

# System Identification of a Farm Vehicle Using Carrier-Phase Differential GPS

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## BIOGRAPHY

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## ABSTRACT

The automatic operation of farm vehicles can have great benefits both in farm productivity and hazardous or impossible operations. Automatic control offers many potential improvements over human control; however,

previous efforts have failed largely due to sensor limitations. Carrier Phase Differential GPS (CDGPS) is an enabling technology that provides a high-bandwidth, low-noise measurement of multiple vehicle states. System identification techniques can then be used to generate a mathematical model for automatic control system design and implementation.

In this work, previous controls research on a large tractor test bed is extended to demonstrate two different methods of system identification. Using *a priori* knowledge of the tractor dynamics, an extended Kalman filter is implemented and demonstrates model parameter identification. A Linear Quadratic Regulator (LQR) controller, based on these parameters, performs closed loop line tracking with a demonstrated error of better than 1.8 cm standard deviation.

The same data is used with the Observer/Kalman Filter Identification (OKID) method, which assumes no *a priori* information about the system dynamics. It is shown that the estimator/controller designed with this system demonstrates equivalent experimental performance. The OKID methodology differs from the extended Kalman filter by utilizing solely the input and output streams to determine the structure and order of the plant model.

## INTRODUCTION

Autonomous guidance of ground vehicles is not a novel idea. Previous attempts have largely failed due to sensor limitations. Some experimental systems require cumbersome auxiliary guidance mechanisms in or around the field [1,2]. Others rely on vision systems that require clear daylight, good weather, or field markers that require deciphering by pattern recognition [3,4]. Since the advent of modern GPS receivers, a single, low-cost sensor is now available for measuring multiple vehicle states. GPS-



Figure 1 — GPS-Equipped Tractor

based systems already have a myriad of uses in land-based vehicles, including meter level code-differential techniques for geographic information systems (GIS) [5-7], driver-assisted control [8], and automatic ground vehicle navigation [9].

Using precise differential carrier-phase measurements of satellite signals, CDGPS has demonstrated centimeter-level accuracy in position measurements [10], and  $0.1^\circ$  accuracy in vehicle attitude determination [11]. System integrity becomes greatly enhanced with the augmentation of the GPS satellite signals by ground based pseudo-satellites [12]. Accurate and reliable measurements of multiple states by CDGPS lend themselves to system identification, estimation, and automatic control. CDGPS-based control systems have been utilized in such varying platforms as a model airplane [13], a Boeing 737 aircraft [10], an electric golf cart [14], and a farm tractor [15].

The process of constructing models and estimating unknown parameters from experimental data is referred to as “system identification.” Complex phenomena can exceed our scientific knowledge and ability to predict plant dynamics. Instead data from carefully-constructed experiments are used to build an adequate mathematical model for control. The goals of system identification are different from physical modeling. Physical modeling attempts to understand the entire process. In contrast, system identification adequately models the plant’s characteristics only to the extent of mapping the input/output behavior sufficient for controlling the plant.

This paper focuses on the system identification and subsequent controller design of a farm tractor using CDGPS as the only sensor of vehicle position and attitude. System identification can provide the mathematical model necessary to implement an automatic control system, by careful analysis of the input and output data of a dynamic system. Two different methods of using the acquired data are presented to generate linear models. These are used to generate automatic control systems, and then tested on a large farm tractor.

## EXPERIMENTAL SETUP

The primary goal of this work was to experimentally demonstrate system identification and precision closed-loop control of a farm tractor using CDGPS as the only sensor of vehicle position and attitude. This section describes the hardware used to accomplish this goal.

**Vehicle Hardware:** The test platform used for vehicle identification and control testing was a John Deere Model 7800 tractor (Fig. 1). Four single-frequency GPS antennas

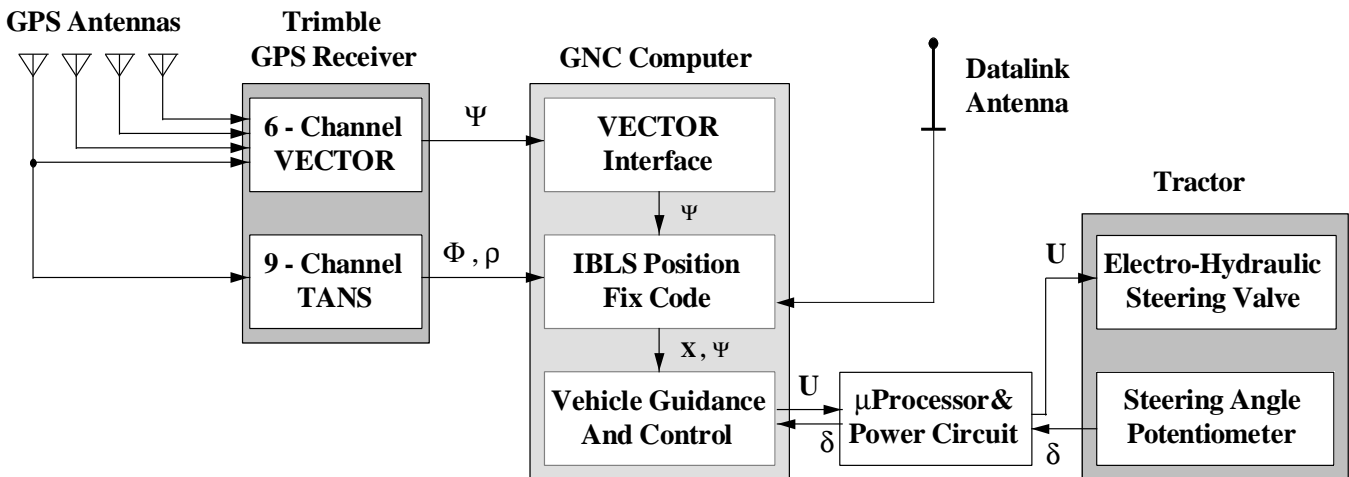


Figure 2 — GPS Hardware Architecture

were mounted on the top of the cab, and an equipment rack was installed inside. Front wheel angle was sensed with a potentiometer—the only non-GPS sensor used in the system—and actuated using a modified Orthman electro-hydraulic steering unit. A Motorola MC68HC11 microprocessor board was the communications interface between the control computer and the steering unit. The microprocessor converted serial commands from the control computer into pulse-width modulated signals which were sent to the power circuits that control the steering valves. Wheel position was sampled and digitized by the HC11 and sent to the controls computer at 20 Hz.

**GPS Hardware:** The CDGPS-based system used for vehicle position is similar to the one used by the Integrity Beacon Landing System (IBLS) [10]. A four-antenna, six-channel Trimble Vector receiver produced attitude measurements at 10 Hz. A single-antenna Trimble TANS PC-card receiver produced code- and carrier-phase measurements—which were used to calculate vehicle position—at 4 Hz. An Industrial Computer Source Pentium-based PC running the LYNX-OS real-time operating system performed the attitude interface, position calculations, data collection and controls calculations using software written at Stanford.

The ground reference station consisted of a Hewlett-Packard 386 PC, a TANS 9-channel PC-card, and software written at Stanford to broadcast phase corrections to the tractor through a Pacific Crest radio modem at 4800 bps. A block diagram representation of the hardware is pictured in Figure 2.

### VEHICLE MODELING AND SYSTEM IDENTIFICATION DATA COLLECTION

The non-linearities of the steering potentiometer and electro-hydraulic actuator have been calibrated in previous work at Stanford [15]. The non-linearities are “linearized” through a look-up table implemented in the Guidance-Navigation-Control (GNC) computer.

Agricultural farm vehicles must be able to operate over various types of terrain and with a variety of implements. While previous work at Stanford has demonstrated closed-loop line following based on a simple kinematic model to a remarkable precision [15], the model is based on assumptions that are known to be false.

The kinematic model (Fig. 3), based on simple geometry rather than inertias and forces, assumes both a constant velocity along the path, as well as no wheel slip. While the velocity may not vary a great deal, it is not constant, and the four wheel drive on the tractor cannot move the vehicle forward without slipping the wheels.

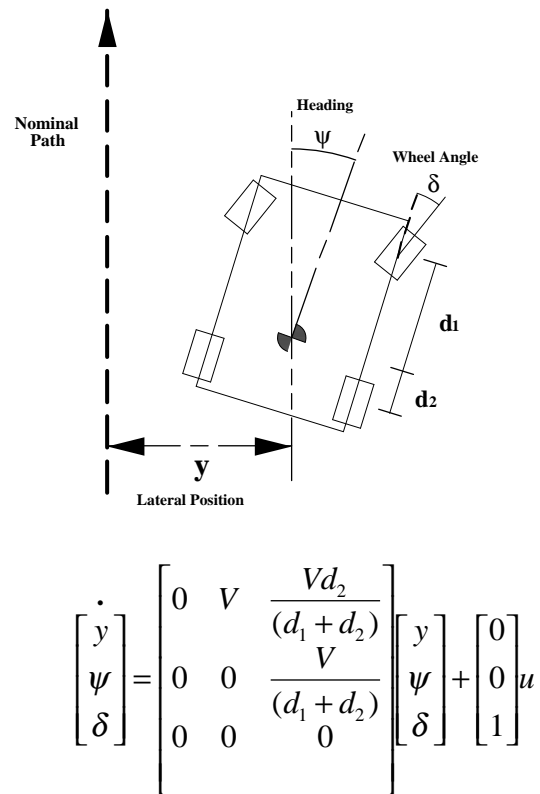


Figure 3 — Simple Kinematic Model

Vehicle conditions can change, and it is expected that time-varying methods or adaptive controls methods will be required to achieve good line-following performance in these changing environments. Furthermore, the vehicle dynamics may change a great deal with different implements and/or soil types. Slopes or ground texture may also affect vehicle performance to the extent that a simple model based on geometric kinematics may not be adequate to control the tractor.

In order to gather data to perform a proper system identification of the farm tractor, a series of ten open-loop line following tests were run in which a human driver, through the GNC computer, caused the steering to either slew left or right at the maximum steering rate. Also, the driver commanded the steering rate to zero through the electro-hydraulic actuator in order to track a roughly straight line. This “pseudo”-random input was designed to apply the maximum power to the tractor through the controls and produce a rich output that would contain information from all modes of interest. A typical pass for system identification is pictured in Figure 4.

These data passes were run without an implement, in first gear, at a forward velocity of approximately 0.33 m/s (0.7 mph), and were subsequently used for calculation and

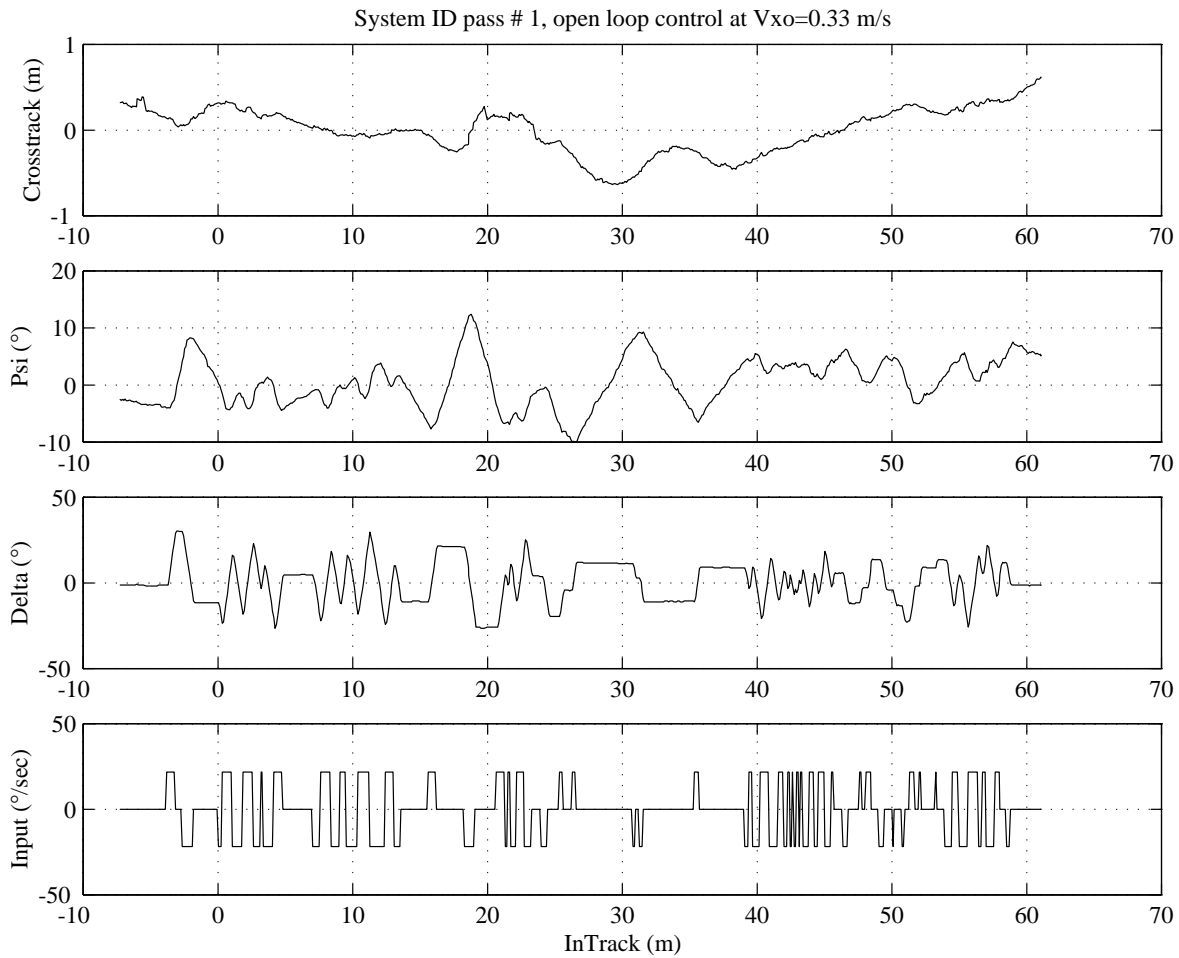


Figure 4 — Typical System Identification Pass

validation of a linear plant model. In this particular research, the data was gathered separately and then processed as a “batch” for identification. Further research will allow the identification process and the data gathering process to occur simultaneously for “on-line” identification.

### THE EXTENDED KALMAN FILTER IDENTIFICATION

A Kalman filter is a computational algorithm that processes measurements of the inputs and outputs to deduce a minimum error estimate of the state of a system. It does this by utilizing knowledge of system and measurement dynamics and assumed statistics of process and sensor noises. An extended Kalman filter is a slightly different computational algorithm for calculating the minimum variance estimate of the state as a function of time and accumulated data for non-linear systems.

In general, an initial guess of the state is propagated through time until the next measurement is available, and then a measurement update is computed using the lin-

earized equations of the system (linearized about the current state) [16]. The appropriate formulae for implementation are summarized in Table 1.

In the case of the tractor, as can be seen from the equations of motion of the tractor [Eqn. 1], there are only three parameters not directly measured in this system:  $V$ —the tractor velocity along track,  $d_1$ —the distance from the front axle to the “center” of the vehicle, and  $d_2$ —the distance from the rear axle to the center of the vehicle.

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & V & \frac{Vd_2}{(d_1+d_2)} \\ 0 & 0 & \frac{V}{(d_1+d_2)} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

<Eqn. 1>

The state of tractor is augmented to include these three

$$\begin{aligned} \dot{x} &= f(x) + \Gamma u_k \\ \dot{P} &= F(\hat{x}_k)P + PF^T(\hat{x}_k) + Q \\ F &= \left. \frac{\partial}{\partial x} f(x) \right|_{x=\hat{x}} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k(y_k - C\hat{x}_k^-) \\ P^+ &= P^-(I - K_kC)P^- \\ K_k &= P^-C^T[CP^-C^T + R]^{-1} \end{aligned}$$

Table 1 — Extended Kalman Filter Equations

parameters, which are then estimated along with the rest of the state. These three parameters are assumed to have no dynamics, i.e. they are constant and do not change with time. At each time step, the state transition matrix is recalculated and propagated forward along with the covariance of the states. The measurements are filtered according to the algorithms in Table 1, and the process stepped through all

of the available data collected during the system identification passes. The results of this filtering can be seen [Fig. 5] to converge from initially poor estimates of these parameters to values that are consistent with the measurements of  $V$ ,  $d_1$  and  $d_2$ .

Note that the values given by the EKF method are slightly different from the values used in previous controllers [15], but are close enough that the original controller still achieved a remarkable line-following precision. This performance, in light of mis-modeling, can be accounted for by the known robust performance of an LQR controller [17].

A new line-tracking controller was designed using the improved estimates of the parameters  $V$ ,  $d_1$  and  $d_2$ , with the same cost functions as the previous controllers. It is important to note that no attempt was made to take advantage of the new information to increase controller performance, and that the only performance gains should be as a direct result of better plant modeling. In addition, a controller based on the EKF results—but including an integral state to offset any constant errors such as steering

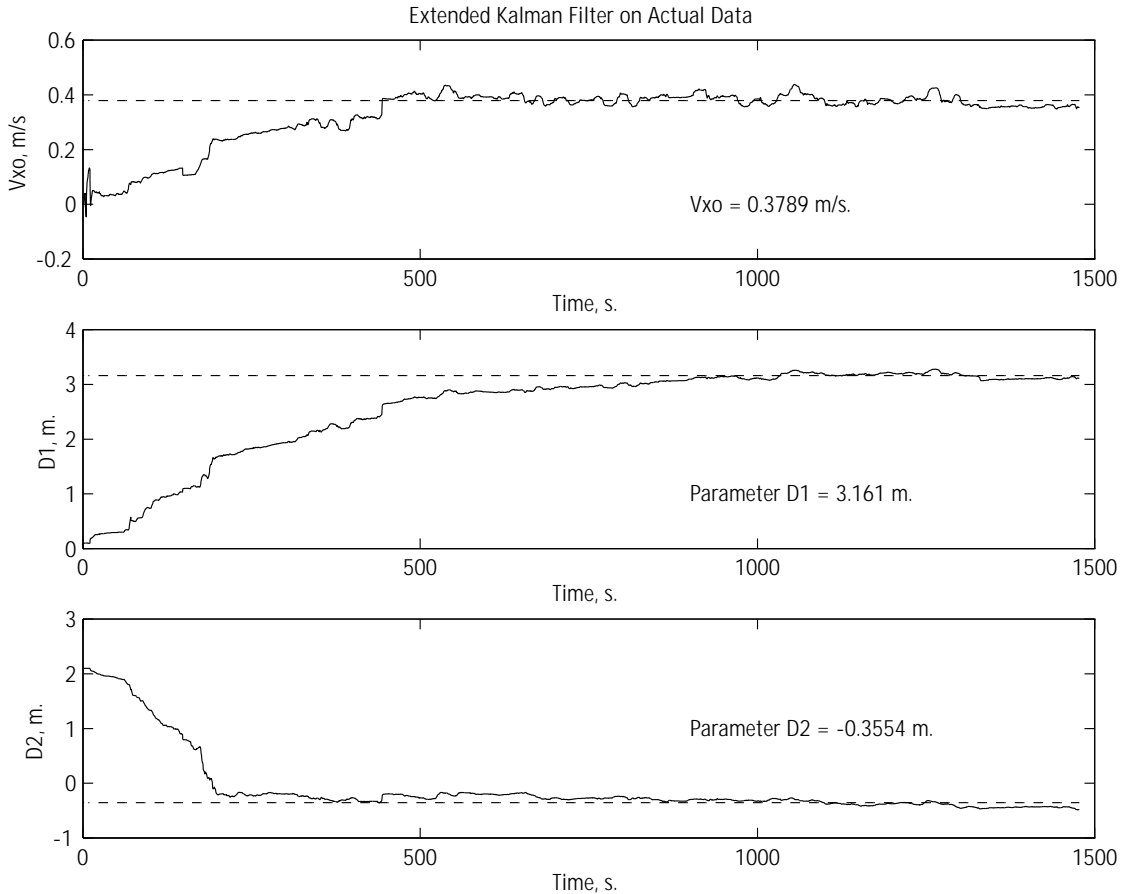


Figure 5— Extended Kalman Filter Convergence

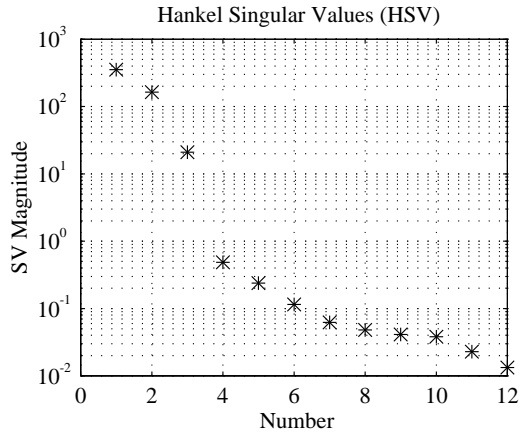


Figure 6 — SVD of Hankel Matrix

bias or ground slope—was implemented to demonstrate increased robustness to disturbances.

Although system identification was performed “off-line”, the method used can be set up into an adaptive scheme whereby the new controller is designed and implemented at every new time-step based on the values of  $V$ ,  $d_1$  and  $d_2$ . “On-line” system identification can be used as a basis for adaptive control methods that will learn the dynamic behavior of the tractor and adapt as a function of time. The varying field conditions and implements that will be attached to the tractor beg for a solution that is adaptive as opposed to a conventional “fixed” controller that is designed and implemented entirely off-line.

### THE OBSERVER/KALMAN IDENTIFICATION PROCESS

A method of system identification that uses only input and output data to construct a state-space realization of the system is the Observer/Kalman Identification (OKID) process, developed at NASA Langley [18]. Given a linear, discrete-time state-space system, the equations of motion can be summarized as follows:

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k \\y_k &= Cx_k + Du_k\end{aligned}$$

Note that the triplet,  $[A,B,C]$  is not unique, but can be transformed through any similarity transform to another set of coordinates. However, the system response from rest when perturbed by a unit pulse input, known as the system Markov parameters, are invariant under similarity transforms. These Markov parameters are:

$$Y_0 = D, Y_1 = CB, Y_2 = CAB, \dots, Y_k = CA^{k-1}B$$

When assembled into the generalized Hankel matrix, the

Hankel matrix can be decomposed into the Observability matrix, a state transition matrix, and the Controllability matrix, thus the Hankel matrix (in a noise free case) will always have rank  $n$ , where  $n$  is the system order.

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+\beta-1} \end{bmatrix}$$

$$H(k-1) = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{\alpha-1} \end{bmatrix} A^{k-1} \begin{bmatrix} B & AB & A^2B & \cdots & A^{\beta-1}B \end{bmatrix}$$

Because noise will corrupt this rank deficiency of the Hankel matrix, the Hankel matrix is truncated by a singular value decomposition at an order that sufficiently describes the system. This truncated Hankel matrix is then used to reconstruct the triplet  $[A,B,C]$  and is referred to as the Eigensystem Realization Algorithm (ERA). A modified version of this algorithm that includes data correlation is used to identify the tractor. A more complete treatment of the subject can be found in [18].

For any real system, however, system pulse response cannot be obtained by simply perturbing the system with a pulse input. A pulse with enough power to excite all modes above the noise floor would likely saturate the actuator or respond in a non-linear fashion. The pulse response of the system can, however, be reconstructed from a continuous stream of rich system input and output behavior. Under normal circumstances, there are not enough equations available to solve for all of the Markov parameters. Were the system asymptotically stable, such that  $A^k=0$  for some  $k$ , then the number of unknowns can be reduced. The identification process would be of little value if it could only work with asymptotically stable systems.

By adding an observer to the linear system equations, the following transformation can take place:

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + Gy_k - Gy_k \quad [\text{add zero}] \\x_{k+1} &= [A + GC]x_k + [B + GD]u_k - Gy_k \\x_{k+1} &= \hat{A}x_k + \hat{B}v_k\end{aligned}$$

$$\hat{A} \equiv [A + GC], \hat{B} \equiv [B + GD \quad -G], \text{ and } v_k \equiv \begin{bmatrix} u_k \\ y_k \end{bmatrix}$$

Thus, the system stability can be augmented through an

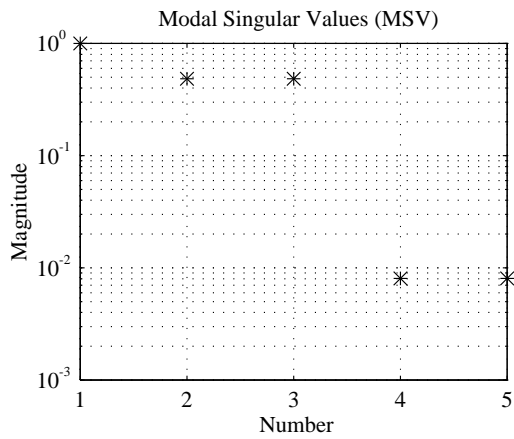


Figure 7 — Modal Singular Values for OKID

observer, and the ideal Markov parameters established through a least-squares solution [19]. It is useful to note that the realization also provides a pseudo-Kalman

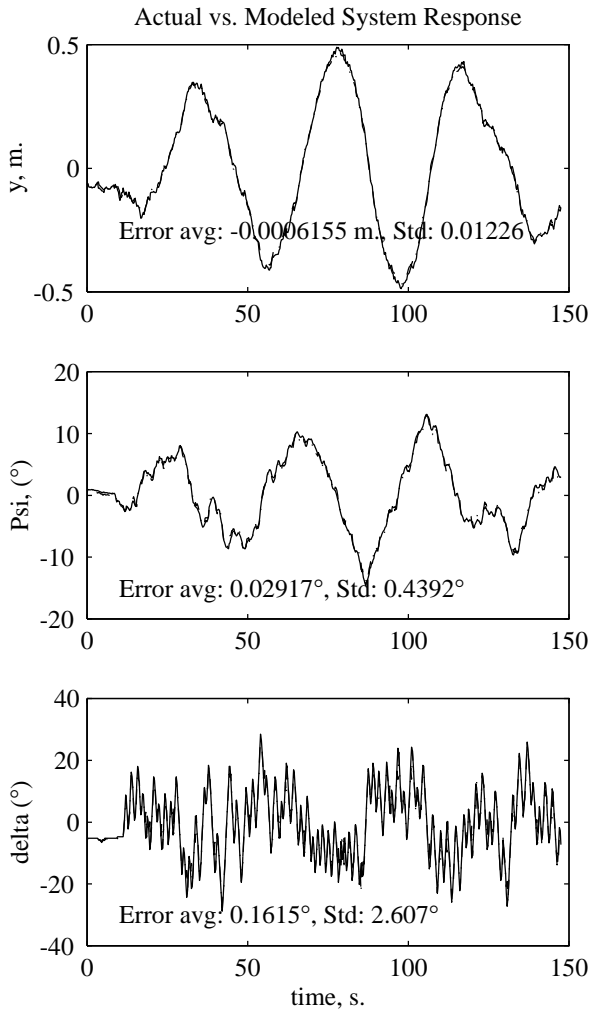


Figure 8 — OKID Reconstruction vs. Actual Data

observer. The observer orthogonalizes the residuals to time-shifted versions of both input and output. This makes controller design a much simpler process. An improved version of the OKID process, which includes residual whitening [20], was used to identify the farm tractor from the experimental data.

A singular value decomposition (SVD) of the tractor Hankel matrix demonstrates a very large drop in the magnitude of the singular values from the third to the fourth, indicating a system order of  $n=3$  [Fig. 6]. In addition, modal singular values of all tractor models of order higher than three exhibit a two order of magnitude drop from the third mode to modes higher than three [Fig. 7]. As an experimental check, the identified model (with observer) was compared to a system identification pass that was not used in the OKID computations [Fig. 8], as can be seen, the match is excellent between the modeled and actual data, with a biases and standard deviations summarized in Table 2.

	bias	one- $\sigma$
Lateral, $y$ (m)	-0.001	0.01
Angular, $\psi$ ( $^\circ$ )	0.030	0.44
Steering, $\delta$ ( $^\circ$ )	0.16	2.61

Table 2 —OKID reconstruction vs. Actual Data

Thus, using the OKID model and estimator, a controller was designed to again match the previous performance of the tractor guidance system [15]. The gain matrix  $[K]$  was calculated using standard LQR techniques, with the quadratic cost function a weighting on the outputs. The results of the OKID method were similarity transformed into a set of coordinates that matched the mapping with

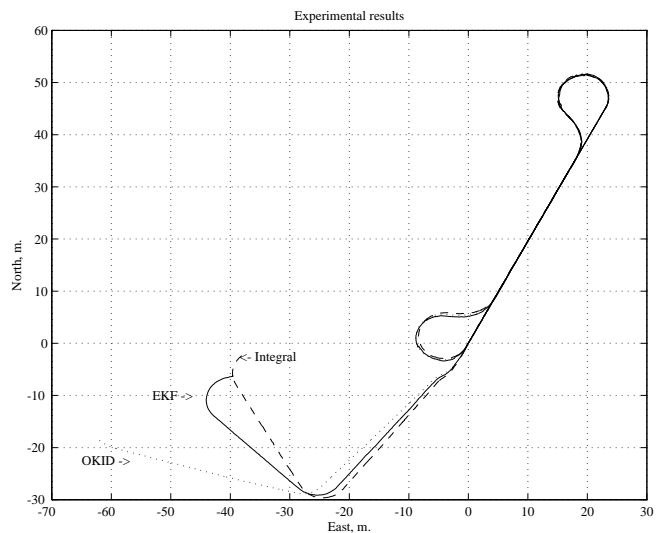


Figure 9 — Overhead View of Tractor Path

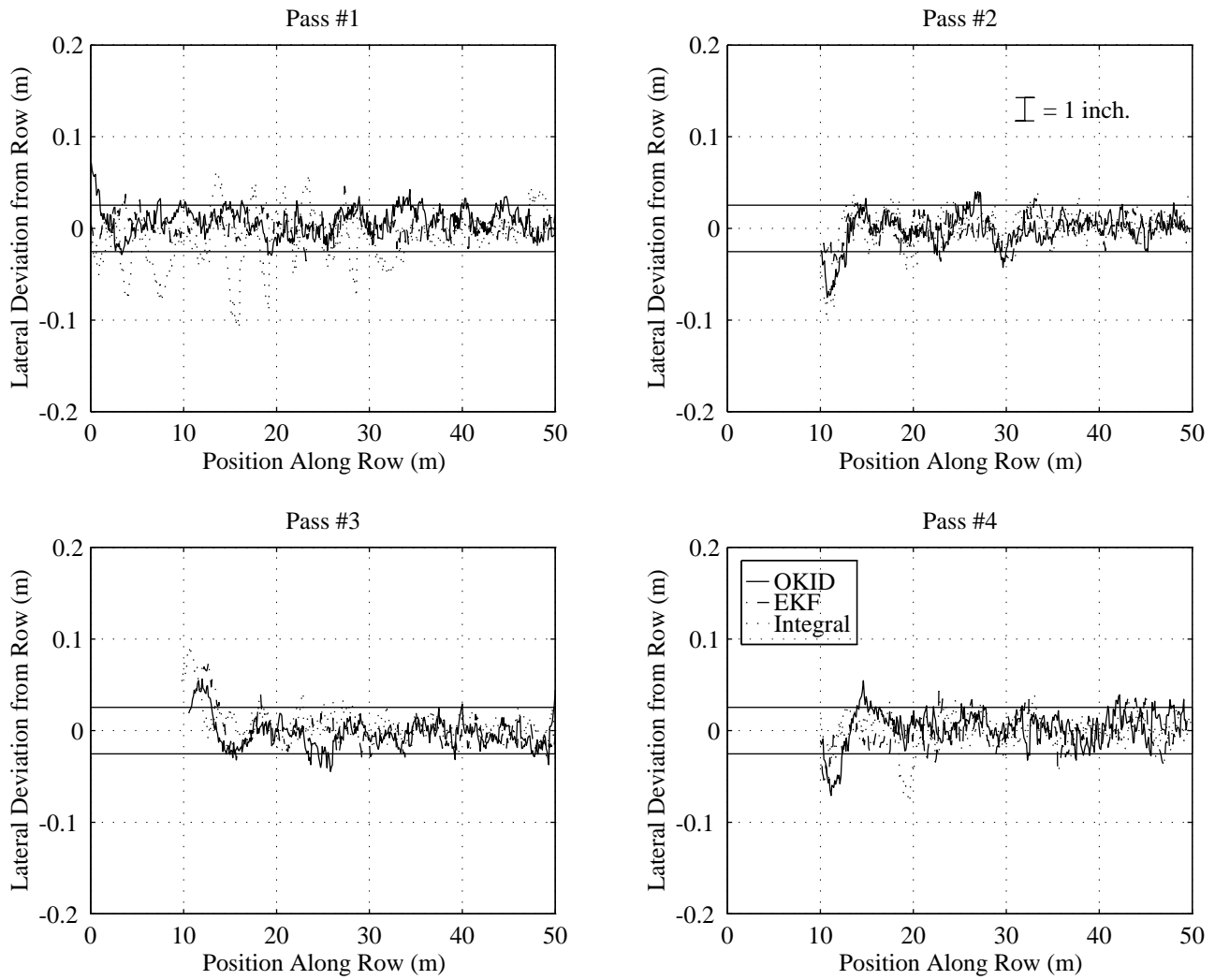


Figure 10 — Close-in View of Tractor Passes

previous work at Stanford. This ensured that the physical intuition as to how the controller was operating would not be lost to mathematically convenient coordinates. The coordinate transformation was such that the states were again directly measured. No attempt was made to tighten the control loops or increase the bandwidth of the controller, merely to demonstrate that a controller could be designed and implemented with little or no *a priori* knowledge of the system dynamics.

## EXPERIMENTAL RESULTS

Three controllers, two based on the extended Kalman filter algorithm (proportional and proportional-integral controllers) and another based on the OKID identification method were implemented on the GPS-equipped farm tractor at Stanford.

The tractor, without an implement, was set into first gear (0.33 m/s) and commanded to follow a “row” 50 meters long, with all tractor guidance under computer control. CDGPS integer ambiguities were initialized by driving the tractor as closely as possible to a surveyed location and manually setting the position estimate. Each of the controllers was used to drive the tractor back and forth four times over the same row. Note that only the “tight line-tracking” was controlled by the newly designed controllers. The bang-bang control used for waypoint navigation, line acquisition, and U-turns remains unchanged from previous work [15].

The results of the three controllers are shown as pictured from overhead in Figure 9. At this scale, the three controllers cannot be differentiated from each other. In order to better visualize the accuracy of the controllers, a closer view of the line is presented in Figure 10, along with lines



<b>bias (cm)</b>	<b>Pass 1</b>	<b>Pass 2</b>	<b>Pass 3</b>	<b>Pass 4</b>
OKID	1.03	-0.22	-0.27	0.31
EKF	0.4	0.12	-0.24	0.3
Integral	-1.01	0.04	1.05	-0.05
<b>1-<math>\sigma</math> (cm)</b>	<b>Pass 1</b>	<b>Pass 2</b>	<b>Pass 3</b>	<b>Pass 4</b>
OKID	1.86	1.85	1.7	1.92
EKF	1.34	1.53	1.77	1.77
Integral	3.16	2.12	1.84	1.79

Table 3 — Summary of Line Following Results

delineating one inch to either side of the desired path. As is clearly visible, all three controllers kept the tractor closely following the line, with rare excursions outside an inch to either side of desired path. The only controller that experienced larger excursions was the integral controller, which is known to be lower performance. The means and standard deviations of the three controllers are summarized in Table 3. The controllers controlled to a  $2\text{-}\sigma$  of less than 5 cm, and excepting the integral controller on pass #1, *maximum excursions from the desired path were less than 5 cm.*

The integral controller is known to possess worse performance, but can mitigate this deficiency by demonstrating an increased steady-state error rejection. Further research along these lines, as well as exploring the areas of adaptive control is currently in progress.

## CONCLUSIONS

As a continuation of work completed at Stanford, this research is significant because it extends the understanding and implementation of a safe, low-cost control system for high-accuracy control of a ground vehicle. Farm tractor data was used for system identification and control synthesis, with a practical demonstration of vehicle control based on data alone—with no *a priori* knowledge of the system. A constant gain controller, based on the identified dynamics, was demonstrated to control the tractor along straight lines with a lateral position error of better than 1.9 cm standard deviation.

These results suggest that the transition from control of a lone tractor to control of the tractor and implement can be accomplished without an accurate physical model of implement-soil interaction. Current research indicates that the use of GPS-derived measurements for system identification and subsequent controller synthesis is an excellent starting point for an adaptive control scheme.

Further research is in progress to explore these adaptive methods, as well as exploring other challenges in the

automatic control of farm vehicles. Pseudolite integration into our system for "on-the-fly" integer ambiguity resolution and integrity enhancement, curved path following, Driver graphical displays, and path planning are all current topics of research.

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