseudorange measurements.

### 1 INTRODUCTION

To overcome the vulnerability of satellite navigation signals in indoor areas and urban canyons, it is necessary to recruit terrestrial transmitters for positioning, because of their high signal power and broad bandwidth, promising deeper indoor penetration and even higher accuracy [1], [2]. However, these terrestrial signals from land-based communication transmitters are not maintained by centralized monitoring and calibration facilities, unlike satellite systems designed for positioning. Thus, there is higher variability in the quality of pseudorange measurements, in particular, due to clock instability and transmission time estimation errors. Furthermore, traveling through more complex ground paths, these signals carry severer multipath errors than what spatial ranging sources experience.

In our previous research, we tested the hybrid positioning system based on the combination of GPS and TV signals in the San Francisco Bay area [1]. Although the hybrid system showed higher availability than the individual systems, many outlying measurements in TV signals were observed, either causing failures in position estimation or producing high position errors. These outliers are the inherent nature of terrestrial ranging sources due to their operating environments, and should be removed for a reliable positioning

service of hybrid systems combining satellite signals with terrestrial signals.

To address this inherent difficulty in the integrated spatial and terrestrial positioning systems, we search for a multi-fault tolerant RAIM algorithm, an algorithm which can handle more than one erroneous pseudorange measurements efficiently, based on the modifications of the three RAIM algorithms: the chi-square test, the horizontal protection level (HPL) test by Brown [3], [4] and the multi-hypothesis solution separation (MHSS) test by Pervan [5], [6]. These conventional RAIM implementations are restructured with iterative fault detection and exclusion steps and their performance is compared based on the collected GPS and TV pseudorange measurements from our previous study [1].

In addition to the problem of multiple outliers, we investigate the issue of the clusterization of land-based transmitters. In contrast to the satellite navigation systems where the locations of the ranging sources are spread evenly to provide best geometries, ground communication transmitters are often clustered in a small circle of an area to take advantage of a tall building or a hill. Thus, even with a large number of transmitters, the resulting geometry could be very poor when they are not spread widely. This concept of transmitter clusterization is incorporated into the RAIM implementations and its effects are examined.

In the later parts of this paper, we introduce the three existing RAIM algorithms and their modifications for hybrid measurements and the concept of transmitter clusterization in Section 2 and compare their performance in Section 3, followed by conclusions in Section 4.

# 2 RAIM ALGORITHMS

There is a rich literature on the subject of RAIM, providing varieties of RAIM algorithms and their implementations [3]–[10] from which we borrow and modify the three algorithms: the chi-square test, the HPL test, and the MHSS test. Since they are designed to handle a single outlier, we add iterative steps to them to remove multiple outliers. Then, the concept of transmitter clusterization is incorporated into our RAIM implementations.

# 2.1 Iterative Fault Detection and Exclusion

First, the  $\chi^2$  test provides the range-domain fault detection comparing a squared norm of least square residuals  $||w||^2$  with a  $\chi^2$  test threshold  $\chi^2_{\rm th}$ , a threshold depending only on the probability of false alarm  $P_{\rm FA}$  and a degree of freedom in measurements, k. Then a disagreement among measurements can be detected by

$$||w||^2 < \chi_{\text{th}}^2(P_{\text{FA}}, k)$$
 (1)

where  $\chi^2_{\rm th}=\{\alpha|P_{\rm FA}=1-F_{\chi^2}(\alpha;k)\}$ . The number of redundant measurements, k—a degree of freedom in a  $\chi^2$ 

Table 1 A degree of freedom in measurements

Operation Mode	Hybrid	GPS	TV
3 dimensional	n-5	n-4	N/A
2 dimensional	n-4	n-3	n-3

distribution—is equal to the number of measurements, n, subtracted by the number of variables in a position estimation equation, described in Table 1. The hybrid mode, using both GPS and TV measurements, has one less redundant measurements because of two clock biases from a GPS receiver and a TV receiver, although synchronization between two receivers could reduce these biases into one bias in a fully integrated GPS/TV receiver. In case of the TV-only mode, 3 dimensional fixes are not considered due to the poor vertical geometries of ground TV transmitters, generating VDOP (vertical dilution of precision) of more than 20.

Second, the HPL test provides protection against outlying position estimates on the basis of HPL compared with a horizontal alert limit (HAL), in addition to the  $\chi^2$  test on the range domain. Because of our focus on pedestrian applications, only horizontal position errors are monitored. HPL is equal to pbiasB multiplied by a maximum slope  $\Delta_{\rm max}$ , where pbiasB represents the worst case range error bias for a given  $P_{\rm MD}$  (the probability of missed detection), a bias on a non-central  $\chi^2$  distribution on which the probability to be below  $\chi^2_{\rm th}$  is equal to a given  $P_{\rm MD}$ , and  $\Delta_{\rm max}$  represents the worst case error propagation from a range error bias to a position error

$$\begin{split} \text{pbiasB} &= \{\beta | P_{\text{MD}} = F_{\chi^2}^{\text{non-central}}(\chi_{\text{th}}^2; k, \beta^2)\} \\ \Delta_{\text{max}} &= \max_i \sqrt{\frac{\sum_{j=1}^2 (G_{ji}^\dagger)}{(P^T P)_{ii}}} \end{split}$$

where  $G^{\dagger}$  is the pseudo-inverse of a geometry matrix G and P is the parity matrix. Then the calculated HPL

$$HPL = \Delta_{max} \cdot pbiasB < HAL \tag{2}$$

is compared with a given HAL for the position domain fault detection [3], [4], and [9].

Our last approach, the MHSS test, is an attempt to directly assess the integrity risk,  $P_{\rm MD}$ , based on multiple hypotheses, each of which assumes a different set of outlying measurements. In a hypothesis,  $H_i$ , where all measurements are assumed to be healthy except the ith measurement  $\rho_i$ , we can calculate a position estimate  $\hat{x}_i$  without  $\rho_i$ . Although hypotheses can be constructed to incorporate more than one outliers, a single outlier (L=1) is assumed per a hypothesis, while multiple outliers are intended to be excluded through the iterations of fault detection and exclusion processes, since the number of hypotheses,  $1 + \sum_{l=1}^{l=L} \frac{n!}{(n-l)! \ l!}$ , explodes as more outliers are supposed. Then, there are n+1 position estimates

 $(\hat{x}_0, \dots, \hat{x}_n)$  of which  $\hat{x}_0$  is based on  $H_0$ , no outlier hypothesis. Now we can estimate the integrity risk on  $\hat{x}_0$  per a given hypothesis

$$P_{\text{MD},i} = \Pr\{||X - \hat{x}_0|| > \text{HAL}|H_i\}$$

where X is a random variable representing a true user position and  $X|H_i$  follows a Gaussian distribution with a mean at  $\hat{x}_i$ . The overall  $P_{\text{MD}}$  is the accumulation of the individual  $P_{\text{MD},i}$  weighted by the probabilities of the corresponding hypotheses,  $(P(H_0), \ldots, P(H_n))$ 

$$P_{\text{MD}} = \sum_{i=0}^{n} P_{\text{MD},i} \cdot P(H_i) < P_{\text{MD,th}}$$
 (3)

which is compared with the threshold of missed detection probability,  $P_{\rm MD,th}.$   $P(H_i)$  is based on the prior knowledge of the probability of a channel failure, for instance set to be  $10^{-3}$  for GPS channels and  $10^{-1}$  for TV channels, relatively high probabilities reflecting the challenging environments of the data collection. Apart from the original MHSS algorithm, we added the  $\chi^2$  test on the range domain to ensure protection against outliers.

While the three tests function as a fault detector, fault identification and exclusion follows the maximum likelihood test by Sturza [7], [8],

$$i^* = \arg\min_i ||p - \hat{p}_i|| = \arg\max_i \frac{w_i^2}{(P^T P)_{ii}}$$
 (4)

finding an outlying measurement minimizing the distance between the parity vector p and its reconstruction  $\hat{p}_i$  based on  $H_i$ . For the interested readers, in case of a multi-outlier hypothesis, it can be extended to

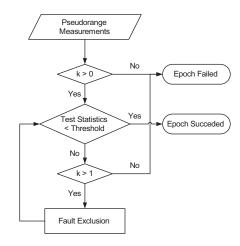
$$(i_1^*, \dots, i_k^*) = \arg\min_{(i_1, \dots, i_k)} ||p - \hat{p}_{(i_1, \dots, i_k)}||$$
 (5)

where  $\hat{p}_{(i_1,...,i_k)} = \tilde{P}\tilde{P}^{\dagger}p$  and  $\tilde{P}$  is a matrix composed of  $(i_1,...,i_k)$ th columns of the parity matrix P.

Using one of the three fault detectors and the common fault identifier, an iterative RAIM can be implemented, illustrated in Fig. 1. Starting with a set of measurements in an epoch, channels are continued to be removed until a subset of channels is found with test statistics below given thresholds, an event when the epoch is declared to be a success (or available). Otherwise, the epoch is called a failure (or unavailable).

# 2.2 Transmitter Clusterization

While navigation satellites are located in coordination to create a best geometry for ground users, TV transmitters are not placed in favor of a good geometry but for good broadcasting of individual stations, thus often located very closely taking advantage of mountains or tall buildings, as illustrated in Fig. 2. This clusteredness is common to



**Fig. 1** RAIM implementation with iterative fault detection and exclusion steps

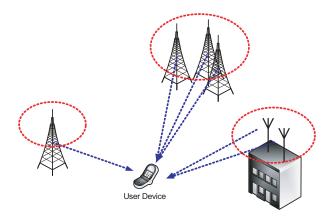


Fig. 2 Clusterization of terrestrial transmitters

most of terrestrial ranging sources, an aspect with double edges—the certain disadvantages are a worse geometry for a given number of transmitters (better if transmitters are scattered properly) and a possible missed detection of outliers due to insufficient geometric diversity, whereas the benefits are the additional assurance on range measurements in agreement and removal of outliers among range measurements from closely located transmitters.

The characteristic advantages and disadvantages of terrestrial ranging sources because of transmitter clusterization are important in RAIM implementations. In particular, we pay attention to the possibility of outlier missed detection. If Fig. 2 is revisited, even though there are 6 transmitters, a user receives signals only from three clusters (in other words three directions) in which several transmitters are located in the closely bounded areas (represented by the red circles). Since each cluster—regardless of the number of transmitters included—is viewed as a single ranging source to the user, there are only, effectively, 3 ranging sources in total, leaving no redundant measurement for fault detection and making a RAIM algorithm vulnerable to an outlier. For example, if an outlier is in a single channel clus-

Table 2 Hybrid data collection in San Francisco bay area

Category	Location	Outdoor	Indoor
Urban	San Francisco	6	4
Suburban	Palo Alto	4	5
Residential	Stanford	8	5
Rural	Halfmoon Bay	5	N/A

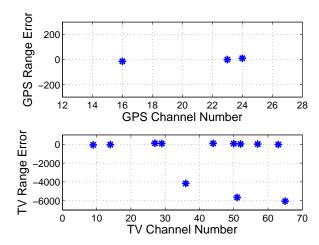


Fig. 3 An urban site in San Francisco downtown (bottom-up view)

ter (leftmost), this outlier cannot be detected reliably by a RAIM algorithm, a case we call a most-hard-to-detect outlier. Hence, for terrestrial ranging sources, instead of the number of channels, the number of clusters should be considered as a criterion determining RAIM detection capability, to reject cases with the insufficient number of clusters. In other words, the reliable RAIM detection can only be guaranteed if the number of clusters exceeds the number of variables. The clusterization effects on positioning results are discussed in the following section.

### 3 RESULTS

This section starts with the examination of the characteristics of the hybrid measurement data collected in San Francisco bay area [1], focusing on the difference between the GPS and TV range measurements, which illustrates the need of multi-fault tolerant RAIM algorithms for terrestrial ranging sources. Then we compare the three candidate RAIM algorithms by the visualization of trade-off curves between availability and accuracy (position errors)—since applications for pedestrian users are the main focus, availability and accuracy are the primary criteria in the performance comparison. From the trade-off curves, a user can navigate through different sets of availability and accuracy and choose one suitable for his/her application. Upon a choice of a point in the trade-off curves, we pursue the detailed performance statistics in categorized areas: outdoor and indoor areas; urban, suburban, residential and rural areas. Finally, the effects of transmitter clusterization is ex-



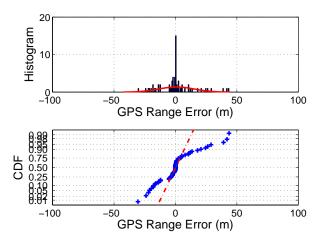
**Fig. 4** Range measurement errors (snapshot view) at an urban site (See Fig. 3)

amined.

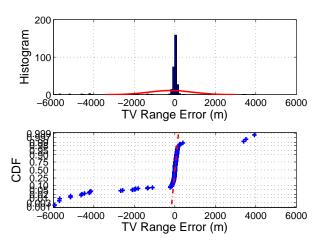
### 3.1 Characteristics of Hybrid Measurements

The fundamental difference between the GPS and TV measurements is the existence of multiple outliers on the TV measurements due to the instability of TV transmitter clocks and the absence of transmission time information (causing estimation errors on transmission time). In addition, although both signals experience multipath errors, the severity of multipath on TV signals is higher than that on GPS signals because of their terrestrial propagation paths. This difference is illustrated by an exemplary data analysis of measurements at an urban outdoor site from San Francisco downtown area (345 California avenue, San Francisco). As shown in Fig. 3, this urban site is filled with high rising office buildings and only a narrow strip of sky is visible from the ground, laying a great challenge for space navigation signals. While this limited sky view allows a marginal number of GPS satellites available for a ground user, there is abundance in the number of TV measurements (See Fig. 4 where only 3 GPS satellites and 12 TV stations were observed).

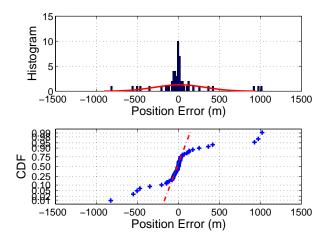
This abundant TV signals, however, cannot be utilized for positioning as they are, due to outliers. Skipping a rigorous mathematical definition of outliers—roughly, outliers can be defined as measurements not originated from an underlying distribution, presumably a zero-mean Gaussian distribution—we still can clearly identify the outlying measurements, disassociated from the rest of the measurements, a phenomenon not observed in the GPS measurements. The existence of outliers is also revealed in the distribution of range errors—the histograms and the CDFs (cumulative distribution function) of the estimated range errors—in Fig. 5 and Fig. 6 where Gaussian distribution fits are tried, marked by red lines. Even though the GPS



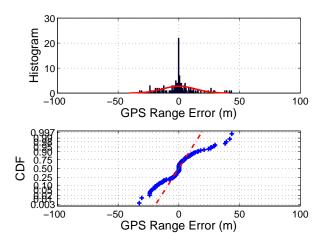
**Fig. 5** Distributions of GPS range measurement errors at an urban site (See Fig. 3) before RAIM applied



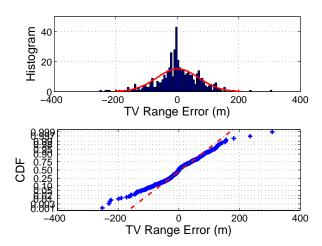
**Fig. 6** Distributions of TV range measurement errors at an urban site (See Fig. 3) before RAIM applied



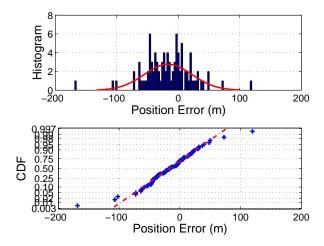
**Fig. 7** Distributions of position estimation errors at an urban site (See Fig. 3) before RAIM applied



**Fig. 8** Distributions of GPS range measurement errors at an urban site (See Fig. 3) after RAIM applied



**Fig. 9** Distributions of TV range measurement errors at an urban site (See Fig. 3) after RAIM applied



**Fig. 10** Distributions of position estimation errors at an urban site (See Fig. 3) after RAIM applied

measurements are not completely homogeneously distributed, the level of deviation from the Gaussian fit is relatively small (less than 50 m); the deviation in the TV measurements is significant, containing large outliers. The consequence of these outstanding measurements is certainly the outlying position errors, as shown in Fig. 7.

Then, a RAIM algorithm (the HPL test) is applied on the same data set and its effects are dramatic. Most of the outliers in the TV range measurements are excluded (compare Fig. 6 before RAIM and Fig. 9 after RAIM), while the cleaner GPS measurements are preserved. Consequently, there is a substantial improvement in the resulting position errors as well, now the maximum error is less than 170 m (which was 1 km before RAIM applied). At the same time, Fig. 9 and Fig. 10 show the good fitting of the resulting distributions to the Gaussian distributions, an evidence to display the RAIM function—removing outliers not belonged to the underlying Gaussian distribution.

The reader should be noted that since our data collection [1], there have been improvements on TV positioning because of the rapid technological advances in TV clock monitoring and transmission time estimation. Therefore, the quality of TV range measurements from a current system is expected to be substantially higher with less outliers than that of the data used in this paper. However, because our data set provides the worst case scenarios, it is actually good for the development of a robust RAIM algorithm and thus, continued to be used within this paper. In this section, we have witnessed the importance of a RAIM algorithm with multi-fault tolerance for the hybrid measurements and now we search for a best RAIM algorithm in the continuing sections.

# 3.2 Availability and Accuracy Trade-off

Although availability and accuracy are competing goals, because their relationship is often not linear, we could search for a trade-off point, a compromise satisfying one requirement with the least sacrifice of the other requirement or present users flexibility between these competing requirements. The trade-off curves are obtained by sweeping the RAIM parameters (for the  $\chi^2$  test,  $P_{\rm FA}=0$ –0.999; for the HPL and MHSS tests, HAL =  $10^1$ – $10^5$  m with the fixed  $P_{\rm FA}=10^{-3}$  and  $P_{\rm MD}=10^{-2}$ ), illustrated in Fig. 11–13, where each point summarizes the position estimation results at all 37 sites. Accuracy is measured in terms of rms (root mean squared) average of horizontal position errors and availability is defined to be

Availability = 
$$\frac{\text{number of succeeded epochs}}{\text{number of total epochs}}.$$

Before RAIM is applied, availability and accuracy are 92% and 1556 m, marked by the rightmost red triangle (by the  $\chi^2$  test with  $P_{\rm FA}=0$ ) in Fig. 11. After RAIM is applied—as we take more conservative approaches—availability decreases but accuracy improves. For the  $\chi^2$  test, there is

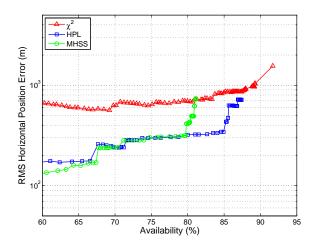


Fig. 11 Trade-off curves between availability and accuracy at all sites

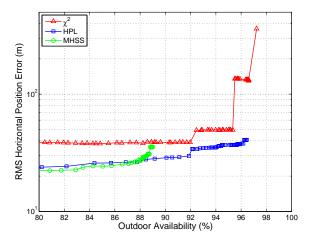


Fig. 12 Trade-off curves between availability and accuracy at outdoor sites

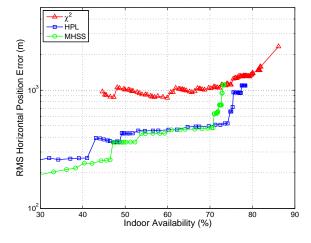


Fig. 13 Trade-off curves between availability and accuracy at indoor sites

only an insignificant improvement even at the sacrifice of availability. From the HPL test results, we can find two distinct phases: the one with improving accuracy without a loss of availability, and the other with deteriorating availability without a noticeable gain on accuracy. The HPL test is sensitive to the outliers in the first phase but not in the second phase, showing that once after the removal of outliers it becomes hard to achieve further improvement on accuracy because the remaining errors are likely to be Gaussian noises. Therefore, it is obvious to choose a point between these two phases, (85% and 344 m), as far as we are interested in high availability and the point can be used as an indicator to the bottom floor of positioning errors. The MHSS test results almost resemble the HPL results, except the 5 % additional loss of availability. In the MHSS test, position estimates from all hypotheses are required to succeed, a requirement being turn out to be more conservative for the clustered transmitters as discussed in Section 2.2. When there are only 3 clusters and one of them is with one channel, a subset (hypothesis) of channels without the single channel cluster has only 2 clusters, becoming incapable to generate position estimates and therefore being declared to be a failure. Consequently, the MHSS test declares the failure of the current set of channels and moves to the next iteration, where the number of clusters only decreases. Thus, this case can never be declared as a success within the MHSS test, even at the absence of outliers and causes dropping of more epoches. Interestingly, in the low availability region, the MHSS test performs better than the HPL test, proving its effectiveness in case of sufficient geometric diversity such as satellite-signal-only cases.

For the outdoor sites, there is little difference among the three methods, all achieving approximately 95% availability with less than 50 m (rms) error, lead by the HPL test. This small gap indicates a smaller number of outliers in the outdoor data than those in the indoor data and even the  $\chi^2$  test, without any consideration for geometries, can perform as well as the others. In the indoor results, in contrast, there is a significant gap between the  $\chi^2$  test and the rest, proving the existence of a large number of outliers.

We select three trade-off points: one for the no RAIM case (92% availability and 1556 m accuracy), one from the HPL test (85% and 344 m under the condition of  $P_{\rm FA}=10^{-2}$ ,  $P_{\rm MD}=10^{-3}$  and HAL = 13183 m), and the last from the MHSS test (80% and 316 m with  $P_{\rm FA}=10^{-2}$ ,  $P_{\rm MD}=10^{-3}$  and HAL = 2089 m). These points are some of the best compromises between availability and accuracy, in particular in indoor areas because the overall position errors are dominated by the significantly higher indoor errors. From Table 3, we can glimpse the improvement by the RAIM algorithms. The accuracy improves in almost an order of magnitude. In particular, the HPL test shows the best efficiency, reducing errors from 361 m to 38 m while losing only 1.5% availability. The improvements by the integration of GPS and TV measurements are also summarized in

**Table 3** Selected trade-off points between availability (%) and accuracy (m)

Location	RAIM	Availability	Accuracy
Overall	No RAIM	91.7	1556
	HPL	84.9	344
	MHSS	79.7	316
Outdoor	No RAIM	97.2	361
	HPL	95.7	37.5
	MHSS	88.7	30.8
Indoor	No RAIM	86.2	2340
	HPL	74.1	524
	MHSS	70.7	481

**Table 4** Effects of GPS/TV integration on availability (%) and accuracy (m) with the HPL test

Location	Signal	Availability	Accuracy
Overall	GPS only	50.8	51.3
	TV only	77.5	449
	Hybrid	84.9	344
Outdoor	GPS only	84.0	47.0
	TV only	80.4	353
	Hybrid	95.7	37.5
Indoor	GPS only	17.6	58.8
	TV only	74.7	551
	Hybrid	74.1	524

Table 4 (results from the HPL test). In overall, the integration brings 34 % gain on availability compared to the GPS-only case, while the gain reaches 56% at the indoor sites. Although the availability increase in the indoor areas comes with degradation of accuracy, we see improvements both in accuracy (47 m to 38 m) and availability (84% to 96%) at the outdoor sites.

The positioning results at these points are further analyzed with detailed statistics in the categorized areas: urban, suburban, residential, and rural areas. First, the availability of the two methods—the HPL test and the MHSS test at their selected trade-off points—is compared in Fig 14, whereas the no RAIM case provides the upper limit on availability. The HPL test shows a very stable outdoor availability, with a just average 1.5% loss from the no RAIM case, 99.6% and 99.8% respectively at the suburban and residential areas where both TV and GPS command high availability, 96% at the rural areas where TV has a weaker coverage. and 87% at the urban areas where GPS signals are substantially blocked by buildings. Even though the hybrid system still inherits the weakness of the GPS and the TV navigation systems at urban and rural areas respectively, the integrated GPS/TV range measurements provide the significantly enhanced availability compared to the individual systems. The MHSS test follows the HPL test closely but the gap is widened in more densely populated areas,

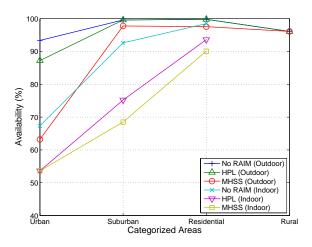


Fig. 14 Availability in categorized areas

in particular at the urban sites where the gap reaches 24%. This availability loss reminds us of the conservatism of the MHSS test on clustered transmitters, as seen in the trade-off curves.

The tendency of availability—low in urban and high in other areas—is more clearly observed in the indoor results. While urban indoor areas remain as the most challenging environments with the availability of 54% for both tests, we see increases up to 94% as we move to the suburban and residential indoor sites. The availability of the hybrid system depends entirely on the TV availability at the urban and suburban sites where the GPS availability is 0% and 2% respectively, except the residential sites where the GPS works in approximately half of the time.

Second, the accuracy plots are given in terms of a 67 percentile circular error probable (CEP) and a 95 percentile CEP, respectively in Fig. 15, Fig. 16 and Fig. 17. There is no noticeable difference between the accuracy of the HPL test and that of the MHSS test, positioning the HPL test as a better performer because of its higher availability given the equivalent accuracy. In the 67 percentile CEP where outlying position estimates are not expected to show up, the poor accuracy of the no RAIM case is less noticeable in Fig. 15. But we can observe an order of magnitude accuracy improvement by the RAIM algorithms in the 95 percentile CEP and the rms position errors (see Fig. 16 and Fig. 17).

Beyond the comparison between the RAIM algorithms, we can envision the measurement environments through these test statistics: availability and accuracy. All indoor and urban areas prove themselves to be challenging environments, resulting in low availability as well as low accuracy, more evident as we move to densely populated areas, residential to suburban and suburban to urban areas. This environmental complexity remains as a great challenge and emphasizes the need for integration among various kinds of (space and terrestrial) ranging sources for improvements in both availability and accuracy.

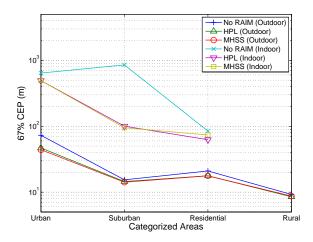


Fig. 15 Circular error probable (67%) in categorized areas

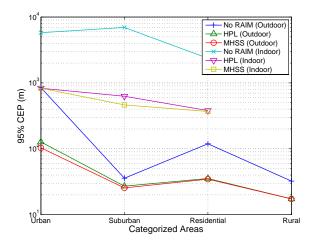


Fig. 16 Circular error probable (95%) in categorized areas

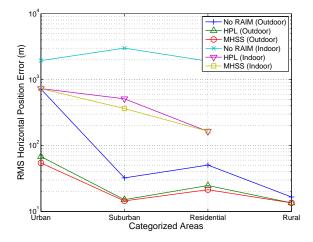


Fig. 17 Horizontal position error (rms) in categorized areas

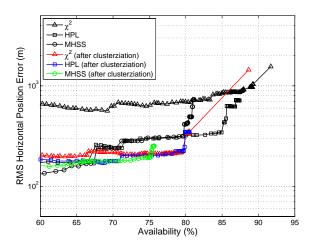


Fig. 18 Trade-off curves between availability and accuracy after transmitter clusterization

**Table 5** Effects of transmitter clusterization on availability (%) and accuracy (m) with the HPL test

Location	Clusterization	Availability	Accuracy
Overall	before	84.9	344
	after	79.7	225
Outdoor	before	95.7	37.5
	after	95.5	36.7
Indoor	before	74.1	524
	after	63.9	341

# 3.3 Transmitter Clusterization

By rejecting cases with the insufficient number of clusters, higher level of protection against outlying position fixes can be achieved, shown in Fig. 18 and summarized in Table 5. While there is little change in the outdoor results because there is a higher chance to collect signals with favorable geometric diversity (from more than 3 clusters), there is a significant reduction, at the indoor sites, of position errors (524 m to 341 m) but at the cost of availability (74% to 64%), revealing that many indoor sites experience severe path loss and blockage and thus there are more marginal cases of signals reception from 3 or 4 clusters. The consequences of the clusterization—the improved accuracy and the degraded availability—are because of the enforcement of the stricter requirement (geometric diversity in terms of the number of clusters) and can be understood as a part of trade-offs in RAIM implementations for terrestrial ranging sources.

# 4 CONCLUSION

Our previous study had shown a significantly improved availability when the GPS signals are combined with the TV signals:

• GPS only: 51% (availability) and 52 m (accuracy)

• Hybrid: 92% and 1556 m

, based on GPS/TV hybrid data collected at sites including urban canyons and indoor locations. However, the unsatisfactory accuracy have lead us to realize and implement the two important aspects in adoption of terrestrial ranging sources: a multi-fault tolerance by iterative fault detection and exclusion steps; and transmitter clusterization to prevent missed detection of outliers. After these two aspects have been implemented as parts of a RAIM algorithm and tested on the same hybrid data, the best compromises between availability and accuracy have been chosen from their trade-off curves generated by sweeping RAIM parameters. First, the iterative fault exclusion process has been shown to be effective in removal of outlying measurements and generating reliable position estimates even at the presence of multiple (more than 3 or 4) outliers.

• Hybrid + HPL: 85% and 344 m

Second, the clusterization—grouping closely located transmitters as a cluster and using the number of clusters as an index to fault detection capability—have disclosed many undetected outlying cases where transmitters were clustered and in lack of geometric diversity.

• Hybrid + HPL + Clusterization: 80% and 225 m

In both applications, the improved accuracy have accompanied the inevitable decreases of availability because of the stricter screening of outliers.

Along with these two developments in RAIM algorithms for terrestrial sources, the rapid improvements in TV clock monitoring and transmission time estimation are expected to bring the hybrid GPS/TV positioning closer to us.

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