Improving GPS Coverage and Continuity: Indoors and Downtown

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BIOGRAPHIES

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Per Enge is the Director of the GPS Laboratory at Stanford, where he is an Associate Professor of Aeronautics and Astronautic. Per has received many awards, including the Burka Award for the Best Paper in the Journal Navigation, the Thurlow Award from the Institute of Navigation, the Joseph Satin Distinguished Fellowship for Excellence in Teaching and Research, and the Johannes Kepler Award for contributions to the field of satellite navigation.

Rod Fan is the founder and currently the Chief Technology Officer of @Road Inc., an information services company, headquartered in Fremont, California, which provides location-enhanced wireless Internet solutions form mobile commerce. Prior to @Road, as Technical Director at Space Systems Loral, he led an engineering team that designed and deployed on satellites the first GPS attitude receiver for orbital control. Rod has a Bachelor’s degree from Brigham Young University and an M.S. in Electrical Engineering from Northwestern Polytechnic University.

Wallace Mann is a Research Engineer at Enuvis, Inc. Previously he worked at Sapient as a Technical Architect responsible for design and implementation of internet sites, as well as technical course designer and People Strategist for the San Francisco office. He holds a Ph.D. in Electrical Engineering from Stanford for which he implemented a system to do 3-D interpretation of gray-scale images.

Anant Sahai is a Research Scientist at Enuvis, Inc. He holds Master’s and PhD degrees in Electrical Engineering and Computer Science from MIT and a Bachelor’s degree from the University of California at Berkeley. Previously, he has worked at Xerox's Palo Alto Research Center (PARC) on approximation algorithms and properties of NP hard problems. Anant has received several awards, including NSF and DOD fellowships, the MIT Ernst Guillemin Best Masters Thesis Award, the UC Berkeley Certificate of Distinction, the UCB EECS Department Citation for the best graduating student, and the UC Berkeley Engineering Henry Ford Award for the highest academic standing in the engineering school.

Jesse Stone is a Research Engineer at Enuvis, Inc. He received his Ph.D. in elementary particle physics from Princeton in 1998, where he collected and analyzed experimental data that resulted in the discovery of the rarest observed subatomic decay event. He also holds a Bachelor's degree in physics from the University of Chicago. He has worked as a quantitative research analyst at D.E. Shaw & Co, and he most recently was a director of product marketing for an online financial services startup.

Anil Tiwari is Director or Systems Architecture at @Road. Prior to joining @Road, he worked at Cisco Systems as a senior engineer in the wireless group. Previously, he was at Trimble Navigation where he led the team that designed the first GPS-based mapping product. He holds an M.S. in Electrical Engineering from Stanford University.

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ABSTRACT

These days, GPS use is being extended into areas where hitherto it simply was not available. For years, it has served society remarkably well for survey, aviation use, maritime use and a host of other applications where the receiver is in the open and the satellites are clearly visible. Now however, society wants more utility from GPS. We want to get position fixes in urban environments and even inside. This requires the GPS receiver to operate at very low signal to noise ratios, integrate every imaginable source of aiding information, and combat multipath. @Road and Enuvis are responding to this new GPS challenge.

@Road has deployed a GPS assistance reference network (GARNET) that provides GPS assistance data. In the fullness of time, this network will provide a data stream that replaces the satellite ephemeris and clock coefficients contained in the GPS navigation message. An alternate source of this data will be most welcome, because signal blockages wreak havoc with the relatively fragile navigation message. GARNET will also replace the Z count in the navigation message, so the receiver can determine the most significant bits in the measured pseudoranges. Finally, it will develop information that will increase receiver sensitivity by reducing the search area and enabling the use of longer averaging times.

Enuvis has designed algorithms that significantly enhance the sensitivity of the GPS receiver, while being robust to uncertainties in the actual signal environment including multipath. In addition, the Enuvis algorithms readily incorporate many sources of aiding data where the quality of this side information may be uncertain. These algorithms can be realized either on the mobile station (thick client) or a network server (thin client).

This paper has two goals. First, it begins the development of formal assessment metrics for these complicated systems. In time, these metrics will be used to measure the performance of GPS based algorithms indoors and in urban environments. Second, we use some of these preliminary metrics to describe the performance of our prototype system. Our system is still evolving and so the performance will change relative to what is shown here. In addition, we only show a subset of the current results, because our page count is limited.

By the way, @Road and Enuvis have no formal business relationship, and their collaboration is currently limited to the authorship of this paper.

1 INTRODUCTION

1.1 The Urban and Indoor Challenges for GPS

The Global Positioning System (GPS) is a widely acknowledged, success story. For over 20 million users, it provides worldwide position fixing, in all weather, at all times of day. It provides this service to users at sea, in the air and in space. In general, these users have a clear view of the sky and can receive all satellite in view with little difficulty. These fortunate users enjoy position-fixing accuracies of 10 meters or better. With differential corrections, they obtain accuracies of better than 1 meter.

To date, GPS has struggled to serve users in cities or other environments with obstructions. These users may not be able to receive signals from the four satellites required for three-dimensional position fixing. Indoors, the user may struggle to find a clear sightline to just one satellite. Yet these urban and indoor users also need to know where they are.

For example, emergency (E-911) callers would like to automatically deliver their estimated position when they call for help. When they place an emergency call, they are under stress and may be unable to provide a clear and accurate description of their location. The public safety service that receives the call from a mobile phone may use the reported position for a second purpose – keeping the lines open for a second or third simultaneous emergency. With position reports, they can compare the location of a new call to the locations from earlier calls. If they are close, they can block the new call, and leave the phone open for callers reporting different emergencies.

The challenge for urban or indoor use of GPS can be partially quantified by the signal to noise ratio (SNR) of the GPS signals. If the user is under an open sky, then typical GPS SNRs are shown in Figure 1, which shows the received SNR for 11 satellites versus the elevation angle of the satellite. As shown, the SNRs range from 19 dB-Hz to 47 dB-Hz, but the majority of the SNRs are above 37 dB-Hz. With a few exceptions, satellites above 15 degrees had SNRs above 34 dB-Hz. In general, even the very low satellites had SNRs above 25 dB-Hz.

Under foliage, the SNRs degrade, and Figure 2 is for a receiver under foliage in Golden Gate Park. As shown, the SNR degrades relative to Figure 1 for an open sky. Under the foliage, many satellites have SNRs below 25 dB-Hz even when they are 40 degrees above the horizon. Inside a hotel, the SNRs really suffer, and Figure 3 shows the SNRs for a receiver located deep inside the W Hotel in San Francisco.

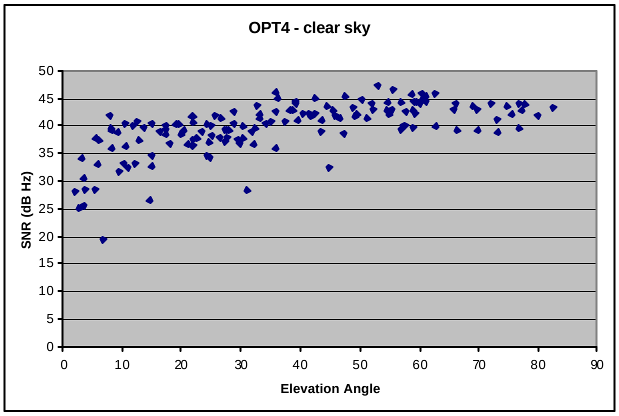

Figure 1: SNR vs. elevation for 11 satellites viewed from the roof of a building

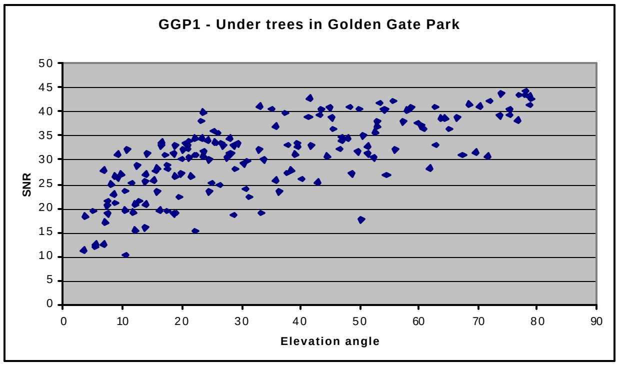

Figure 2: SNR vs. elevation for 11 satellites viewed from under foliage in a park

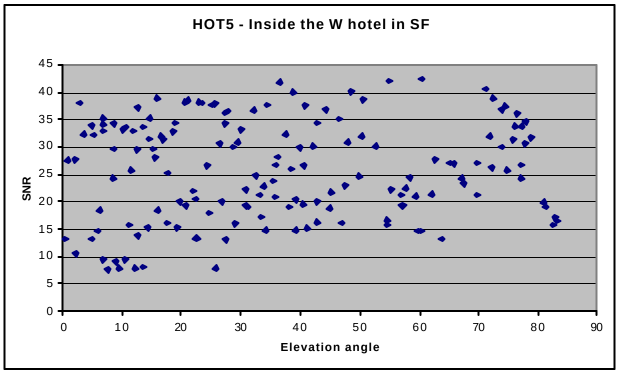

Figure 3: SNR vs. elevation for 11 satellites viewed from inside the W Hotel in San Francisco

The urban and indoor challenge is compounded by multipath. As the name implies, the signal from the satellite has followed multiple paths to the receiver. In addition to the direct path, the signal has arrived after one or more reflections. In open environments, the reflected signals are almost always weaker than the direct signals, but this is not always the case in cities and indoors. Reflections from buildings and other structures are commonplace, and this multipath can have any of three undesired effects. The reflected ray may destructively interfere with the direct ray and *fade* the composite signal power. The reflected ray may have approximately the same power as the direct ray and distort the correlation peak used by the receiver to make the GPS measurements. The reflected ray may be much stronger than the direct ray and cause the receiver to assume that the reflected ray is the direct ray. This last effect introduces the largest measurement errors.

The challenges posed by weak signals and multipath are appreciable. To date, the vast majority of GPS applications have enjoyed open sky reception of the satellite signals with SNRs of 35 dB-Hz or greater and comparatively weak multipath. As shown in Figures 2 and 3, indoor position fixing will require operation at SNRs below 20 dB-Hz, and as we shall discover that multipath is a real challenge.

1.2 Meeting the Challenge Using Satellite Navigation

Satellite navigation will overcome these challenges and become a key component for urban and indoor positioning. We feel that the following initiatives guarantee this forecast:

1. GPS data assistance can be communicated from GPS reference receivers to the mobile stations (MSs). The GPS reference receivers are at known locations that are electro-magnetically quiet and have a clear view of the sky. They provide information to improve the performance of any GPS mobile that may be in an obstructed environment. The aiding information includes differential corrections and the navigation message that normally comes from the satellites. The content and format of this aiding information has been standardized by industry. Such a network is shown in Figure 4.
2. Altitude aiding can be very helpful for GPS users in cities. An accurate altitude estimate from a database is equivalent to an extra satellite at zenith, and reduces the number of GPS satellites that are required for position fixing.
3. The arrival time of terrestrial radio signals at the MS can be measured and used to aid the GPS position fix. For example, signals from some cell phone base stations are synchronized to GPS time and are therefore capable of providing GPS frequency and time to the mobile station (MS). Even if the base stations (BS) are not synchronized to GPS, the arrival times of two BS signals can be differenced by the MS to derive a hyperbolic line of position that can be added to the suite of GPS measurements. This latter innovation does require synchronization between the base stations.

4. Advances in MS hardware handset hardware and robust signal processing algorithms enable acquisition of GPS signals in much more difficult environments than previously possible.
5. Future GPS satellites will broadcast three civil signals rather than just one. For the most part, civil use of GPS today is based on only one frequency, f_{L1}=1575.42 MHz. In the future, two new signals at f_{L2}=1227.60 MHz and f_{L5}=1175 MHz will be available. These new signals will begin to be available starting in 2003 or so. They will provide so-called frequency diversity, which will help to mitigate the effect of multipath. In other words, multipath may interfere with one signal, but is much less likely to simultaneously degrade all three signals. In addition, the new signal at f_{L5} has been designed to be more powerful and to give better performance in multipath environments.
6. In time, Galileo will be the European counterpart to GPS. It will place approximately twenty satellites in medium earth orbits (MEO) and geostationary orbits (GEO). The signals from Galileo are being designed with use in cities and indoors as prime objectives. Galileo and GPS will be two components of a worldwide Global Navigation Satellite System (GNSS). Taken together, they will virtually ensure that several satellite signals are available even to users in tough environments.

Of these innovations, we feel that algorithms are key. The algorithms must intelligently process GPS signals and integrate assistance data from various sources such as GPS reference networks, cellular networks, and altitude data bases. Optimal processing of such information is computationally prohibitive. A major challenge posed by indoor operation of GPS receivers is the design of efficient algorithms that make near-optimal use of available information. This paper concentrates on algorithms.

1.3 Objectives and Outline of This Paper

In this paper, we use the assessment framework shown in Figure 5 to describe the performance of Enuvis algorithms that increase the sensitivity of GPS user equipment and combat urban multipath. As shown, a formal assessment requires a clear description of the aiding information that is assumed, and the configuration of the system that hosts the algorithms. These items are further discussed in Section 2. Our assessment continues by considering acquisition and code phase accuracy in a noise-only environment. These topics are developed in Sections 3 and 4 respectively. Our evaluations of multipath performance and sensitivity to user and clock dynamics are discussed in Sections 5 and 6 respectively.

The metrics described in this paper do not encompass every dimension of algorithm performance. We simply do have not space in this short paper for a complete description of all that we have done. For example, we routinely evaluate the compute time and memory required to run our algorithms, but that work is not described herein.

In addition, our algorithms and our assessment metrics continue to evolve. Even the metrics described in this paper vary with respect to maturity. The metrics for assessing acquisition and code phase accuracy in noise are rather mature. In contrast, our assessment of multipath and sensitivity to user dynamics is informal at this time. With more work and feedback from the community, our assessment framework will become more streamlined and complete.

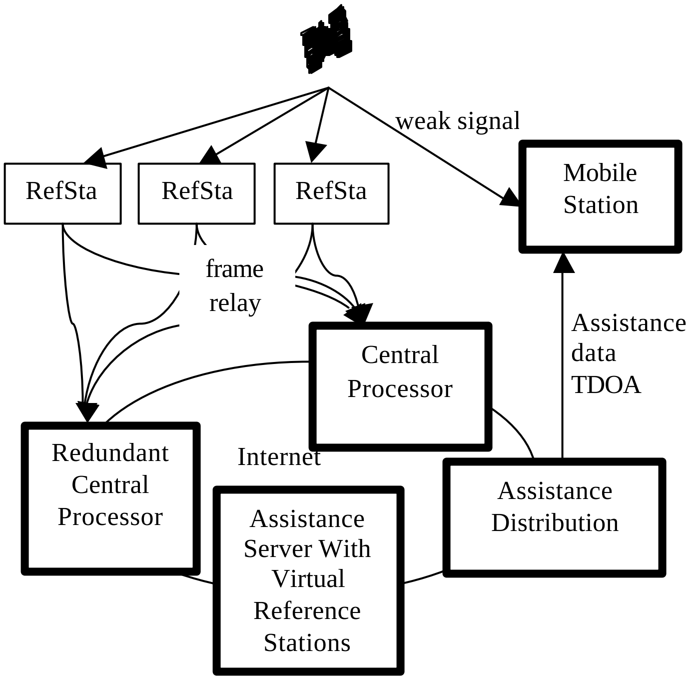

Figure 4: GPS Assistance Reference Network

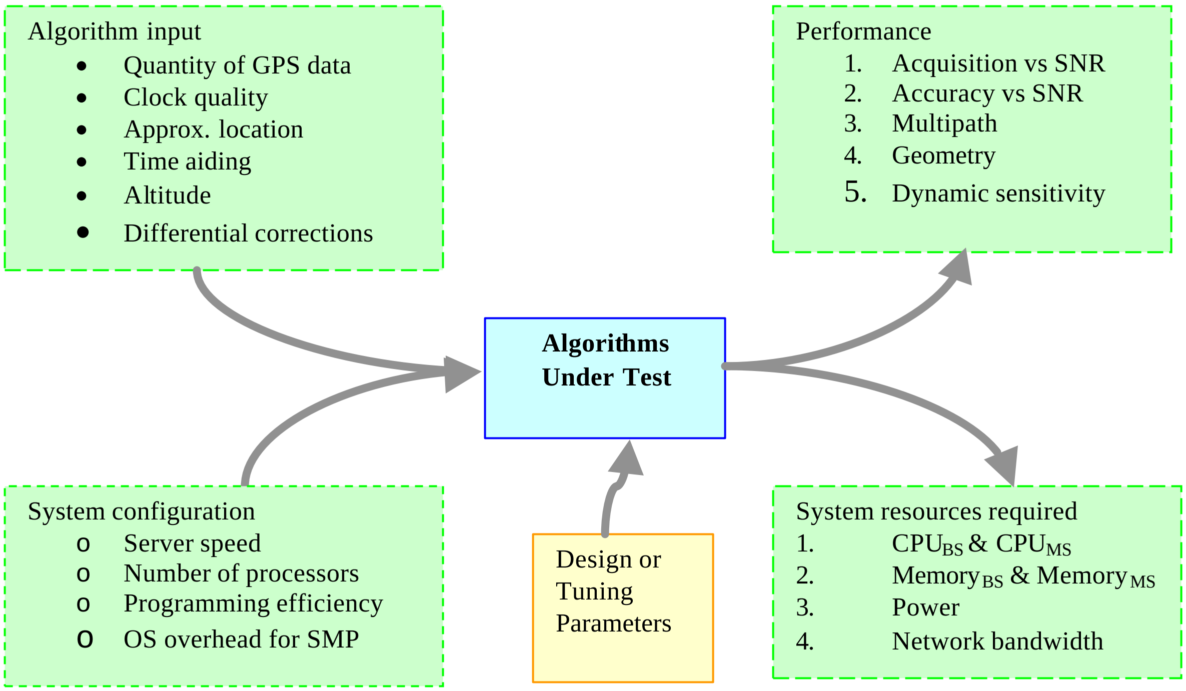

Figure 5: Current Assessment Framework

2 ALGORITHM INPUT AND SYSTEM CONFIGURATION

Enuvis' current algorithm is quite flexible in operating under different assumptions regarding inputs, and can be deployed on a wide variety of computational platforms. In order to be concrete in our evaluation of the algorithm we will make for the purpose of this report specific assumptions regarding the algorithm inputs and computational platform.

The algorithm inputs and pertinent assumptions are:

1. GPS signal. The algorithm is given 1.024 seconds of a GPS signal sampled at 4.096MHz, with 2-bit quantization. (The signal is first mixed to an intermediate frequency and then band-pass filtered to a 2MHz bandwidth.)
2. Clock stability. The clock used in mixing and sampling the GPS signal is accurate to 1.0ppm.
3. Approximate location. The algorithm is provided a location within 4km of the true receiver location.
4. Time aiding. The algorithm is given a time stamp identifying the time of the first GPS signal sample. The error of this time stamp is known to be less than 100ns.
5. Altitude aiding. The algorithm is given an estimate of receiver altitude. The RMS error of this estimate is 100m.
6. Differential corrections. The algorithm is given differential corrections that are accurate to within 1m.

It is worth noting that these are strictly *working assumptions* that identify precisely the inputs to the algorithm that we will evaluate herein. They are *not requirements* of Enuvis' technology. In particular, the algorithm settings can be adjusted to accommodate different assumptions on the input.

Motivated by the thin mobile station deployment, we will consider computations executed on a server architecture. Characteristics of the platform include:

1. Server speed: 500Mflops.
2. Number of processors: 8.
3. Programming efficiency: 80%.[1]
4. Operating system overhead for SMP: 84% utilization.[2]

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[1] The programming efficiency represents the fraction of CPU cycles spent executing the core signal processing operations on an optimized implementation running on an idealized single processor machine.

[2] The operating system overhead for SMP represents the extent to which the operating system can make the real multiple processor box behave as a faster idealized single

It is worth mentioning that Enuvis' current algorithm can also be deployed on a DSP chip, instead of the computational platform described above.

3 ACQUISITION PERFORMANCE

To maximize coverage in urban environments, a GPS algorithm should be designed to acquire signals with low signal to noise ratios (SNR). We define the SNR as the signal power divided by the white noise power in a 1 Hz band. The signal and noise are both taken to be at the input to the A/D converter. This analog signal has realized all the gains and losses associated with the antenna and RF deck, but not the A/D converter or the signal processing algorithm.

Before describing our metrics for acquisition performance, let us offer some perspective on the range of SNR of interest to indoor GPS applications. Figure 6 plots histograms of SNR values of GPS signals acquired by Enuvis' algorithm in the middle of the first story of a five story office building. There are three histograms. The first provides a distribution of the maximum SNR among satellites observed at any given time. The second provides a distribution of SNR for the second strongest satellite signal observed. The third histogram aggregates SNR values among all remaining acquired satellite signals. Note that in this environment, a GPS algorithm should be able to operate effectively at SNR levels in the 15dB-Hz to 20dB-Hz range in order to acquire two satellite signals regularly. To acquire three or more satellite signals, the GPS algorithm should be designed for the 10dB-Hz to 20dB-Hz range. Based on our experiments, these SNR ranges are representative of those observed in difficult indoor environments of interest.

Let us now discuss how we propose to measure acquisition performance. We first note that acquisition of one or more signals can aid in the acquisition of additional, weaker signals if the algorithm can take advantage of the information provided by the first acquisition. Hence, to rigorously assess acquisition performance, one should take into account the number of satellites that have been acquired prior to the satellite of interest. This can lead to a complicated and cumbersome metric. For the sake of simplicity, instead of considering acquisition performance for each possible number of previously acquired satellite signals, we only consider the cases of a first acquisition and a second acquisition – that is, acquisition when no other satellite signals have been acquired and acquisition after one satellite signal has been acquired. It turns out that the marginal benefit of acquiring a single satellite signal is much higher that that of acquiring additional satellite signals, so the performance of the second acquisition approximates that of subsequent acquisitions.

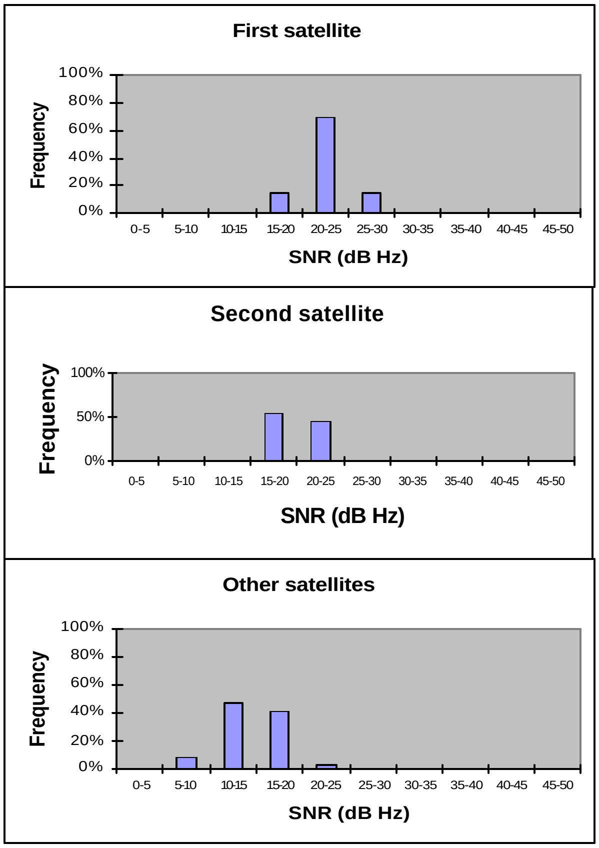

Figure 6: SNRs in the Middle of a Five Story Office Building

As a metric of acquisition performance, we propose plots of the quantity of GPS data that must be processed in order to acquire a satellite with any given level of SNR, with a probability of a false alarm no greater than 0.00001, and a probability of detection of 0.5. The probability of false alarm is defined to be the probability that an acquisition is reported with error larger than a single code chip. The probability of detection is the probability that the signal is acquired given the specified SNR and quantity of data. We propose plots of such performance curves for cases where the satellite is (1) the first to be acquired or (2) the second to be acquired.

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processor machine. Enuvis' algorithms are fundamentally very amenable to parallel execution.

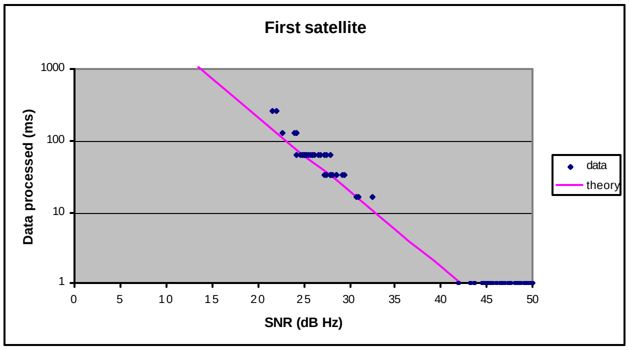


Figure 7: Data Required to Acquire First Satellite (Pr(fa)=0.00001, Pr(md)=0.5)

Figures 7 and 8 show such plots with theoretical predictions and real data. Figure 7 is for the first satellite to be acquired and Figure 8 is for subsequent satellites. Each data point represents an SNR estimated based on a real signal versus the amount of data used by Enuvis' algorithm to acquire (with the probability of false alarm set to 0.00001). Enuvis' algorithm is currently implemented to generate results only for data set durations that are powers of two, rather than for every possible duration of data (though this aspect of the algorithm can easily be modified). Hence, we would expect the real data to round any point on our theoretical curve up to the next power of two milliseconds. Indeed, empirical results from Enuvis' algorithm in field tests demonstrate this sort of behavior.

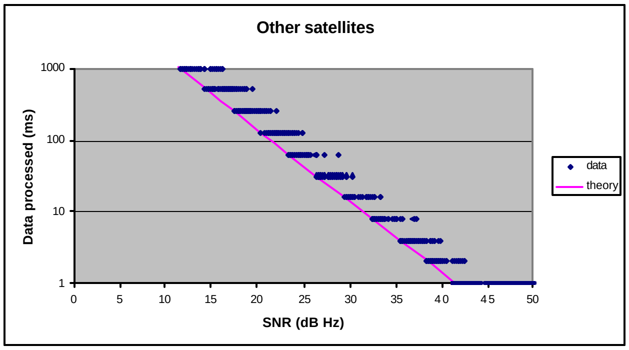


Figure 8: Data Required to Acquire Subsequent Satellites

Note that some empirical data points in Figures 7 and 8 exhibit use of amounts of data beyond rounding up to the next power of two milliseconds, relative to the theoretical curves. This should be expected, because the probability of false alarm is limited to 0.00001 and the probability of detection for the theoretical curve is 0.5. Consequently, the algorithm occasionally considers a signal as not being acquired until more data is used. Similarly, the algorithm occasionally is able to detect a signal with a slightly lower SNR than what is suggested by the theory.

4 Accuracy of Code Phase Estimates

To assess the accuracy of code phase estimation, we propose a plot of RMS error as a function of the amount of data processed, for different levels of SNR. Such curves reflecting performance of Enuvis' algorithm are provided in Figures 9 and 10. These curves show theoretical curves as well as empirical results. The data points plotted alongside the theoretical curves represent estimated RMS error of code phase estimates over data taken in field tests. The bars around these points represent one-standard-deviation confidence intervals for these estimated RMS estimates.

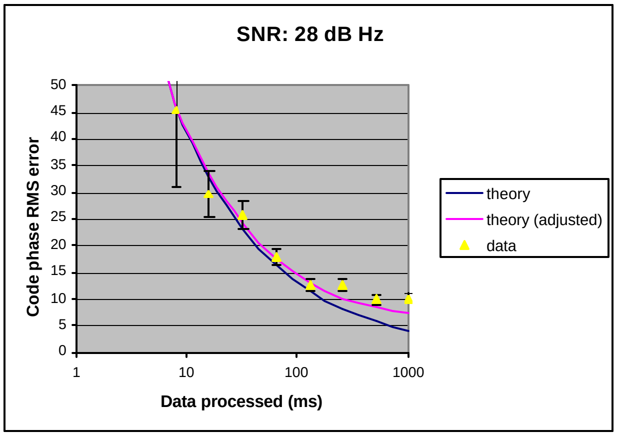


Figure 9: Code Phase Accuracy Versus Data Processed for an SNR of 28 dB-Hz.

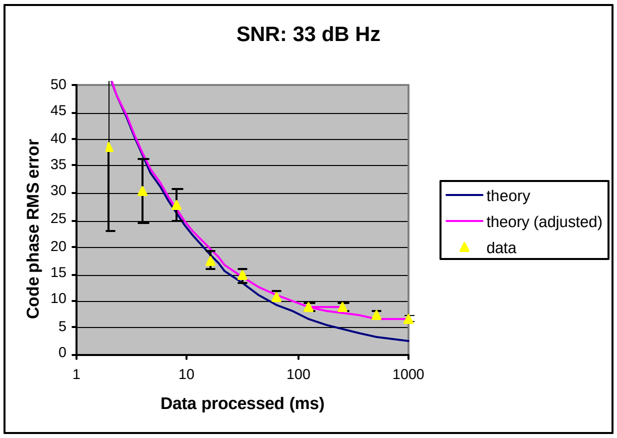


Figure 10: Code Phase Accuracy Versus Data Processed for an SNR of 33 dB-Hz.

The theoretical curves are derived under an assumption that errors are measured relative to the true location (e.g., within a meter) and the exact time (e.g., within nanoseconds) at which data is captured. Such quantities are not available, and errors in the quantities used in our computations give rise to irreducible residual errors – the estimated RMS errors do not converge to zero. If we assume that RMS error associated with code phase estimation converges to zero, it is possible to estimate the residual portion of the error. Figures 9 and 10 also plots theoretical curves adjusted to take into account residual errors estimated in this way. These adjusted curves (labeled "adjusted") appear to match up quite well with empirical results.

5 MULTIPATH

Code phase error introduced by random noise, as discussed in Section 4, is not sufficient as a metric for location accuracy. First, multipath frequently contributes a larger error to pseudorange than random noise. Second, pseudorange accuracy is diluted when mapped into the position domain. The dilution of precision (DOP) depends upon the number and geometry of satellites in view, which depends in turn on the configuration of obstructions in the environment. This section concentrates on the first effect - multipath.

In the fullness of time, multipath performance assessment will require simulation and the use of standard benchmarks based on real data samples from urban environments. We are currently in the process of developing these assessment methods. In the meantime, to provide some sense of the efficacy of Enuvis' algorithm in challenging environments, we present an example in which Enuvis' proprietary methods for dealing with signal distortions improve performance. The environment is an urban setting in Downtown San Francisco, surrounded by tall buildings. Pictures of the site are presented in Figure 11.

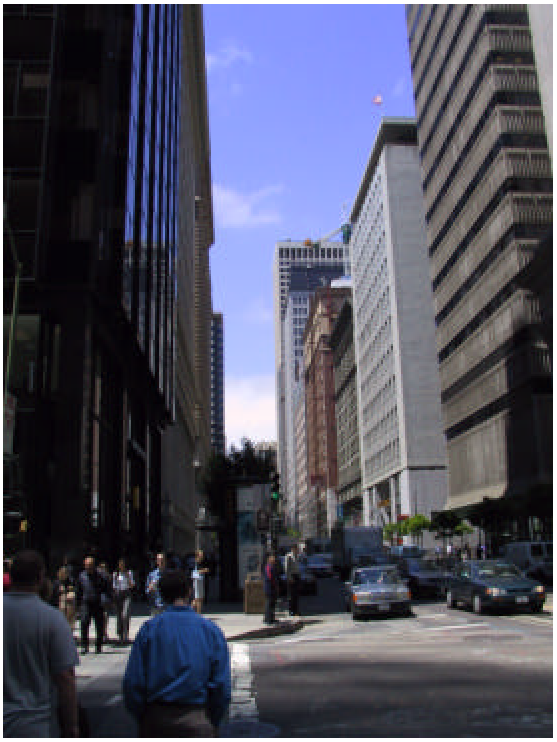
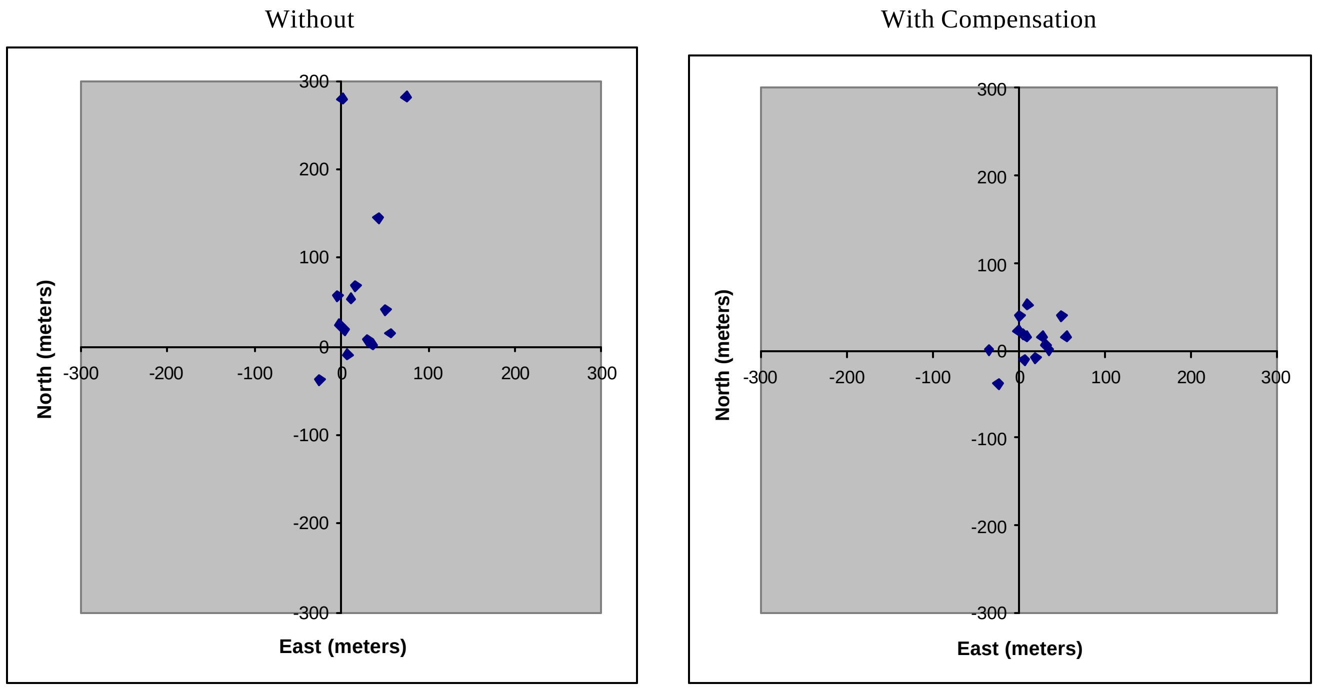


Figure 11: Test Environment for Figures 12 and 13

To demonstrate performance of Enuvis' algorithm we present two sets of results in Figures 12 and 13. In both figures, these results on the left represent the performance of Enuvis' algorithm without our multipath compensation, and the results on the right include our compensation. Figure 12 contains a pair of scatter plots in which each data point represents a location estimate generated by the algorithm upon processing 256ms of GPS signal for a given data capture. The center of the scatter plot corresponds to the true location of the antenna. Figure 13 illustrates acquisition and accuracy of location estimates for various quantities of GPS data processed by the algorithm. It shows the percentage of fixes with the specified accuracy or better as a function of the data quantity. As shown, our compensation improves accuracy significantly.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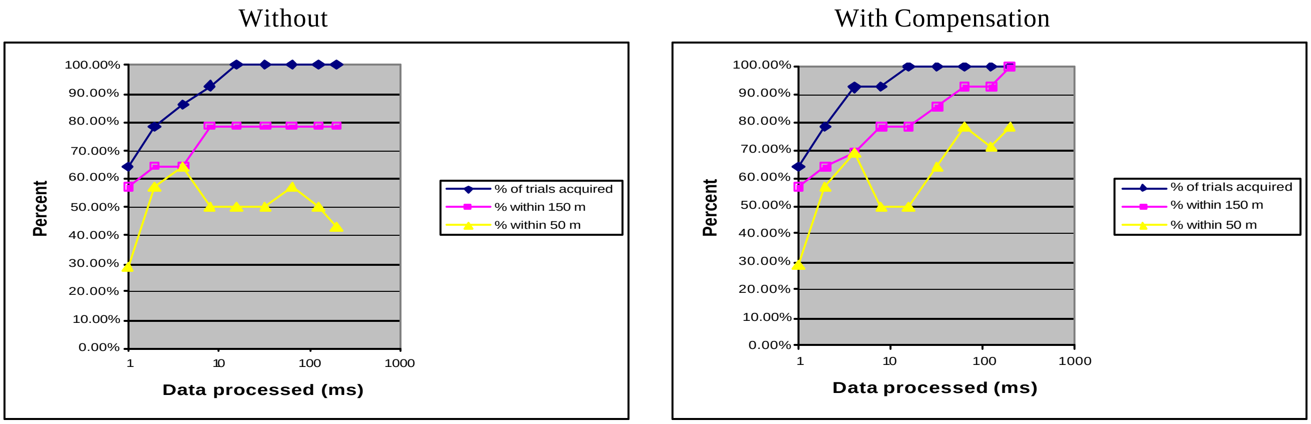
With Compensation

Figure 12: Multipath Performance Shown With and Without Enuvis Multipath Compensation


Without

With Compensation

Figure 13: Multipath Performance

6 ROBUSTNESS TO USER MOTION

A complete assessment must measure sensitivity to the motion of the user and the quality of the mobile station clock. After all, mobile stations are seldom absolutely stationary, and even modest motions of the GPS antenna can have a large effect on the performance of algorithms that are not prepared. Systems that require the user to be stationary or the clock to be very stable are simply unrealistic. The data presentation in Figure 13 partially addresses this need, because it reports accuracy and acquisition performance as function of the amount of GPS data processed. If the duration of signal required is short, then the system will be less vulnerable to unmodeled user motion or instabilities in the user clock.

Looking forward, we intend to offer more formal models of user and clock dynamics and metrics for assessing performance losses relative to such dynamics. Models for these dynamics must take into account statistics of typical usage and must also be correlated appropriately with environment. For example, if a receiver is traveling at 80km per hour, it is unlikely that the receiver will be in a building or even a dense urban environment.

Enuvis' algorithm is designed to accommodate dynamics, and therefore, our expectation is that Enuvis' algorithm will perform favorably in the presence of dynamics.

Figures 14 and 15 depict Enuvis performance while driving a car at approximately 35mph. For every data sample, acquisition was possible with 1ms of GPS data. The road traveled straight from North to South, so the longitude along the road was approximately constant. Figure 14 plots longitudinal RMS error as a function of the amount of data used by the algorithm. Figure 15 is a scatter plot of estimated locations generated during the drive, based on 256ms of data from each signal capture. Note that the points are roughly lined up in the North-South direction. These results point to the efficacy of Enuvis' algorithm when the receiver is in motion.

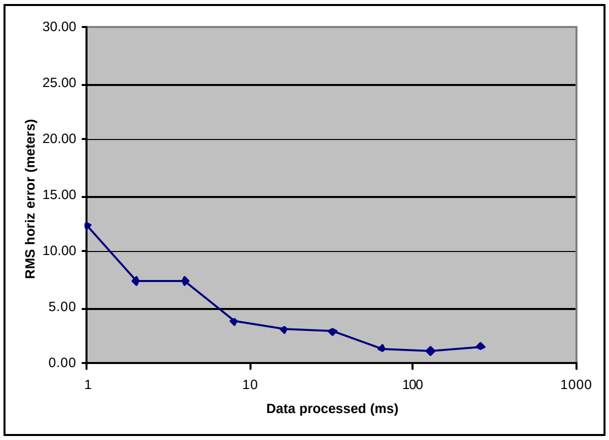


Figure 14: Cross-track Error Versus the Quantity of Data Processed for an Automobile Traveling at 35 mph.

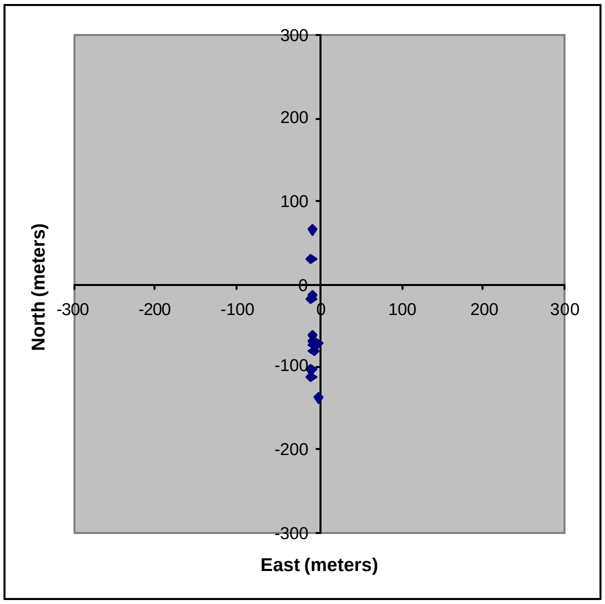


Figure 15: Horizontal Scatter Plot for an Automobile Heading North at 35 mph.

7 SUMMARY

We have reviewed the signal challenges associated with using satellite navigation indoors and downtown. In these environments, the satellite signals are blocked, weakened and reflected. @Road has developed a GPS assistance reference network (GARNET) to provide aiding information from receivers at favorable sites to receivers in these difficult environments. Enuvis has developed algorithms that increase the sensitivity of the receiver, ward of multipath and readily incorporate any aiding information that might be available. This paper *begins* the formal assessment of those algorithms.

Our assessment of acquisition performance is captured by Figures 7 and 8. These figures quantify the amount of GPS data required to acquire a GPS signal as a function of SNR. Figure 7 assumes that the signal is the first to be acquired, and Figure 8 assumes that the signal is the second to be acquired. The accuracy of code phase estimates is assessed based on Figures 9 and 10, which plots accuracy versus the amount of GPS data processed, for various SNR. These figures contain theoretical curves and empirical results that validate the theory.

We have also presented results on the multipath performance of our algorithms, and Figure 12 shows a horizontal scatter plot with and without the user of our multipath compensation. Our sensitivity to user motion is assessed in an informal setting. Indeed, Figure 14 shows the cross-track position error of an automobile traveling at 35mph. As shown, our algorithms provide accuracy better than 5 meters when 10 milliseconds of data are processed.

Our assessment metrics continue to evolve. Even the metrics described in this paper vary with respect to maturity. The metrics for assessing acquisition and code phase accuracy in noise are rather mature. In contrast, our assessment of multipath and sensitivity to user dynamics is informal at this time. With more work and feedback from the community, our assessment framework will become more streamlined and complete.

Our algorithms are also evolving and so the performance will change relative to what is shown here. For example, the results shown herein assume that our algorithms are running on a server. However, they can also be deployed on a DSP chip, instead of the computational platform described above. Such a deployment would offer comparable performance in terms of acquisition and code phase accuracy. Finally, we only show a subset of the current results, because our page count is limited.