Section 6 results and Figures 17-22 are currently being revised.
No results are included for “strobe correlators”.

Detecting Anomalous Signals from GPS Satellites

Per Enge, Eric Phelts and Alexander Mitelman
Stanford University
October 12, 1999

Abstract: The international civil aviation community is developing two systems to augment the global navigation satellite services (GNSS). With such augmentation, GNSS will serve as the primary means of air navigation. Both augmentations include ground networks of reference receivers and a data link to the aircraft. The first system is called a local area augmentation system (LAAS), and it places reference receivers at a single airport and uses a VHF data link to communicate with aircraft approaching that airport. Each LAAS serves aircraft within 40 miles of the instrumented airport and the intrinsic capability of the system includes all categories of precision approach including auto-land. Wide area augmentation systems (WAAS) deploy reference networks that span continents and use geostationary satellites to communicate with the aircraft. They support enroute flight across continental areas, terminal area operations, and provide vertical guidance for approaching aircraft.

To the international community, a LAAS is known as a Ground Based Augmentation System (GBAS), because it uses a terrestrial data link. In contrast, the WAAS is known as a Space Based Augmentation System (SBAS), because it uses a satellite data link to the user. In any event, these two systems complement each other, and the Federal Aviation Administration (FAA) is deploying both types of system in the United States.

Both designs have been accompanied by arduous safety analyses directed at guaranteeing that all system faults are detected and isolated. This paper reports on one part of the overall safety analysis – the detection of anomalies in the signals from the GNSS satellites. If a signal anomaly could generate hazardously misleading information (HMI) within approved receivers, then it is called an *evil waveform* and the ground system must include a signal quality monitor (SQM) to detect the troubled satellite.

1 Introduction

Today, global navigation satellite services (GNSS) provide position and time information to over eight million users. These services are primarily based on signals from the Global Positioning System (GPS) deployed by the United States. However, some applications make use of signals from a partial constellation of satellites belonging to a Russian system called GLONASS. Moreover, Europe is contemplating the development of a GNSS called Galileo. This paper will focus on GPS, but our results may well be applicable to Galileo and GLONASS.

A mobile user can use the Global Positioning System (GPS) to estimate position to within 100 meters of truth, and time to within 1 microsecond of an international standard clock. Two receivers can operate in a differential mode and estimate their relative position to within a few centimeters and their clock difference to within a few nanoseconds. All of these capabilities are global, available 24 hours a day, and are nearly instantaneous.

GPS is being augmented to serve safety-critical applications such as harbor entrance or aircraft navigation. To serve these demanding applications, the augmented system must meet stringent requirements on *accuracy, integrity, continuity* and *availability.* Indeed, the accuracy of a stand-alone, civilian user of GPS is approximately 100 meters, but a ship entering harbor may require an accuracy of 10 meters, and the accuracy required for a precision approach of an aircraft in zero visibility is better than 1 meter.

Integrity requires the navigation system to provide an accurate estimate of its own performance in real time. Pilots must be warned within seconds if the accuracy of the system has degraded below normal, expected levels. After all, the pilot may not have another reliable estimate of aircraft position, and so the navigation system must continuously assess its own performance. This assessment is called a protection level (PL), and is continuously compared to the alarm limit (AL) required for the operation. If the PL is smaller than the AL, then the operation may proceed. If not, then the operation cannot be initiated or must be aborted.

The PL must overbound the true position error with high probability. If the true error is greater than the protection level, then the pilot may attempt an operation that is overly ambitious and unsafe. On the other hand, the PL cannot be too conservative. If so, the intrinsic capability of the system is not being fully utilized and operations may be needlessly aborted or avoided. If an increase in the PL causes an operation to be aborted after it has been initiated, then the continuity of the system has failed. Continuity requires the navigation system to reliably support a critical operation once it has been initiated.

1.1 Local and Wide Area Augmentation Systems

The Federal Aviation Administration and the international aviation community have responded to these challenges by designing two high-integrity, differential GPS (DGPS) systems. In general, DGPS places a GPS reference receiver at a precisely surveyed location, and measures the difference between the current GPS measurements and the theoretical measurements implied by the known reference location. These differences are then broadcast to the roving user as corrections to each satellite location. The user equipment applies the corrections to each of his GPS measurements. This differential technique reduces all errors that are spatially correlated provided the corrections are delivered promptly. Two DGPS systems are being developed for aviation. In the United States, these systems are called a local area augmentation system (LAAS) and a wide area augmentation system (WAAS).

A LAAS places reference receivers at a single airport and uses a VHF data link to communicate with aircraft approaching that airport. LAAS service is limited to the area around the instrumented airport, but the intrinsic capability of the system includes all categories of precision approach including auto-land. In contrast, the reference networks for wide area augmentation systems (WAAS) span continents and geostationary satellites are used to communicate with the aircraft. This system supports enroute flight across continental areas, terminal area operations, and provides vertical guidance for approaching aircraft.

To the international community, a LAAS is known as a Ground Based Augmentation System (GBAS), because it uses a terrestrial data link. In contrast, the WAAS is known as a Space Based Augmentation System (SBAS), because it uses a satellite data link to reach the user. In any event, these two systems complement each other, and the Federal Aviation Administration (FAA) is deploying both types of system in the United States.

Both systems have been the subject of many studies and flight trials, and these efforts are well documented in the burgeoning literature on LAAS and WAAS. The lion’s share of the current effort is directed at providing integrity without unduly sacrificing continuity or availability. The protection levels (PL) provided by LAAS and WAAS must be prompt, sharp and sensitive to real changes in the system performance. The safety analyses must ensure that the user is appropriately informed of any performance changes and this guarantee must be robust to any plausible failure mechanism or fault.

1.2 Evil Waveforms

This paper is directed at one part of the overall fault analysis. It concerns itself with the impact of GPS signal anomalies that disturb the shape of the correlation function used by the GPS receivers. This analysis is partially motivated by the one and only known occurrence of such an event. In March of 1993, the signal from GPS satellite PRN 19 resulted in an anomalous correlation peak and introduced 3 to 8 meter position errors into differential GPS systems (Edgar, Czopek and Barker, 1999).

If these disturbances are specific to one satellite and are not removed by differential processing, then hazardously misleading information (HMI) could result. These signal anomalies do not refer to the normal effects of band pass filtering on the correlation peak – this smoothing is common to all the satellites used by the aircraft and does not result in an undetected degradation. Nor do these anomalies refer to effects that are common to the reference and airborne receiver – these common mode errors are removed by the normal differential processing.

The anomalies of concern here effect only one satellite and are not completely removed by differential processing. Some of these effects only exist when the reference receiver and the roving receiver use different *correlator spacings*. However, some exist even when identical receivers are used on the ground and in the air. Example disturbances are shown in Figures 1, 2 and 3, which show the anomalous signal in the top trace and the resulting correlation function in the bottom trace. Figure 1 shows a waveform with lag in the falling edge of the spectrum spreading code. As shown, such a falling edge lag results in a correlation function with a flat top. Figure 2 shows a waveform with nearly undamped ringing – the corresponding correlation function has many false peaks. Figure 3 shows a waveform with highly damped oscillation – its correlation function suffers visible distortion.

1.3 Outline

Section 2 of this paper recounts the public history of PRN 19 and early attempts to model this signal failure. These early failure models range from simple to elaborate. One of the simple models is based on the PRN 19 signal spectrum and another is based on the hypothesis that the PRN 19 signal was reflected by a mismatch in the satellite’s transmission line. The most elaborate models are based on the worst case waveforms from the theory. These most evil waveforms introduce the largest error into the airborne measurement given the monitoring system used by the ground. However, these extraordinarily pernicious signals cannot be generated by any identifiable mechanism on the satellite.

Consequently, Section 3 proposes a model that we feel strikes the right balance between realism and potential for danger, and Section 4 develops closed form expressions that greatly simplify the computations required for Sections 5 and 6.

Section 5 studies the performance of a simple signal quality monitor (SQM) in the face of the threat model from Section 3. In general, the SQM must flag all anomalous waveforms that might yield hazardously misleading information in an approved airborne receiver. Our simple prototype compares the range measurements made by correlator pairs operating at different locations on the correlation peak. The performance of any SQM is sensitive to the nominal effects of noise, interference and multipath. We make no attempt to analyze these limits in this paper. Rather, we use the minimum detectable errors (MDEs) that have been derived by Shively, Brenner and Kline, 1999. Our prototype SQM is used to segregate the waveforms in our threat model into detectable and undetectable subsets.

Section 6 computes the aircraft pseudorange error due to the anomalies that were not detected by our prototype SQM. These errors are functions of: the threat model parameters, the SQM design and three design parameters for the avionics. The maximum error over the set of undetected waveforms is plotted versus the key design parameters for the avionics. These errors are compared to the maximum errors (MERRs) that an aircraft conducting a Category I precision approach can tolerate. The results of this comparison are used to constrain the design parameters allowed for the avionics.

Noise and multipath determine the ability of the SQM algorithm to detect anomalous waveforms and they determine the aircraft’s tolerance of undetected waveforms. However, these dependencies are not explicitly treated in this paper. Rather, they are reflected solely in the MDEs and MERRs provided by our collaborators at MITRE and Honeywell (Shively, Brenner and Kline, 1999 and Shively, 1999).

Section 7 is a brief summary. Appendix A provides a tutorial on the nominal correlation function assumed by the GPS receivers, and Appendix B contains the derivation of the worst case waveforms from the theory. Appendix C proves that the order of correlation and convolution can be reversed. This result is key to the analyses in Sections 4 through 6.

2 GPS Satellite PRN 19 and Some Early Threat Models

2.1 PRN 19

GPS satellite PRN 19 was launched on October 21, 1989 and within a few weeks it was declared operational. PRN 19 served without reported incident until March of 1993, when it was found to have an effect on *differential* GPS user equipment during flight trials conducted at the Oshkosh air show in 1993 (Aviation Week, July 26, 1993). When PRN 19 was included in the navigation solution, position errors of 3 to 8 meters occurred. Without PRN 19, the differentially corrected accuracy was better than 0.5 meter. This degradation raised concern, because undetected errors of 8 meters are intolerably large for an aircraft conducting a precision approach. The troubled system employed reference and airborne receivers with different correlation techniques.

Trimble Navigation Limited sent an official request to investigate this anomaly to the USAF Space Command 2SOPS in July of 1993, and this request invoked a remarkable effort to restore the full function of PRN 19. This effort involved many cooperating agencies and individuals, both civilian and military (Edgar, Czopek and Barker, 1999). Civilian participants included Trimble, NASA's Jet Propulsion Laboratories and the University of Leeds. Military participants included USAF 2SOPS, Rockwell International, The Aerospace Corporation and other supporting contractors and test facilities.

Specifically, the University of Leeds scrutinized the signal from PRN 19 by training a one-meter dish antenna on the suspect satellite and analyzing the spectrum (Riley and Daly, 1993). They recorded the spectrum of a healthy satellite and compared it to the spectrum of PRN 19. As shown in Figure 4, the PRN 19 spectrum includes a spectral spike at or near the center frequency. This anomaly is conspicuous because the GPS signal has a normally suppressed carrier.

The restoration process included two separate corrective actions in October of 1993 and in January of 1994. Taken together, these two actions apparently restored the full health of the signal from PRN 19. The PRN 19 spectrum no longer included a spectral spike and differential position accuracy returned to a nominal 50 centimeters even when PRN 19 was included (Aviation Week, February 15, 1994).

In January of 1998, PRN 19 returned to center stage, because the Federal Aviation Administration (FAA) asked for a general analysis of possible satellite signal anomalies. The FAA is deploying local and wide area augmentation systems (LAAS and WAAS) and both will support Category I aircraft approach operations. To serve this purpose, they must flag all faults that cause the vertical error to exceed a vertical alarm limit (VAL) of 10 meters within 6 seconds. Future LAAS systems will provide guidance for Category II and III operations and will have even more stringent requirements on the VAL and time to alarm.

The analysis effort has been greatly assisted by Working Groups 2 and 4 within RTCA Special Committee 159. Special Committee 159 is responsible for GPS avionics and Working Groups 2 and 4 specialize in WAAS and LAAS respectively. Both groups also contribute to the design of the associated ground systems. This report owes much to the members of both groups.

2.2 Candidate Models for the Degraded Signal

Two simple signal models were proposed to explain the SV19 anomaly at the January 1998 RTCA meeting of Working Group 4. The first simply added a sine wave at the carrier frequency, and so the transmitted signal would be

$$\begin{aligned} s(t) &= \sqrt{2C_X}\,D(t)X(t)\cos\left(2\pi f_{L1}t+\theta\right) \\ &+\sqrt{2C_I}\,\sin\left(2\pi f_{L1}t+\phi\right) \end{aligned}$$

In this equation, the first line corresponds to the nominal C/A code signal and the second line contains the perturbing sine wave. The powers in the two components are C_X and C_I respectively, $D(t)$ is the navigation data, $X(t)$ is the C/A code and f_{L1} is the center frequency including Doppler shift. This model is certainly appealing because it directly explains the spectral data collected by the University of Leeds. However, it does not readily explain the observed position errors of 3 to 8 meters. If such a disturbed signal was input to a single receiver the resulting pseudorange error would be

$$\Delta\tau \approx \sqrt{\frac{C_I}{C_X}}\frac{d}{2\sqrt{2}N_{C/A}}\ \text{chips} = \sqrt{\frac{C_I}{C_X}}\frac{106d}{N_{C/A}}\ \text{meters}$$

As shown, the error is proportional to the square root of the ratio of the two powers, the correlator spacing, d, and inversely proportional to the length of the C/A code, $N_{C/A}=1023$ chips. Even if C_I/C_X=100, the resulting error is only 0.52 meters. Such a power imbalance would prevent the receiver from locking to the C/A code unless the total power transmitted by the satellite was greatly increased. In short, the satellite would need to devote enough power to the additive signal to overcome the processing gain of the C/A code against narrow band interference. No likely mechanism for such a power increase has been hypothesized. For this reason, the sine wave threat model has not been pursued.

The second early threat model hypothesized that the signal contained a delayed replica of the C/A code signal as follows

$$\begin{aligned} s(t) &= \sqrt{2C_X}\,D(t)X(t)\cos\left(2\pi f_{L1}t+\theta\right) \\ &+\sqrt{2C_M}\,D(t-\tau)X(t-\tau)\cos\left(2\pi f_{L1}t+\phi\right) \end{aligned}$$

Such a disturbed signal could result if the signal path on the satellite included a mismatched transmission line that caused onboard reflections. In this case, the transmitted signal would contain delayed replicas of the nominal signal with amplitude controlled by the reflection coefficient of the mismatched line. Such a signal would readily explain position errors of 3 to 8 meters. After all, the spread-spectrum processing gain does not attenuate the disturbance, because the additive disturbance is also modulated with the C/A code. This signal would have an effect very similar to multipath except that the reflection would be coherent – there would be no differential Doppler shift and no performance gain from averaging of carrier smoothing. However, this model simply does not explain the spectral data collected by the University of Leeds, so it too has been abandoned.

The next threat model used signal theory to generate the worst case disturbances to a differential GPS positioning system where the reference and airborne receivers sample the correlation function at different locations. The resulting waveforms are worst case, and so they are called most evil waveforms or MEWFs. The signal derivation is summarized in Appendix B and an example MEWF is shown in Figure 5. A set of correlation functions for a different MEWF is shown in Figure 6.

The MEWF shown in Figure 5 is superposed on a single rectangular chip from a healthy C/A code signal. This MEWF was generated for an SQM with two pairs of correlators. The first pair generates the reference pseudorange that is processed by the ground system to form the differential correction, and the second generates a pseudorange solely for the purpose of signal quality monitoring. The reference and monitor have correlator spacings of 0.1 and 0.2 respectively. If the reference and monitor pseudoranges difference is other than nominal, then the SQM removes the satellite from service. The waveform shown in Figure 5 is worst case for an aircraft using an intermediate spacing of 0.15 chips.

In contrast to Figure 5, the SQM considered in Figure 6 has three pairs of correlators. This SQM has a reference spacing of 0.2 chips and two additional ground monitors spaced at 0.6 and 1.0 chips. In addition, Figure 6 shows the resulting correlation functions – not the underlying waveforms. The top, triangular curve is the nominal correlation in the absence of any evil disturbance. The remaining curves include the effect of the MEWF and are parameterized by the relative amplitude of the nominal and disturbed signals. Excluding the top curve, the remaining curves have good to bad amplitude ratios of 1.414 (+3 dB), 1 (0 dB), 0.717 (-3 dB) and 0. The monitor and reference samples occur at the locations shown by the vertical lines. As shown, the MEWF has greatly disturbed the overall correlation function, but not at the sample locations shown. The early sample at any of the monitor spacings is equal to the late sample at the same spacing and so no detection occurs.

These most evil waveforms come from the theory alone and have some extraordinarily unlikely properties. First, they are non-causal relative to the edge of the corresponding C/A code chip. Second, they are phase and amplitude modulated to avoid detection while still causing maximum errors for an airborne receiver with an unmonitored correlator spacing. Indeed, the waveforms represented in Figures 5 and 6 are carefully crafted to avoid detection by the monitors, but the resulting phase and amplitude modulation is unlikely to be generated on a real satellite. For this reason, these most evil waveforms do not form the basis for our SQM requirements or designs.

Even so, these theoretical waveforms did serve an important service – they caused the aviation community to articulate the signal effects that might cause a LAAS or WAAS to output hazardously misleading information (HMI). These effects are:

- Dead zones: If the correlation function loses its peak, then the receiver’s discriminator function will include a flat spot or dead zone. If the reference receiver and airborne receiver settle in different portions of this dead zone, then HMI results.
- False peaks: The –3 dB correlation function shown in Figure 6 has several false peaks. If the reference receiver and airborne receiver lock to different peaks, then HMI could exist.
- Distortions: The +3, 0 and –3 dB correlation functions shown in Figure 6 are distorted. They are symmetric relative to the reference values when sampled at the prescribed monitor locations. However, an aircraft that uses a correlator spacing other than the one used by the reference or monitors may well suffer HMI.

The threat model described in the next section is also capable of generating these effects and it is much simpler than the model required to generate MEWFs.

3 A Proposed Threat Model

In this section, we introduce our preferred threat model, which is depicted in Figures 7 and 8. Our model has three parts and enjoys a number of virtues. First and foremost, it creates the three correlation peak pathologies that concern the aviation community – dead zones, false peaks and distortions. In fact, our preferred model was used to generate the waveforms and correlation functions shown in Figures 1, 2, and 3. Second, the model has only three parameters and so computation and testing are not overly complex. Third, the threat waveforms are causal and can be more readily simulated or generated.

Finally, our model corresponds to the subsystems in the C/A code signal path on a GPS satellite. As shown in Figure 8, the GPS signal generator includes a Navigation Data Unit (NDU) followed by a cascade of analog signal processing units. The NDU outputs baseband signals for the navigation data, the C/A code, and the P(Y) code. The analog processing includes frequency upconverters, intermediate and high power amplifiers, antenna beam forming and finally the antenna.

Our three part model is certainly not intended to model every possible failure of a system as sophisticated as the GPS navigation payload. Even so, it is complete in a very important sense – it generates all of the correlation aberrations discussed earlier.

3.1 Threat Model A: Lead/Lag Only

Threat Model A consists of the normal C/A code signal except that all the positive chips have a falling edge that leads or lags relative to the correct end time for that chip. An example of this waveform is shown in Figure 1, and such waveforms would appear at the two outputs labeled *Threat Model A* in Figure 7. Notice these waveforms do not pass through the second order system shown in the figure. As shown in Figure 8, this threat model is associated with a failure in the NDU that introduces possible lead or lag in the falling edge of the baseband signal, but does not include any failure of the analog sections.

Threat Model A has a single parameter Δ, which is the lead ($\Delta < 0$) or lag ($\Delta > 0$) expressed in fractions of a chip. The proposed range for this parameter is

$$\boxed{\begin{array}{c}\text{Model A: lead / lag anomalies only}\\ -0.12 \le \Delta \le 0.12 \Leftrightarrow 0.0 \le \Delta \le 0.12\end{array}}$$

Values of lead or lag larger than 12 percent of a chip are not required, because the signal quality monitors described in Section 5 readily detect waveforms with $|\Delta| > 0.12$.

Section 4 will derive the correlation function for Threat Model A. This function will include dead zones similar to the one shown in the bottom of Figure 1 provided that $\Delta \neq 0$. Figure 1 uses a value of Δ outside of our recommended range simply to make the dead zone more visible. Section 4 will also discover that waveforms with lead need not be tested, because their correlation functions are simply advances of the correlation functions for lag. Hence the HMI threat is identical.

3.2 Threat Model B: Amplitude Modulation Only

Unlike Threat Model A, Threat Model B introduces amplitude modulation. More specifically, it consists of the output from a second order system when the nominal C/A code baseband signal is the input. An example waveform is depicted in Figure 2, and such waveforms appear at the output labeled *Threat Model B* in Figure 7. As shown, the nominal C/A code signal, $x_{nom}(t)$, is the input to the second order system, because this model does not include any lead or lag of the baseband signal. As shown in Figure 8, Threat Model B assumes that the degraded satellite subsystem can be described as a linear system dominated by a pair of complex conjugate poles. These poles are located at $\sigma \pm j2\pi f_d$ where σ is the damping factor in nepers/Msecond and f_d is the resonant frequency with units of cycles/second.

Two parameters, σ and f_d specify the location of the dominant poles of the degraded circuitry. They can also be used to specify the impulse response $h_{2nd}(t)$ or the unit step response $e(t)$. The unit step response of a second order system is given by

$$e(t) = \begin{cases} 0 & t \le 0 \\ 1 - \exp(-\sigma t)\left[\cos\omega_d t + \dfrac{\sigma}{\omega_d}\sin\omega_d t\right] & t \ge 0 \end{cases}$$

$$\omega_d = 2\pi f_d$$

This step response will soon serve us well, because we will seek step responses for second order systems in Section 4 and we will integrate these step responses. In anticipation, we now provide the integral of *e(t)*.

$$E(t) = \int_0^t e(\alpha)d\alpha$$
$$= \begin{cases} 0 & t \le 0 \\ t - \dfrac{2\sigma}{\sigma^2 + \omega_d^2} + \dfrac{\exp(-\sigma t)}{\sigma^2 + \omega_d^2}\left[2\sigma \cos \omega_d t + \left(\dfrac{\sigma^2}{\omega_d} - \omega_d\right)\sin \omega_d t\right] & t \ge 0 \end{cases} \quad (1)$$

Threat Model B allows the following ranges for the parameters defined above

Model B: 2nd order anomalies only
$\Delta = 0$
$4 \le f_d \le 17$
$0.8 \le \sigma \le 8.8$

Smaller values of f_d are not included, because they would correspond to failures that also effect the military signal from the satellite. The military signals are more vigilantly tested before launch and during on-orbit acceptance. In addition, they are more thoroughly monitored after the satellite is declared operational. Smaller values of σ are not included, because they would place the system poles close to the $j\omega$ or in the right half plane. The resulting signals would be either oscillatory or unstable. Larger values of σ are not included, because they result in waveforms that are either detected by the signal quality monitors in described in Section 5 or because the undetected waveforms do not generate new constraints on the avionics. Within these parameter ranges, Threat Model B generates distortions of the correlation peak as well as false peaks. The false peaks in Figure 2 result when $\left(\sigma = 0.8, f_d = 8.0, \Delta = 0\right)$.

3.3 Threat Model C: Lead/Lag and Amplitude Modulation

Threat Model C introduces both lead/lag and amplitude modulation. More specifically, it consists of outputs from a second order system when the C/A code signal at the input does suffer from lead or lag. This waveform is a combination of the two effects described above, and assumes that a failure of the NDU can be associated with a simultaneous degradation of the analog section of the satellite. An example of such a waveform is shown in Figure 3, and these signals appear at the two outputs labeled *Threat Model C* in Figure 7.

This model includes all three parameters described above with the following ranges

Model C: both lead / lag and 2nd order anomalies
$-0.12 \le \Delta \le 0.12 \Leftrightarrow 0.0 \le \Delta \le 0.12$
$7.3 \le f_d \le 12$
$0.8 \le \sigma \le 8.8$

The range for f_d is smaller than for Threat Model B, because the likelihood that a failure causes both non-zero lead/lag ($\Delta \neq 0$) *and* amplitude modulation is small. Within the parameter ranges shown above, Threat Model C generates dead zones, distortions of the correlation peak, and false peaks. The waveform shown in Figure 3 corresponds to the point $(\sigma = 8, f_d = 5.0, \Delta = 0.12)$, which is just outside the threat space for Threat Model C.

4 Signal Quality Monitoring (SQM)

The SQMs considered in this paper use multiple samples of the main correlation peak to detect anomalous signals. This paper emphasizes SQMs that use pseudorange differences to form decision statistics, but we also mention a strategy that uses the sampled correlation values.

4.1 Tests Based on the Maximum Pseudorange Difference

The $\Delta\tau_{\text{nom}}(d_1, d_2)$ are the pseudorange differences measured when the nominal healthy signal is present. These nominal measurements are computed as follows

$$\tau_{\text{nom}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{nom}}(\tau + d/2) - \tilde{R}_{\text{nom}}(\tau - d/2) = 0\right\}$$
$$\tilde{R}_{\text{nom}} = h_{\text{pre}} * R_{\text{nom}}$$
$$\Delta\tau_{\text{nom}}(d_1, d_2) = \tau_{\text{nom}}(d_1) - \tau_{\text{nom}}(d_2)$$

In contrast, the $\Delta\tau_{\text{a}}(d_1, d_2)$ are the pseudorange differences measured in real time while seeking an anomalous waveform. These test statistics are given by

$$\tau_{\text{a}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{a}}(\tau + d/2) - \tilde{R}_{\text{a}}(\tau - d/2) = 0\right\}$$
$$\tilde{R}_{\text{a}} \in \left\{h_{\text{pre}} * R_{\text{A}}, h_{\text{pre}} * R_{\text{B}}, h_{\text{pre}} * R_{\text{C}},\right\}$$
$$\Delta\tau_{\text{a}}(d_1, d_2) = \tau_{\text{a}}(d_1) - \tau_{\text{a}}(d_2)$$

In these equations, the subscripts “**nom**” and “**a**” denote nominal and anomalous measurements respectively.

Some SQM algorithms simply compare a set of pseudorange differences to their nominal values. If any of these differences exceeds a specified threshold, then that satellite is removed from service by the LAAS or WAAS ground facility. The RTCA community considered two such SQMs – sparse and dense sampling. Sparse sampling is depicted in the upper portion of Figure 16 and reliably flags a given waveform as anomalous provided that one of the pseudorange differences exceeds the corresponding minimum detectable error (MDE). In other words, the anomalous waveform is reliably detected if and only if

$$\beta_B = \max\begin{bmatrix} \dfrac{\Delta\tau_a(0.1,0.15) - \Delta\tau_{nom}(0.1,0.15)}{MDE(0.1,0.15)} \\ \dfrac{\Delta\tau_a(0.2,0.15) - \Delta\tau_{nom}(0.2,0.15)}{MDE(0.2,0.15)} \end{bmatrix} \geq 1$$

As shown in Figure 16, dense sampling uses a picket fence of samples and reliably detects anomalous satellites provided

$$\beta_C = \max\begin{bmatrix} \dfrac{\Delta\tau_a(0.05,0.1) - \Delta\tau_{nom}(0.05,0.1)}{MDE(0.05,0.1)} \\ \dfrac{\Delta\tau_a(0.15,0.1) - \Delta\tau_{nom}(0.15,0.1)}{MDE(0.15,0.1)} \\ \dfrac{\Delta\tau_a(0.2,0.1) - \Delta\tau_{nom}(0.2,0.1)}{MDE(0.2,0.1)} \\ \dfrac{\Delta\tau_a(0.25,0.1) - \Delta\tau_{nom}(0.25,0.1)}{MDE(0.25,0.1)} \\ \dfrac{\Delta\tau_a(0.3,0.1) - \Delta\tau_{nom}(0.3,0.1)}{MDE(0.3,0.1)} \end{bmatrix} \geq 1$$

The MDE is the smallest pseudorange difference that can be detected without exceeding the false alarm and missed detection rates that are specified for Category I precision approach. Importantly, they are not the test thresholds themselves. The assumed MDEs are shown in Table 1, and they are for the measurements $\Delta\tau_a(d_2,0.1)$. In other words, they are for pseudorange difference measurements relative to a reference correlator with a spacing of 0.1. They are small, because they assume that the monitor samples of the correlation peak are taken at the same time as the reference samples. This simultaneity causes the majority of the multipath and noise errors to cancel. Without simultaneity, the MDE would be much larger.

Table 1: Minimum Detectable Errors (MDE) from Shively, Brenner and Kline, 1999

Spacing (d_{mon})	MDE for satellites at 5 degrees (meters)	MDE for satellites at zenith (meters)
.05	1.83	0.09
.15	1.62	0.09
.20	3.14	0.18
.25	4.57	0.26
.30	5.91	0.34
.95	17.48	0.99
1.00	18.04	1.02

1.05	18.56	1.05

The sparse and dense SQM algorithms find their origins in two very different philosophies. The picket fence was designed to detect any significant departure from nominal in the satellite signal. If no such departure is detected, then the waveform should be the nominal and expected GPS waveform. It should be safe to use by any variety of avionics – even if the avionics used a correlator scheme that was unanticipated in 1999. On the other hand, the picket fence costs more. Even though multi-correlator technology is certainly available today (mid-1999), these receivers have not yet found widespread application in equipment certified for aviation use.

In sharp contrast, the sparse algorithm was designed to protect avionics that used correlator spacing close to those used by the sparse algorithm itself. In other words, it was designed to protect *neighborhoods* around spacings of $0.1T_C$, $0.15T_C$ and $0.2T_C$. As such, its founding intention offers no protection to the arbitrary design of the future. The picket fence strategy is not further discussed in this paper, because the actual performance of these two algorithms is not as different as these very disparate philosophies would suggest.

The undetected points for the sparse detector and Threat Model A (lead/lag anomalies only) are shown in Figure 17. As shown, large leads or lags $\left(|\Delta| \geq 0.04 = 12 \text{ meters}\right)$ are readily detected on the ground. The undetected points in this threat space have small leads or lags and closely resemble the nominal signal. In addition, the location of undetected points is symmetric about the $\Delta = 0$, because the correlation function for a signal with lag is simply a time shift of the correlation function for lead.

The undetected points for Threat Model B (second order anomalies only) are shown in Figure 18. In this case, the undetected points have high frequencies ($f_d \geq 15$ MHz). The pre-correlator filters on the ground and in the air remove such high frequency anomalies. So once again, the undetected signals resemble the nominal signal. The undetected points for Threat Model C (lead/lag plus second order anomalies) are shown in Figure 19. Once again, the loci of the undetected points are symmetric about $\Delta = 0$.

The impact of these undetected points on the aircraft receiver performance is discussed in Section 6. However, we first describe another straight-forward SQM.

5.2 Mean Square SQM

The SQM described in this subsection uses one control loop to control the time of all the correlation samples. In the pseudorange difference tests, each correlator pair could be slewed independently. Even though the samples were taken simultaneously and in near proximity to each other, the sampling times were derived from separate control loops.

The mean square SQM combines the information from the different samples as follows:

$$\tau(d_{\text{ref}}) = \arg_{\tau}\left\{\tilde{R}(\tau + d_{\text{ref}} / 2) - \tilde{R}(\tau - d_{\text{ref}} / 2) = 0\right\}$$

$$\tilde{R}(\tau) = h_{\text{pre}}(\tau) * R(\tau)$$

$$z_m = \tilde{R}\left(\tau(d_{\text{ref}}) - d_m / 2\right)$$

$$\gamma = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{z_{\text{a},m} - z_{\text{a},m-1}}{z_{\text{a,prompt}}} - \left(\frac{z_{\text{nom},m} - z_{\text{nom},m-1}}{z_{\text{nom,prompt}}}\right)\right)^2$$

In these equations, $\tau(d_{\text{ref}})$ is the pseudorange estimate derived by the reference correlator and control loop. All other samples $\{z_m\}_{m=1}^{M}$ are synchronized to this reference time. The test statistic simply sums the squares of the differences between the measured slopes and the expected nominal slopes. Like the tests described in the last subsection, the differences cancel much of the noise and multipath effect.

This mean square approach has not been tested yet, but we mention it here to show that many different SQMs can be used to detect anomalous waveforms. These algorithms may or may not use synchronized samples as opposed to independently controlled loops. They may or may not use the so-called prompt sample. In short, we discuss sparse sampling in this paper not because we know it to be best. Rather, we simply wish to establish an existence proof. If sparse sampling provides adequate performance, then other acceptable SQMs certainly exist.

6 Aircraft Pseudorange Error Due to Undetected Signals

We now subject the aircraft receiver to the waveforms from our threat space that cannot be detected by the SQM. These undetected points are denoted $\left\{(\sigma, f_d, \Delta)_n\right\}\Big|_{n=1}^{N}$ and form a subset of the entire threat space. The aircraft pseudorange error is computed for each point in this set as follows

$$\Delta\tau = \left(\tau_{\text{a,air}}(d_{\text{air}}) - \tau_{\text{a,ref}}(d_{\text{ref}})\right) - \left(\tau_{\text{nom,air}}(d_{\text{air}}) - \tau_{\text{nom,ref}}(d_{\text{ref}})\right)$$

$$\tau_{\text{a,air}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{a,air}}(\tau + d/2) - \tilde{R}_{\text{a,air}}(\tau - d/2) = 0\right\}$$

$$\tau_{\text{a,ref}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{a,ref}}(\tau + d/2) - \tilde{R}_{\text{a,ref}}(\tau - d/2) = 0\right\}$$

$$\tau_{\text{nom,air}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{nom,air}}(\tau + d/2) - \tilde{R}_{\text{nom,air}}(\tau - d/2) = 0\right\}$$

$$\tau_{\text{nom,ref}}(d) = \arg_{\tau}\left\{\tilde{R}_{\text{nom,ref}}(\tau + d/2) - \tilde{R}_{\text{nom,ref}}(\tau - d/2) = 0\right\}$$

$$\tilde{R}_{\text{a,air}} \in \left\{h_{\text{air}} * R_{\text{A}}, h_{\text{air}} * R_{\text{B}}, h_{\text{air}} * R_{\text{C}}\right\}$$

$$\tilde{R}_{\text{a,ref}} \in \left\{h_{\text{ref}} * R_{\text{A}}, h_{\text{ref}} * R_{\text{B}}, h_{\text{ref}} * R_{\text{C}}\right\}$$

$$\tilde{R}_{\text{nom,air}} = h_{\text{air}} * R_{\text{nom}}$$

$$\tilde{R}_{\text{nom,ref}} = h_{\text{ref}} * R_{\text{nom}}$$

As shown, the aircraft's pseudorange error, $\Delta\tau$, contains the aircraft's current measurement, $\tau_{\text{a}}(d_{\text{air}})$ minus the current differential correction from the reference receiver on the ground, $\tau_{\text{a}}(d_{\text{ref}})$. The test statistic also removes the nominal difference between the aircraft and reference measurements $\tau_{\text{nom}}(d_{\text{air}}) - \tau_{\text{nom}}(d_{\text{ref}})$, because this common mode term will be present in the corrected measurements for the other satellites. Hence, it will cause errors in the aircraft's clock estimate but it will not effect the position estimate.

The aircraft pseudorange error is plotted as a function of correlator spacing and filter bandwidth in Figures 20, 21 and 22. In fact, the plots provide contours of

$$\max_{\sigma, f_d, \Delta} \Delta\tau(d_{\text{air}}, BW_{\text{air}})$$

The maximum error over the threat waveform parameters (σ, f_d, Δ) is plotted versus the correlator spacing and bandwidth used by the avionics. The pseudorange error contours for 3.5 meters are accented, because this is the maximum error (MERR) that LAAS can tolerate for a satellite at low elevation angle (Shively, 1999). Low-lying satellites are the most difficult for SQM, and so we use the MDEs and the MERRs for satellites at 5 degrees elevation. As shown, an SQM, based on correlator spacings of 0.1, 0.15 and 0.2 chips, protects the design parameters shown by the dark lines.

The contours in Figures 20, 21 and 22 assume that the reference filter is a sixth order Butterworth filter with a 3 dB bandwidth of 16 MHz. The aircraft filter is drawn from a collection of filters including Butterworth filters of order 6, 8 and 12, Tchebyshev filters of order 6 and 8, an elliptical filter of order 9 and a finite impulse response filter with 93 taps. Figure 23 shows the magnitudes of the frequency responses for these filters, and Figures 20 through 22 show the worst case pseudorange error over this family of filters.

The pseudorange error depends on the airborne filter in at least two ways, and these mechanisms are summarized in Figure 24. First, decreasing bandwidth decreases the impact of the signal anomalies. The lowest frequencies under Threat Models B and C are 4 and 7.3 MHz respectively, and these are attenuated whenever the two-sided bandwidth of the airborne receiver falls below 8 and 14.6 MHz respectively.

However, decreasing bandwidth also tends to increase the differential group delay, so the perturbation introduced by the anomaly tends to move relative to the monitored portion of the correlation peak. For this reason, receivers with double-sided bandwidths greater than 7 MHz should be required to have bounded differential group delay.

The absolute value of the differential group delay should be bounded as follows

$$\left| \frac{\partial \phi}{\partial \omega}(\omega_{3\text{dB}} = 2\pi f_{3\text{dB}}) - \frac{\partial \phi}{\partial \omega}(\omega_0 = 2\pi f_0) \right| \leq 150 \text{ nanoseconds}$$

$$f_{3\text{dB}} = \text{3dB cutoff frequency}$$

$$f_0 = \text{center frequency}$$

$$\phi(\omega) = \text{filter's phase response}$$

Figures 25 and 26 show the differential group delays for two families of filters. Figure 25 shows the delays for a set of 6th order Butterworth filters where bandwidth is the parameter. For this filter type, the differential delay never exceeds 150 nanoseconds even for very narrow bandwidths. Figure 26 shows the delays for a family of 8th order Tchebyshev filters with 0.1 dB of inband ripple. In this case, the differential group delay specification is exceeded when the bandwidth is narrow.

7 Summary

The international civil aviation community is developing two systems to augment the global navigation satellite services (GNSS). A wide area augmentation system (WAAS) provides service over continental areas by deploying networks of reference receivers and uses geostationary satellites to communicate with the aircraft. A local area augmentation system (LAAS) places reference receivers at a single airport and uses a VHF data link to communicate with aircraft approaching that airport. LAAS service is limited to the area around the instrumented airport, but the intrinsic capability of the system includes all categories of precision approach including auto-land. Both systems require signal quality monitoring (SQM) to detect anomalies in the signal from the GNSS satellites. One such anomaly effected the signal from GPS PRN19.

This paper considers several possible threat models for the degraded signal, and proposes one preferred model. This model enjoys the following advantages. First, it generates the three correlation peak pathologies that concern the aviation community – dead zones, false peaks and distortions. Second, the model has only three parameters and so computation and testing are not overly complex. Third, the threat waveforms are causal and can be more readily simulated or generated. Finally, the model can be associated with the subsystems in the onboard signal generator actually used by a GPS satellite.

All of the threat waveforms can be envisaged as points within a cube that we will refer to as the threat space or threat cube. The paper identifies the portions of the threat space that are not detected by a signal quality monitor (SQM) and waveforms in those portions are called undetected points. The paper also calculates the airborne errors caused by the undetected points. To speed those calculations, the paper develops closed form expressions for the correlation functions for each threat model.

The paper finds that an SQM, based on correlator spacings of 0.1, 0.15 and 0.2 chips, protects the design parameters of practical interest to the designer of a GPS receiver for aircraft.

Appendix A: The Nominal GPS Signal and Correlation Function

The GPS signal received by civilian aircraft has the following form

$$s(t) = \sqrt{2C}D(t)X(t)\cos(2\pi ft + \theta) \qquad \text{(A.1)}$$

The signal is the product of three different time waveforms: an radio frequency (RF) carrier, $\cos(2\pi ft + \theta)$; the navigation data, *D(t)*; and a *spread spectrum code*, *X(t)*. The carrier for the civil signal is a sinusoid with frequency equal to $f_{L1} = 1575.42$ MHz. This GPS transmission frequency is in the so-called L band and is sometimes called the *L1* frequency. Both the data and the codes use binary phase shift keying (BPSK) to modulate the transmitted carrier. Both are composed of sequences of rectangular waveforms which have duration T_C and amplitude +1 or –1, and these sequences simply flip the polarity of the cosine wave. As shown in equation (A.1), the GPS signal component of interest has amplitude $\sqrt{2C}$, where C is the power in the signal.

As shown in Figure A.1, the expected signal is a sequence of rectangular waveforms. Each of these elemental waveforms is called a chip and the satellite's signature sequence determines whether the chip polarity is a +1 or –1. The civilian code, *X(t),* is known as the Clear/Access or Coarse/Acquisition or C/A code. The C/A code has a chipping rate of 1.023×10^6 chips per second = 1.023 Mcps, and the length of the signature sequence is 1023 chips. Hence, the duration of an individual chip (T_C) is slightly less than a microsecond, and the duration of the overall code is one millisecond.

For the navigation data, each +1 or –1 is called a *bit*, and the bit stream carries the information required to use the given satellite for position fixing. This navigation message carries the data required by the receiver for position fixing. The authoritative description of the format and content of the navigation message is ICD-200. Surprisingly, this data can be sent using only 50 bits per second (bps) which means that the duration of each bit is $T = T_b = 20$ milliseconds. Since the navigation message only requires a rate of 50 bits per second (bps), there are 20 repeats of the code in each data bit.

Bandwidth can also be used to characterize the navigation message and the codes. Bandwidth is a measure of the spectral occupancy of a signal. For GPS, the bandwidths are approximated by considering the Fourier transform of the underlying rectangular waveforms. The Fourier transform of a rectangular waveform is given by

$$A\Pi\left(\frac{t}{T}\right) \underset{\Im}{\longleftrightarrow} AT\frac{\sin \pi Tf}{\pi f} = AT\mathrm{sinc}\left[\pi Tf\right]$$

This waveform takes a maximum value at *f=0* and falls to 0 at *1/T*. Hence, the *null-to-null bandwidth* is *2/T* , which is 2.046 MHz for the C/A code.

The receiver correlates the incoming signals from the satellites with internal replicas of those signals, and the GPS codes were selected for their auto- and cross-correlation properties. Correlation measures the similarity of two waveforms or sequences, and the GPS receiver uses correlation to estimate the time offset between the incoming signal and the replica. The maximum likelihood estimate of the arrival time corresponds to the time when the correlation is greatest.

The auto-correlation function for the code *X(t)* is (Sarwate and Pursley, 1983)

$$R_{X,X}(\tau) = \int_0^T x(t)x(t-\tau)dt$$

where $T=NT_C$ is the period of the code. Auto-correlation multiples *x(t)* by shifted versions of itself and integrates the product. If *x(t)* resembles a shifted version of itself, then the auto-correlation will be large. However, $R_{X,X}(\tau)$ will most certainly take its maximum value when $\tau = 0$, where

$$R_{X,X}(\tau) \leq R_{X,X}(0) = \int_0^T x^2(t)dt = T = NT_c$$

The GPS codes are crafted to have small auto-correlation values at all shifts greater T_C, and so the peak at $\tau = 0$ is both unique and sharp. As shown in Figure A.1, most receivers use a delay lock loop (DLL) to track the arrival time of the received signal. This loop attempts to straddle the peak of the correlation function with an early sample on the leading edge and a late sample on the falling edge as shown in Figure A.1. This loop strives to track the peak of the correlation function by slewing the samples such that their difference is null. In fact, the difference between the early and late samples is called the *error function* or the *discriminator function* as shown in Figure A.2. The receiver identifies the zero crossing of the discriminator function as the best estimate of the time offset between the replica and received signals.

The separation in time of the early and late samples is a key design parameter denoted as *d*. Typical values of this spacing *d* range from ten percent of a chip duration ($0.1T_C$) for a so-called narrow correlator design to $1.0T_C$ for a wide or standard correlator spacing. This parameter has a significant role in determining the sensitivity of the receiver's performance to noise, interference and multipath. In addition, it is typically hard-wired at the time the receiver manufacturer produces an application specific integrated circuit (ASIC) for their receiver and is generally not variable. This correlator spacing plays a central role in this paper, because it, along with two other parameters, determines the size of the measurement error introduced by an anomalous waveform.

In the absence of filtering, the central lobe of the correlation function looks like a triangle as shown in Figure A.1. However, all GPS receivers places a filter before the correlation process to mitigate the effect of radio interference. This filter delays the correlation peak and smooths the corners of the correlation function. This delay and smoothing does not normally introduce any position errors, because all the signals from all of the GPS satellites pass through the same filter. Hence, they all are identically smoothed and shifted. The resulting bias in the location of the zero crossing of the discriminator is identical to all of the measured arrival times, and this common bias is attributed to the receiver's time estimate rather than the position estimate.

Appendix B: Derivation of Most Evil Waveforms

This appendix uses signal theory to find the additive signal perturbations that maximize the pseudorange error in a differential GPS system that employs reference and rover receivers with mismatched correlator spacings. These waveforms have been called most evil waveforms or MEWFs. They induce maximum position errors for an airborne user of differential GPS but are completely indistinguishable from nominal signals at the reference station, even when the resulting correlation peak is monitored on the ground by additional correlator pairs (*monitors*).

The first portion of this appendix provides a mathematical characterization of the standard early minus late discriminator. The second portion outlines a procedure for constructing an MEWF for a given set of correlator spacings on the ground and correlator spacing in the air. The final section shows some plots of the resulting waveforms and their associated correlation peaks.

The time domain representation of a standard early--late discriminator is shown in Figure B.1. In the diagram, $x(t)$ is the internal replica of the code used by the receiver; $d = 2\alpha$ is the correlator spacing; T is the integration interval used by the accumulator (typically, one epoch of C/A code, or approximately 1 msec); and $\hat{\tau}$ is the current code phase estimate.

The early minus late samples from the correlator shown in the top of Figure B.1 can also be generated by sampling the output of the matched filter shown in the bottom of Figure B.1 (Ziemer and Tranter, 1976). In the frequency domain model, $X(f)$ denotes the Fourier transform of $x(t)$.

A differential GPS system consists of at least the two processing channels shown in Figure B.2, which uses the matched filter from Figure B.1 as a building block. As shown, the differentially corrected system is modeled as the difference of two matched filters. The aircraft and ground reference use correlator spacings $d_{\text{air}} = 2\alpha_{\text{air}}$ and $d_{\text{ref}} = 2\alpha_{\text{ref}}$, respectively. These spacings are, in general, unequal. The error function $z_{\text{air}}(T)$ is obtained by integrating the system output over an interval of length T and sampling the result.

In general, the MEWF for a given ground configuration is the signal which gives rise to a (distorted) correlation peak with the following properties:

- The correlation peak is identical to a nominal, error-free correlation peak when viewed at any of the reference; and
- The code phase error due to this correlation peak is maximum when viewed at the aircraft's correlator spacing..

In other words, the effect of evil is invisible to the all correlators on the ground, but maximally affects the code phase estimate seen at the aircraft.

When no monitors are used ($M=0$), the input waveform $J(f)$ which maximizes $z_{\text{air}}(T)$ is given by

$$j_0(t) = \arg\max_{j(t)} z_0(T) \xleftrightarrow{\Im} J_0(f)$$

$$J_0(f) = X(f)\left(\sin 2\pi\alpha_a f - \sin 2\pi\alpha_r f\right)$$

where this last result is from the Schwartz_inequality. In other words, the system error is maximized when the Fourier transform of the input is the complex conjugate of the overall transfer function.

To detect anomalous satellite signals, a differential GPS system like LAAS or WAAS can use multiple monitors of the correlation peak. A model for such a ground station is shown in Figure B.3, and the MEWF for this new system can be found by modifying our earlier result for no monitors. With M monitors, the MEWF is defined to be the signal that generates the largest error $z_{\text{air}}(T) = \langle J_M, Y_{\text{air}} \rangle$ subject to the constraint that it does not cause any response from any of the monitors. This latter constraint can be articulated as follows:

$$\left\{\langle J_M, Y_m \rangle = 0\right\}_{m=1}^{M}$$

$$\left\{Y_m(f) = X^*(f)\left(\sin 2\pi\alpha_{\text{m}} f - \sin 2\pi\alpha_{\text{ref}} f\right)\right\}\Big|_{m=1}^{M}$$

where $2\alpha_m$ is the correlator spacing used by the monitor m, and the set $\left\{Y_m(f)\right\}_{m=1}^{M}$ is the basis created by the monitors. In summary, the MEWF maximizes the differentially corrected pseudorange error subject to the constraint that it must fall in the null space of the monitor basis. Such waveforms can be found using the Schwartz inequality together with Gramm-Schmidt orthogonalization.

This definition of MEWF is not entirely realistic, because practical monitors have a non-zero threshold due to the nominal effects of noise and multipath. Such non-zero thresholds are accounted for in the analysis of the second order threat model discussed in Sections 3, 4 and 5.

For one monitor (*M=1*), the MEWF is

$$J_1(f) = Y^*_{\text{air}}(f) - \frac{\langle Y^*_{\text{air}}, Y_1 \rangle}{|Y_1|^2} Y_1(f)$$

where $Y_1(f)$ is the basis function for the one and only monitor. For *M* monitors, the MEWF is given by

$$J_M(f) = Y^*_{\text{air}}(f) - \sum_{m=1}^{M} k_m Y_m(f)$$

$$k_m \text{ is chosen such that } \langle J_M, Y_m \rangle = 0 \quad \forall m = 1 \cdots M$$

The MEWF shown in Figure 5 was generated using the *M=1* formulas with correlator spacings of 0.1, 0.2 and 0.15 for the reference, monitor and aircraft respectively. It also assumed that the ground and aircraft pre-correlator filters are both sixth order Butterworth filters with 3 dB bandwidths of 16 MHz. The correlation function shown in Figure 6 is for a system with 2 ground monitors *(M=2)* spaced at 0.6 and 1.0 chips, and with a reference spacing of 0.2 chips.

Appendix C: Exchanging the Order of Correlation and Convolution

Both cascades, shown in Figure 9, produce the correlation function $R_{X,Z}(\tau) = \langle x(t), z(t+\tau) \rangle$ where

$$z(t) = h * x = \int_{-\infty}^{\infty} h(t-\alpha)x(\alpha)d\alpha$$

In this equation, *h(t)* models the cascade of the second order system ($h_{2nd}(t)$) and the pre-correlator filter ($h_{pre}(t)$) shown in Figure 9. The proof is as follows

$$\langle x(t), z(t+\tau) \rangle = \int_0^T x(t)z(t+\tau)dt$$

$$= \int_0^T x(t) \int_{-\infty}^{\infty} h(t+\tau-\alpha)x(\alpha)d\alpha dt$$

Substituting $\theta = \alpha - t$ yields

$$\begin{aligned}\langle x(t), z(t+\tau)\rangle &= \int_0^T x(t) \int_{-\infty}^{\infty} h(\tau-\theta) x(\theta+t) d\theta dt \\ &= \int_{-\infty}^{\infty} h(\tau-\theta) \int_0^T x(t) x(\theta+t) dt d\theta \\ &= \int_{-\infty}^{\infty} h(\tau-\theta) R_{XX}(\theta) d\theta \\ &= h * R_{XX}\end{aligned}$$

which completes our short proof.

References

1. Aviation Week, Filter Center, July 26, 1993
2. Aviation Week, Filter Center, February 15, 1994
3. C. Edgar, F. Czopek, and B. Barker, “A Co-operative Anomaly Resolution on PRN-19”, Proceedings of IONGPS99, The Institute of Navigation, September 14-17, 1999
4. D. Luenberger, *Optimization by Vector Space Methods*, Wiley and Sons, New York, pp. 53-58, 1969.
5. C. Shively, M. Brenner and P. Kline, “Multiple Ground Tests Protecting Against Satellite Correlation Symmetry Faults in LAAS (Revision 4)”, private communication, September 13, 1999
6. C. Shively, “Acceptable Error Limits for Satellite Signal Faults in LAAS”, Proceedings of IONGPS99, The Institute of Navigation, September 14-17, 1999
7. R. Williamson, R. Crowell and H. Trotter, *Calculus of Vector Functions*, Prentice-Hall, New Jersey, third edition, pp. 160-166, 1972.
8. R. E. Ziemer and W. H. Tranter, *Principles of Communications: Systems Modulation and Noise*, Houghton Mifflin, Boston, pp. 315-317, 1976.