Galileo-GPS RAIM for Vertical Guidance

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ABSTRACT

In the next ten years the number of pseudorange sources and their quality is expected to increase dramatically: The United States is going to add two new civil frequencies (L5 and L2C) in the modernized GPS, and the European Union is planning to launch Galileo, which is planned to be fully operative before 2015, also with multiple frequencies. By combining two frequencies, users will be able to remove the ionospheric delay which is currently the largest error, thus reducing nominal error bounds by more than 50%. This reduction in nominal error bounds together with the large number of satellites is not only going to increase the accuracy of the positioning, but more importantly, it is going to increase the robustness against satellite failures (or other range errors), even without augmentation (e.g., Inertial Reference Unit (IRU), baro-altimeter). Preliminary studies suggest that, using Receiver Autonomous Integrity Monitoring (RAIM), it might be possible to provide a 50m Vertical Alert Limit (VAL) worldwide, with a bound on the maximum error, even in the event of one satellite failure, one constellation failure or a multiple satellite failure.

The purpose of this work is to investigate which VALs could be achieved with RAIM under conservative failure assumptions. This paper also summarizes previous work concerning RAIM algorithms and compares their results against a common standard. First, in light of the experience with the Wide Area Augmentation System (WAAS), a threat space for a dual frequency Galileo-GPS constellation is defined. This threat space is necessary in order to achieve a low VAL, as it does not suffice to assume single failures only. Second, RAIM methodologies adapted to the threat space are compared, and the most practical one was found to be a multiple hypothesis approach. Finally, the performance results of the chosen RAIM scheme with a Galileo-GPS dual frequency constellation are presented. It was found that an unaided Galileo-GPS constellation yielded Vertical Protection Level (VPL) values under 20m for the combined dual system. This optimistic conclusion indicates that it will likely be possible to provide vertical guidance to aircraft without the need for any additional augmentation when the future GPS and Galileo constellations are operational.

INTRODUCTION

This work aims to evaluate the performance of an unaided Galileo-GPS constellation from a vertical integrity standpoint (e.g. for aviation precision approach). A multitude of algorithms or methods were proposed for RAIM over time, both for GPS alone and more recently for combined Galileo-GPS constellations. However, the presented results were hard to compare between the different papers due to the lack of a standardized threat model and also the different assumptions made by each author. This paper seeks instead to establish a Satellite Failure Threat Space that is general enough to allow testing different algorithms and assumptions against a standard model. To accommodate the different assumptions existing in the literature, parametric studies are conducted on factors external to the integrity monitor, such as the satellite failure probability and the expected User Range Accuracy (URA). Three of the existing algorithms, called Least Squares (LS), Maximum Solution Separation and Multiple Hypothesis Solution Separation (MHSS) were implemented as part of the current study. The resulting VPL values from using these different algorithms are then compared and the origin of the inherent differences is discussed. The most practical algorithm for use with the dual constellation will be adopted. Based on the final results with this algorithm, a conclusion is drawn on the capabilities of the unaided combined constellation and direction for future work is laid out. The current work evaluates what is the maximal threat space against which it is possible to offer protection, and does not involve Fault Detection and Elimination (FDE).

SATELLITE FAILURE THREAT SPACE

The *threat space* is a consistent and complete set of assumptions about the environment in which a RAIM algorithm is applied. A standardized threat space can be regarded as a general test case against which each individual algorithm may be applied. It has to be universal enough such that it can constitute a frame in which to apply the particular set of assumptions of each particular algorithm, and it should include all considered threats. The threat space is in fact the sample space of all failure modes, including the "no failure" case or nominal conditions. A *failure mode* is the outcome of each of the navigation beacons (i.e. satellites or space vehicles (SVs)) being in a "healthy" or "failed" binary state, with a certain probability. *Nominal conditions* contain the expected modes of behavior from the satellites, propagation medium, and user receiver with its surrounding environment. Under these conditions, the users achieve their expected level of performance. The *failed state* is an anomalous condition that can threaten the accuracy and integrity of the system when undetected, and the continuity and availability when it is detected. Such failures should be infrequent and short in duration. The threat model places limits on the extent and behaviors of fault modes. The threat space needs to be all-inclusive, such that all feared events are taken into account, including events introduced by the algorithm itself. Each method can be different with respect to its vulnerability to various fault modes.

For the purposes of this study, dual frequency full GPS and Galileo constellations will be assumed. Therefore, ionospheric threats will not be considered (In future studies, second order TEC delays and scintillation will be investigated). The clock and ephemeris errors are assumed to be normally distributed $\boldsymbol{N}(0, \sigma_{URA})$ under nominal, healthy satellite conditions. The troposphere model will match the one in the WAAS MOPS and is assumed to be bounded by the confidence level provided in [7]. Receiver noise and multipath are also bounded by the provided iono-free sigma term. Note that, like GBAS and SBAS, receiver failure and excessive multipath terms are not explicitly put into the threat space. However, RAIM offers some protection against such fault modes where no ground augmentation can. Previous literature seems to be much in agreement on a theoretical way to describe errors at the user. For that reason, the latest model in [4] was considered appropriate. The position error variance for satellite *i* is described there by:

$$\sigma_i^2 = \sigma_{URA}^2 + \sigma_{i,tropo}^2 + \sigma_{i,iono\text{-}free}^2 + \sigma_{L1L5}^2.$$

Multiple independent faults will be considered in the combined constellation. This algorithm can be expanded to handle different probabilities of multiple failures, but the independent fault model was adopted in light of possible satellite clock failures as main threat sources. Additionally, separate constellation failure modes will be considered for the case where correlated faults exist across either the GPS or Galileo constellations but not both. Furthermore, a particular user can only receive information from a subset of the SVs, specifically the ones at an elevation above a predefined mask angle. Thus, it is practical to consider only the satellites in view from the location of each specific user.

The *a priori* probability of satellite failure will be discussed and an analysis will be conducted for the relevant number of failures. The *satellite failure* is defined here as the behavior of a SV when its corresponding range error cannot be overbounded with a Gaussian $\boldsymbol{N}(0, \sigma_{URA})$. This seems to be a more natural way to describe a failure for this algorithm. Although it is a good binary discriminator, the customary method of setting a failure threshold for pseudorange error does not help identify systematic errors when they are just below the threshold. The practicality of detecting failures according to this new definition needs nonetheless some further scrutiny in future studies.

The total error budget for providing Hazardous Misleading Information (HMI) is strictly limited for the case of aviation precision approaches, such that the maximum allowable integrity risk is 10^{-7}/approach. This budget needs to be divided between all the possible failure modes, and the resulting VPL will be very sensitive on the allocation of this integrity budget. Normally, in applying any RAIM algorithm, multiple failures are neglected, for modes which are less likely than a certain threshold. The reason why certain improbable failure modes need to be neglected is that the entire threat space is extremely large and impractical to compute. Therefore, it is imperative to limit the computation of the position error only to the most dangerous events from an integrity point-of-view. In the same time, one can afford to conservatively assume the worst case scenario (i.e. failure generating HMI) for the remaining threats, as they have a small enough probabilistic impact on the total error or the total integrity anyway.

RAIM ALGORITHMS

All the algorithms used in this paper have previously been proposed by other authors [1, 2, 6], and were slightly adapted in order to be compared with each other before they were tested against the proposed threat model. For each algorithm, the most natural way to compute the VPL is to search for a range of position errors which ensures a Pr(HMI) at most equal to the given integrity risk. Concurrently, every attempt will be made to minimize the VPL interval centered on the all-in-view position solution. Ultimately, a simple comparison with the VAL corresponding to the user's needs will be needed for making a decision on whether to proceed with the desired operation. As a matter of fact, this approach is similar to the calculation of integrity risk at the Alert Limit (AL) proposed for Galileo integrity [5], which method has already received careful consideration for use in integrity monitoring in that context.

As a slight aside, instead of the all-in-view solution, a different, optimal position solution can be chosen [6], on which to center the VPL at each time step. This could help achieve a slightly lower protection bound. Nevertheless, there is a tradeoff here, since the process of computing the optimal position is more computationally complex and it may also not lead to a smooth position

No. of faults more likely than 1e-8	0	1	2	3	4	5	6	7	8	9	10
Failure Prior per satellite <	4.20 E-10	6.00 E-06	1.70 E-04	9.90 E-04	3.00 E-03	6.50 E-03	1.10 E-02	1.90 E-02	2.80 E-02	3.90 E-02	5.20 E-02

Table 1. Prior Probability of Failure thresholds (based on binomial independent trials).

solution over time. In consequence, the decision was made to give preference to the smoother all-in-view solution while saving the algorithm some additional computational time.

Classical LS RAIM algorithm

The same LS algorithm described in [2, 8] was used here for vertical integrity. However, as discussed above, no failure decisions are made directly by the RAIM algorithm. Instead, the VPL is set large enough to include the largest position error. Users can thus make the decision themselves, by comparing the VPL with their required VAL. Instead of a decision threshold, the adapted LS method actually uses the Sum of Square Errors (SSE) test statistic. This approach is still conservative, since it allows the total SSE along any of the lines of sight (LOS) as the worst case scenario. The test statistic is subsequently multiplied by the maximum vertical position error vs. test statistic slope to yield the Approximate Radial-Error Protection (ARP) value [8], the primary ingredient of the VPL. The ARP interval is in fact based on the total error from all satellites occurring in the worst possible mode (i.e. along the LOS most sensitive to error).

Solution Separation algorithm

The name Solution Separation (SS) is used here to identify the RAIM algorithm which combines position solutions from different subsets of the satellites in view to compute a Protection Level (PL), particularly the VPL. By artificially assuming one or more SVs to be faulty and eliminating them from the position equation, one can obtain a partial position solution based only on the remaining satellites. Among all the partial subsets of satellites, the existence of at least one fault-free subset is thus guaranteed, and consequently the correct position solution will certainly be included in the range of positions that constitutes the VPL interval. Instead of the ARP, the difference between the estimated and actual position or *navigation error* is computed for each satellite subset (see Appendix), such that a VPL (or error interval) can actually be associated with each partial position solution. Based on the assumed SV probability of failure and the average number of satellites in view, a decision will be made on what is the maximum number of simultaneous failures that needs to be considered. This represents an advantage over the LS algorithm, which can only handle the assumption of one SV failure at a given time. The computational load of the algorithm will however also increase exponentially with the number of assumed simultaneous failures.

When computing the VPL for both the above algorithms, for each position solution (all-in-view, or subsets of satellites in view for SS), a 5.33σ (viz. $1\text{-}10^{-7}$ probability), confidence interval is generated and added on top of the vertical error. In the SS case, the minimum VPL is half of the size of the union of all error intervals corresponding to subsets of satellites in view. Nonetheless, if we want smooth changes in the position solution for the SS algorithm, we can center the VPL interval around the all-in-view solution, but we need to make sure we select a large enough VPL value to include the above union of error intervals.

Multiple Hypothesis SS algorithm

The MHSS algorithm described here is a generalization of

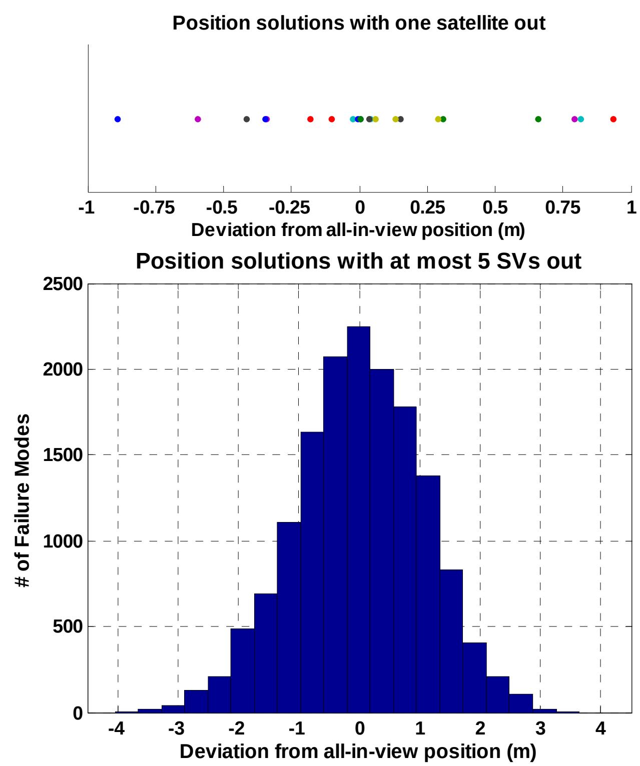

Figure 1. Spread of Position Solutions for URA = 1m, Prob. of SV failure: 6.5 * 10^{-3}/approach

the algorithm proposed in [6] for use in conjunction with LAAS. This algorithm was already tested against a CAT III VAL requirement of 5m and demonstrated to achieve low VPLs. Furthermore, its assumptions are general enough such that it can be used for any RAIM-type integrity computation. In the current study, multiple SV failures were considered in order to cover all possible failure modes included in the threat space. The probability of occurrence of each failure mode is taken into account and a search is made for the VPL which most closely makes use of the entire integrity budget available.

In applying the MHSS algorithm, modes with more than a certain number of failures are neglected from the position calculations when that number of SV failures is less likely than 10% of the total budget, or 10^{-8}/approach for at most 24 satellites in view (Table 1). As an example, for a 10^{-4} probability of failure it is necessary to consider up to two satellites out, while for any probability larger that $1.7*10^{-4}$ three or more failures will be taken into account. The set of neglected modes will constitute an unknown failure mode, and its corresponding integrity risk will be accounted for and diminish the total integrity allocation. Additionally, an a priori probability of failure of 10^{-7} per each approach will be associated to each possible constellation failure. This last failure mode was not considered for the previous two algorithms, as those algorithms make a conservative worst-case scenario assumption for each possible failure, thus making it impossible to provide a PL if a constellation failure is even considered. An example of the distribution of position solutions from failure modes not being neglected can be seen in Figure 1.

The MHSS algorithm assumes the fault-free case (no known satellite failures) and considers all possible, yet undetected failure modes. The integrity risk is computed based on satellite geometry and the partial position

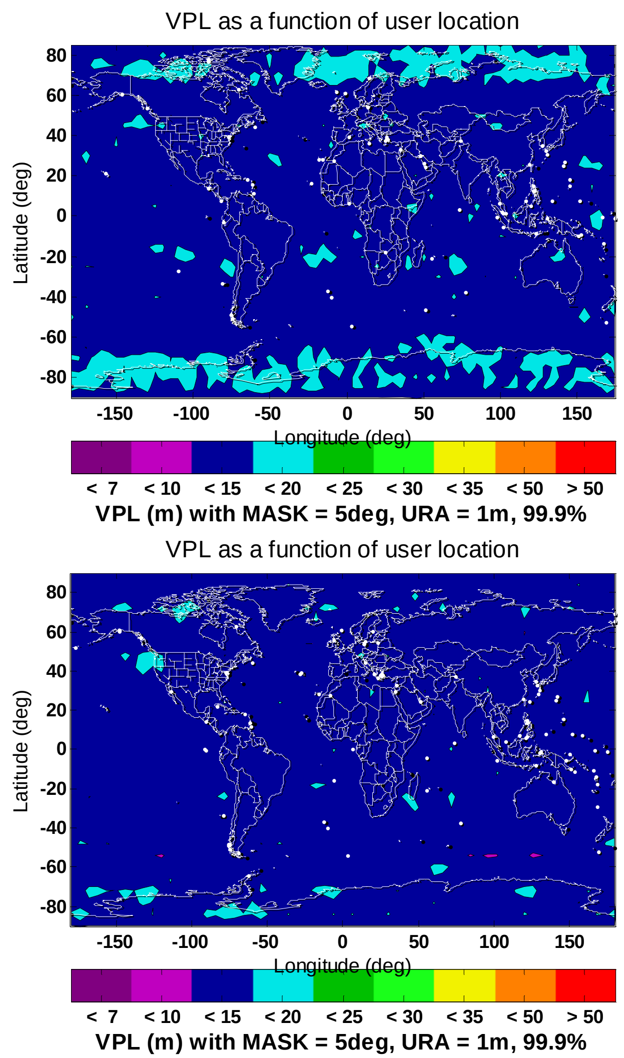

Figure 2. URA of 1m, Mask angle of 5 degrees. Top: LS RAIM, Bottom: SS with one SV failure considered.

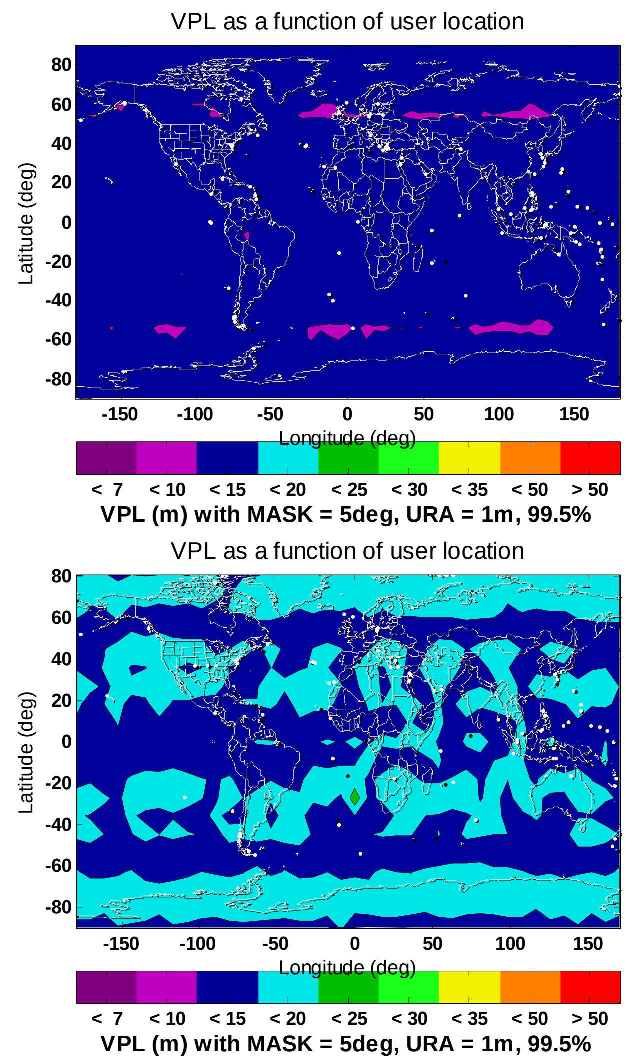

Figure 3. URA of 1m, Mask angle of 5 degrees. Top: SS with one SV failure, Bottom: SS with two SV failures.

solutions, but the prior probabilities of failure are fixed and not updated conditional on the measurements. These probabilities of failure can be assumed to be lower if the user has the possibility to run a χ^2 check and detect a satellite fault or has access to external information such as integrity flags that may be broadcasted by the Galileo satellites or an external augmentation system (e.g. WAAS GEOs). The MHSS algorithm can also be applied after excluding such faulty satellites. Another reason why one would want to employ satellite elimination is improving availability for the navigation solution.

COMPARISON OF LS AND SS RAIM SCHEMES

Simulations were performed in order to test the RAIM algorithms against the same threat model. Each simulation lasts 72h, with at least 500 time steps and 200 users over the entire world. According to WAAS MOPS [7], 24 GPS and 30 Galileo SV constellations are assumed. At each user location, the 99.5th percentile VPL over the simulation period is mapped. The world maps are then colored by interpolation between grid points.

A first observation is that SS performs better than basic LS RAIM under an identical threat model (Figure 2). There is a good explanation for that. In classical LS RAIM, information is lost in the process of forming the SSE statistic, in the respect that the total error is summed over all geometrical directions or LOS. In the meantime, the SS algorithm clearly associates each position error with the corresponding LOS, or the geometry of a particular satellite subset, and does not assume that the maximum total error can be generated entirely by a single satellite.

Furthermore, as it was expected, single satellite elimination provides better position accuracy than multiple satellite elimination. Although one can discard

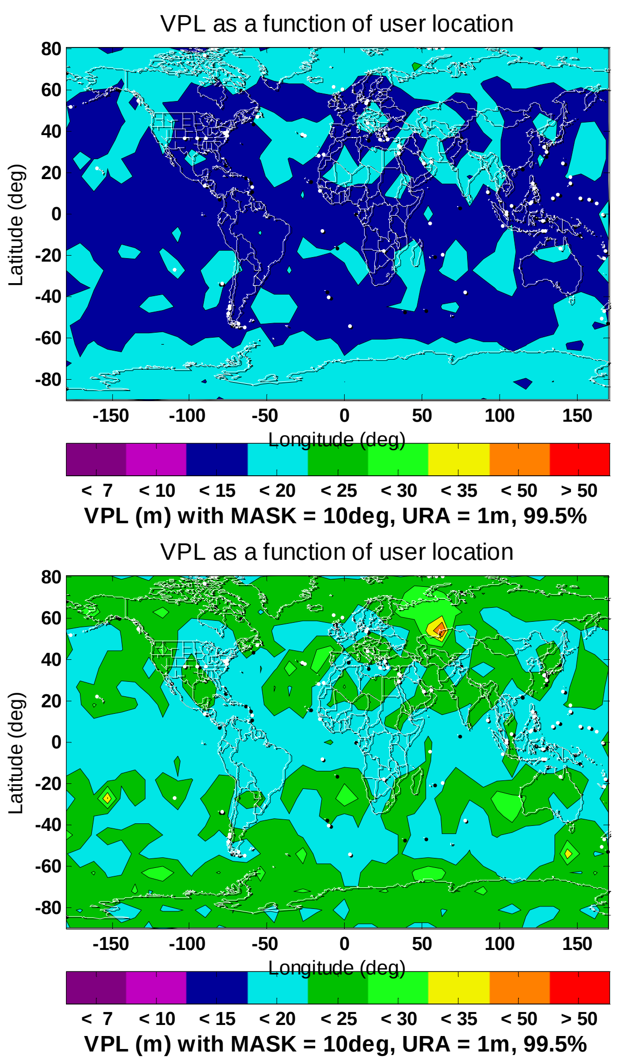

Figure 4. URA of 1m, Mask angle of 10 degrees. Top: SS with one SV failure, Bottom: SS with two SV failures.

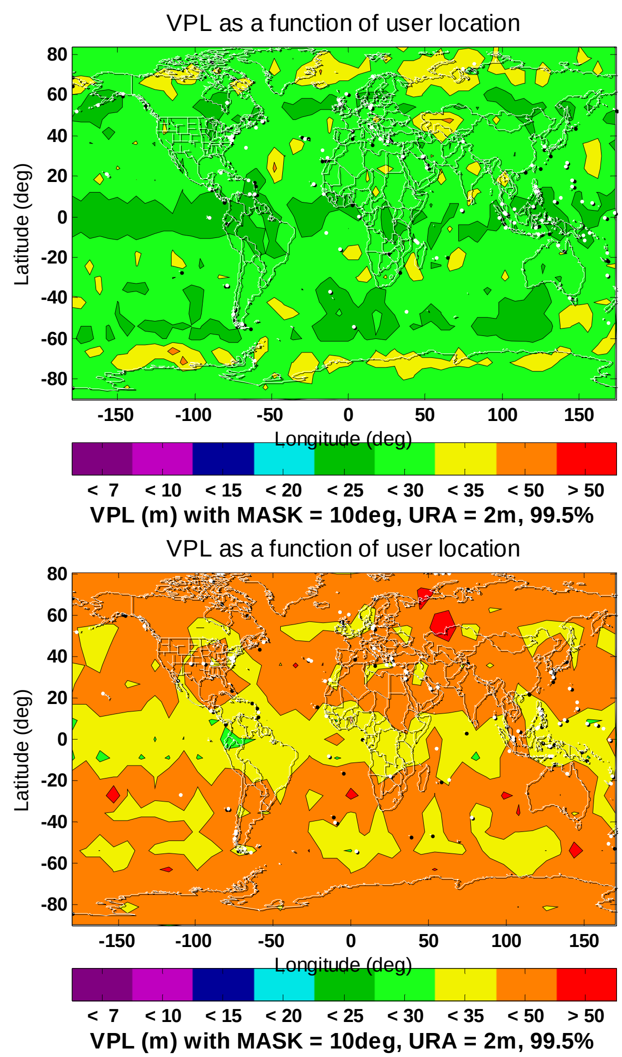

Figure 5. URA of 2m, Mask angle of 10 degrees. Top: SS with one SV failure, Bottom: SS with two SV failures.

several large positioning errors by eliminating multiple satellites from the position equation, this operation also leads to a loss of good geometry (e.g. high elevation satellites), thus in fact degrading performance (Figure 3).

If the mask angle is increased in the simulations, less SVs will be in view on the average, causing the satellite geometries to deteriorate and the VPL to increase. Another expected result was for VPL values to increase proportionally with the assumed URA. It is seen that there is not a linear relationship between the two; however there is a clear correlation, as it can be seen by comparing Figures 4 and 5.

The final conclusions from comparing LS and SS algorithms are also summarized in a more quantitative format in Figure 6. Since classical LS RAIM does not apply to cases where more than one failure needs to be considered, it is impossible to make a direct comparison

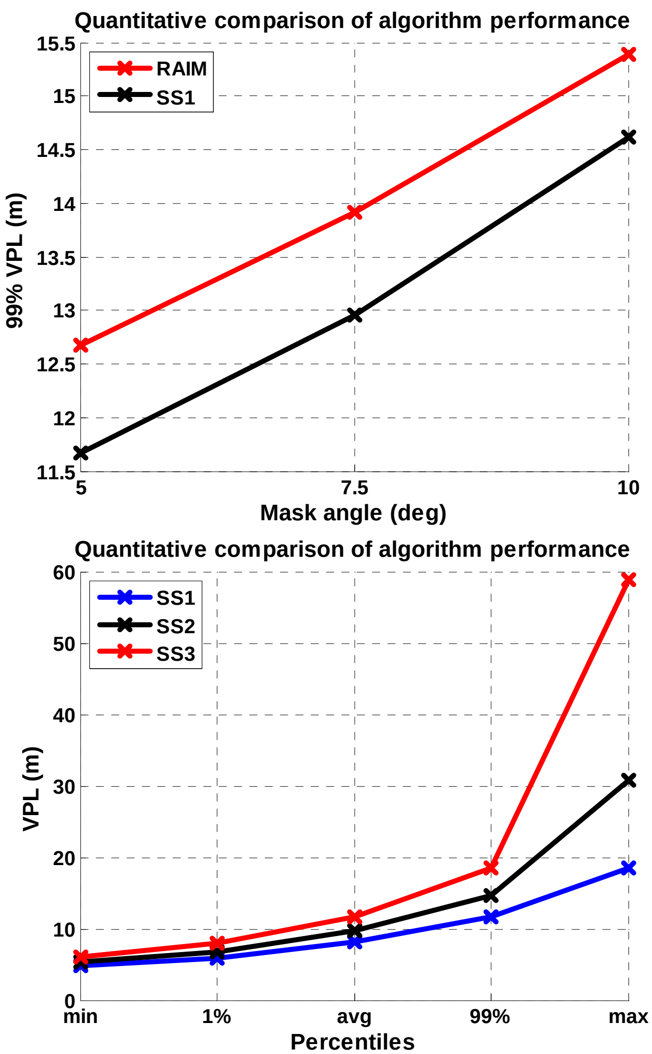

Figure 6. Comparison of LS and SS (URA=1m). Top: LS and SS vs. mask angle, Bottom: SS algorithm assuming 1-3 maximum failures (5 deg mask angle).

with the results from SS with more than one satellite out. However, it is evident that the premise of multiple failures causes the VPL values to degrade rapidly, especially for the worst-case geometry. This could be a reason why previous work has chosen to ignore multiple failures based on their very small probability of occurrence. In the case of a dual constellation, however, the probability of multiple failures is increased. Consequently, a method that can address these issues appropriately needed to be examined.

FINAL RESULTS WITH THE MHSS ALGORITHM

After careful consideration, the MHSS method was found to be the most appropriate for combined Galileo-GPS RAIM. Intuitively, this algorithm brings an added advantage, since it does not conservatively assign equal weights to all considered failure modes. More insight is gained by examining the results from the MHSS simulation. Compared to the previously examined

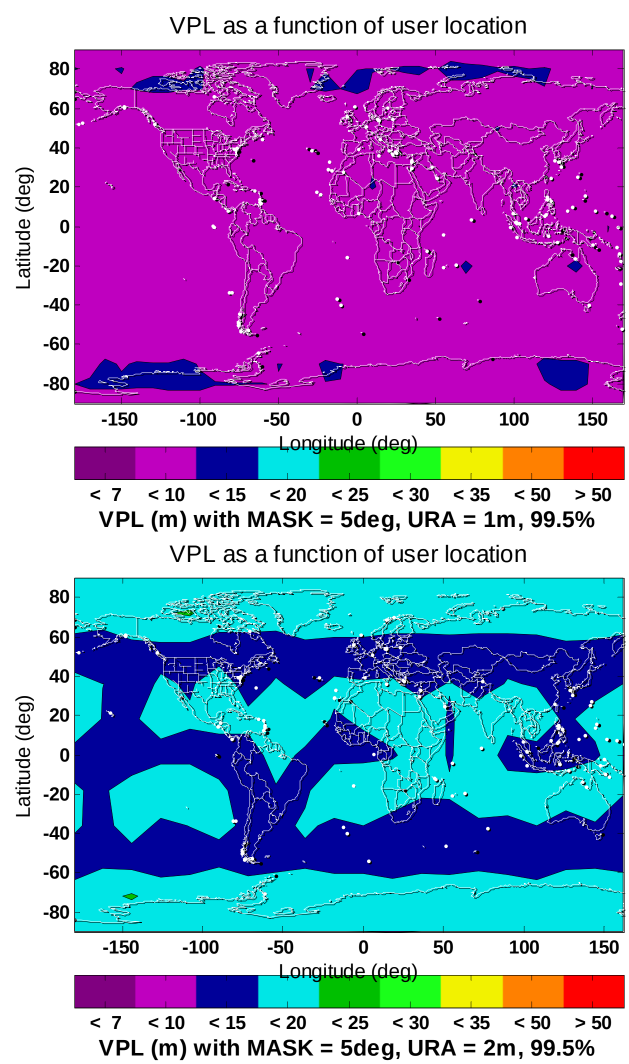

Figure 7. VPLs from MHSS algorithm. Top: URA of 1m, Bottom: URA of 2m.

algorithms, a lower VPL will be generated for the same geometry and there is a visible improvement in the overall performance.

Comparing the fully operational mode, with an average of 18 Galileo-GPS SVs in view (Figure 7) to the degraded mode where only one of the constellations is available (Figures 8, 9), for a SV failure prior of 10^{-4}, there are obvious geometry effects at equator and poles versus mid-latitudes that should be noted here. For example, one extra satellite will be visible on the average at the poles, however the corresponding VPL will still be higher due to poorer geometries, not having any SVs available at high elevations. The situation gets reversed in the case of the degraded modes, when the average VPL is slightly lower at the poles and equator due to the fact that an extra satellite visible proves to be important for the overall geometry most of the time.

Comparing the two results for the degraded modes, it is possible to notice that the proposed MHSS method works really well for an average of 10 or more SVs in view (e.g. the case of the Galileo constellation only), as there will be enough redundant satellites in the position equation. However, things start to break down when less SVs are in view. For the GPS-only case, when only an average of 8 satellites are in view, the availability is impacted when partial solutions need to be formed, such that the 50m VAL can only be guaranteed 95% of the time.

The SV failure prior and URA were chosen to be the variable parameters for this study since their values are determined outside RAIM, depending on the particular constellation characteristics. Figure 10 presents the summary of all MHSS results. The reason why some points have been left out on these plots for prior probabilities greater than 0.003 (viz. 5 or more failures

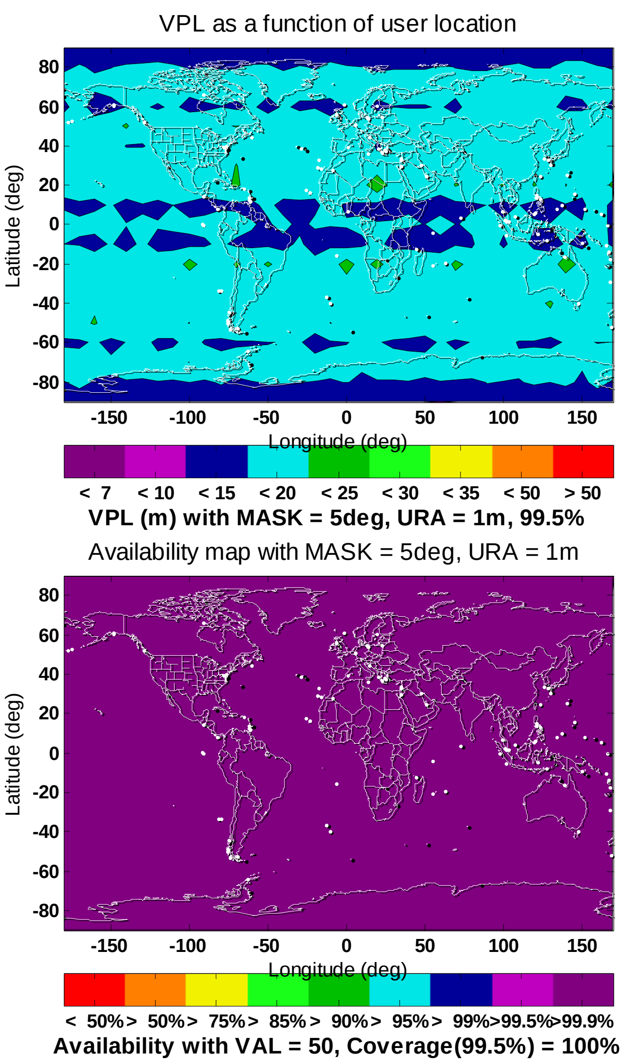

Figure 8. MHSS algorithm in degraded operation mode (only Galileo constellation). Top: VPL map, Bottom: Availability for VAL of 50m.

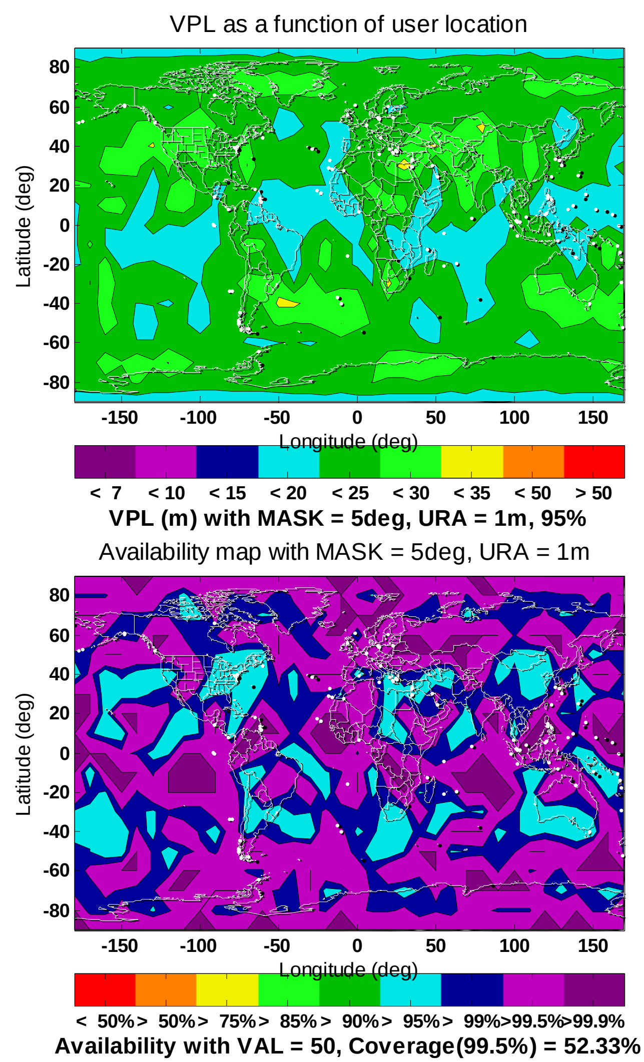

Figure 9. MHSS algorithm in degraded operation mode (only GPS constellation). Top: VPL map, Bottom: Availability for VAL of 50m.

considered) is that those points were not statistically comparable with the rest of the data on the plot. Due to the excessively long simulation times, less statistical samples were collected in these extreme cases, such that it is impossible to accurately represent those VPL values with an occurrence frequency of less than 1%.

It might look surprising at first sight that the average VPL is quite insensitive to the chosen failure priors. One way to explain this result is by the fact that multiple failures are much less likely than single failures and are always weighted accordingly. Adding more possible failure modes into consideration does not change the probability distribution of the position solution significantly. Therefore, the limits of the VPL interval, outside of which the real position is less likely than 10^{-7} to lie, do not change by much. What changes with the failure

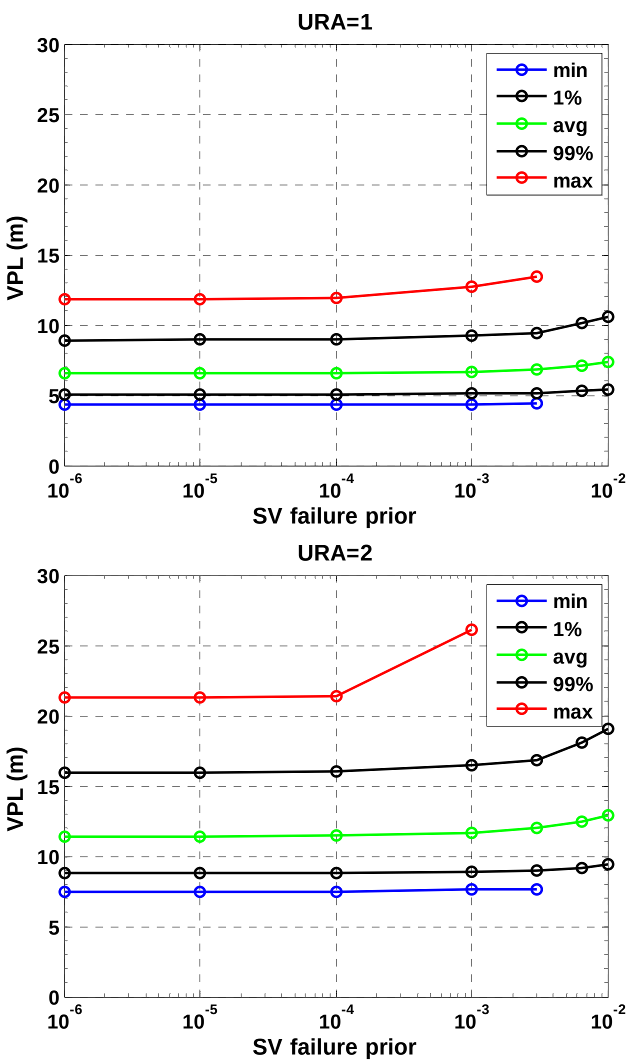

Figure 10. Parametric dependence of VPL on URA (1m, respectively 2m) and probability of SV failure (between 1 and 6 simultaneous failures considered).

probabilities, however, are the tails of the distributions, making the worst case more extreme, as critical satellites for the geometry are more likely to fail.

CONCLUSIONS

Overall, the results obtained here are very encouraging for the performance of an unaided combined Galileo-GPS constellation. We were able to get VPL values of under 20m for the combined dual constellation and still less than 35m for the degraded operation modes. Thus, vertical guidance seems achievable without any external augmentation.

The threat models need to be investigated in greater detail. Additional error sources need to be considered, which were not included in the original threat space (e.g. SQM, receiver and antenna biases), as the Gaussian characterization of the errors might not be sufficient for providing convincing evidence on the combined constellation integrity performance. Although the case of multipath was included in the error terms, a more detailed discussion of this threat is needed as well.

In light of future developments, the MHSS algorithm used here is suited for the implementation of a FDE scheme, since it allows the separation of the effects of each subset of satellites on the overall position solution. Satellites could then be removed and tested one at a time in order to remove the faulty measurement and reduce the VPL.

With these considerations in mind, the authors still expect that it will be possible to meet at least the Low-Precision Vertical (LPV) level VPL requirements after all the above adjustments are made.

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APPENDIX

The way in which one can compute the estimated navigation position error for SS-type algorithms will be described here in more detail. It is important that this procedure does not require actual pseudorange measurements (not available in simulation), as the RAIM algorithms provide a PL based only on the relative geometry between the user and the Galileo and GPS constellations.

As in the case of LS RAIM, one starts out with the linearized measurement equation for a number **n** of satellites in view:

$$\mathbf{y} = \mathbf{G} \cdot \mathbf{x} + \boldsymbol{\varepsilon} \qquad (1)$$

The linearization took place around the estimate minus actual *position deviation vector* **x**, which is five-dimensional for the case of the dual constellation (North, East, Up and one time coordinate for GPS and Galileo each). A simplifying assumption could be made by considering a fixed, known Galileo-GPS clock bias, but the choice was made not to use that assumption here, in order to maintain generality. The other terms above are the **n** × 1 *measurement vector* **y** containing the differences between the expected ranging values and the raw pseudorange measurements to each of the **n** satellites, the **n** × 5 geometry or *observation matrix* **G**, and the **n**-dimensional *measurement error* **ε**. For the simulation purposes, the error along each satellite LOS was taken to be zero-mean Gaussian noise with the σ_i^2 variance defined earlier in this paper.

The weighted LS solution for **x** is given by:

$$\mathbf{x\_est} = (\mathbf{G}^T \cdot \mathbf{W} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^T \cdot \mathbf{W} \cdot \mathbf{y} \equiv \mathbf{K} \cdot \mathbf{y} \qquad (2)$$

where **K** is called the weighted pseudoinverse of **G** and the *weighting matrix* **W** is the inverse of the measurement noise covariance matrix **Σ**. For simplification, it was assumed that the error sources are uncorrelated between all the different SVs. Therefore, **Σ** is a diagonal **n** × **n** matrix:

$$\boldsymbol{\Sigma} = \sigma_i^2 \cdot \mathbf{I}_{\mathbf{n} \times \mathbf{n}} \qquad (3)$$

While the independence assumption may not be strictly true, it should be a reasonably good approximation. The equations subsequently derived do not depend on this assumption, which only makes them easier to implement in practice.

At this point, in the case of the LS algorithm, one would estimate the measurement noise:

$$\boldsymbol{\varepsilon}\mathbf{\_est} = \mathbf{w} \equiv (\mathbf{I} - \mathbf{G} \cdot \mathbf{K}) \cdot \mathbf{y} \qquad (4)$$

in order to be able to compute the weighted SSE metric. Instead, the SS RAIM employs the residuals from estimating the actual position error:

$$\boldsymbol{\delta}\mathbf{x} = \mathbf{x\_est} - \mathbf{x} = \mathbf{K} \cdot \boldsymbol{\varepsilon} \qquad (5)$$

for the all-in-view solution. For the partial solutions, **x_est** and **x** will be replaced with the corresponding vectors based on a partial set of measurements. Of interest here is only the third element in **δx**, the vertical component of the navigation error. This element will be called $\mathbf{x}_v$, in agreement with the notation in [6] used for describing the MHSS algorithm. For the ordinary SS algorithm, a partial VPL range will be computed for each partial position solution:

$$\mathrm{VPL} = \mathbf{x}_v \pm \mathbf{k}_v \cdot \boldsymbol{\sigma}_v \qquad (6)$$

where $\mathbf{k}_v$ is the number of standard deviations equivalent to the required integrity confidence interval ($\mathbf{k}_v$ = 5.33 for a 10^{-7} integrity risk) and $\boldsymbol{\sigma}_v$ is a measure of vertical accuracy derived from the covariance of the position estimate:

$$\boldsymbol{\sigma}_v^2 = [(\mathbf{G}^T \cdot \mathbf{W} \cdot \mathbf{G})^{-1}]_{3,3} \qquad (7)$$

Then, the overall VPL for the SS algorithm will be chosen such that it defines an interval around the all-in-view estimated position including all the partial solution ranges.

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