Methodology and Case Studies of Signal-in-Space Error Calculation

Top-down Meets Bottom-up

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Outline

• Introduction – Signal-in-space error
• Methodology – Top-down
• Methodology – Bottom-up
• Case Studies
  – Planned satellite position outage, PRN 10, Day 39 of Year 2007
  – Unplanned clock anomaly, PRN 07, Day 229 of Year 2007
Error Sources of GPS Signals

Signal in space error
- Satellite position
- Clock
- Other

Propagation error
- Ionosphere
- Troposphere

Ionosphere Delay

Troposphere Delay

Receiver and local environment error
- Receiver clock
- Multipath
Motivation & Prior Work

**Motivation**
- signal-in-space, propagation and receiver errors have been well studied, but better understanding is still required
- Essential for GPS integrity
  - Satellite failures are identified if the signal-in-space errors exceed 4.42*URA (User Range Accuracy)
  - The statistics of signal-in-space errors are useful for evaluating URA

**Prior work of signal-in-space error calculation**
- KAIST, Jiyun Lee. GEAS presentations since early 2009
- Ohio Univ., Frank Van Grass. GEAS presentation 2009
- FAATC, Tom McHugh for WAAS PAN report
- Aerospace, Karl Kovach, presented at SCPNT in Nov. 2008
- David L. M. Warren and John F. Raquet, Broadcast vs. precise GPS ephemerides: a historical perspective, GPS Solutions, 2004
Signal in Space (SIS) Errors

• Main errors
  – Satellite position
  – Satellite clock

• Other
  – code-carrier incoherence
  – signal deformation
  – Inter-signal errors
  – satellite antenna phase center variation
  – satellite antenna group delay center variation
  – relativistic correction errors
Methodology Overview: Top-down vs. Bottom-up

**Top-down**
- Signal in space error
  - Satellite position
  - Clock
  - Other
- Propagation error
  - Ionosphere
  - Troposphere
- User receiver error
  - Receiver clock
  - Multipath

**Bottom-up**
- Signal in space error
  - Satellite position
  - Clock
  - Other
- Ionosphere error + clock error

Signal in space error = total pseudo-range error
- receiver clock error
- multipath error
- ionosphere error
- troposphere error
Bottom-up Methodology, Flow Chart

- Start
- Pick proper broadcast ephemerides based on the time of the truth
- Propagate broadcast satellite positions to the time of the truth
- Propagate broadcast satellite clock error to the time of the truth
- Calculate the difference between the propagated broadcast ephemerides and the truth
- Project the ephemeris error to a certain receiver on Earth
- End

Bottom-up

Signal in space error
- Satellite position ✓
- Clock ✓
- Other ✗

Signal in space error = satellite position error + clock error
Top-down Methodology, Data Source

Data Source: Wide Area Augmentation System (WAAS) / National Satellite Test Bed (NSTB) Network
- 38 stations in North America, with 3 receivers per station
- Data update rate: 1 Hz
- Output pseudo-range measurements and navigation messages
Bottom-up Methodology, Data Sources

Broadcast ephemeris: International GNSS Service (IGS) network

Precise ephemeris: National Geospatial-Intelligence Agency (NGA) network
# Methodology Comparison: Top-down vs. Bottom-up

<table>
<thead>
<tr>
<th></th>
<th>Top-down</th>
<th>Bottom-up</th>
</tr>
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<tbody>
<tr>
<td><strong>Data Source</strong></td>
<td>WAAS &amp; NSTB</td>
<td>IGS &amp; NGA</td>
</tr>
<tr>
<td><strong>Control of data source</strong></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Data update rate</strong></td>
<td>High, every 1 sec</td>
<td>Low, 15 min</td>
</tr>
<tr>
<td><strong>Depend on post-processed truth</strong></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Include all SIS errors</strong></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Receiver glitches</strong></td>
<td>No for WAAS</td>
<td>Yes</td>
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<tr>
<td><strong>Remove all non SIS errors</strong></td>
<td>No</td>
<td>Yes</td>
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<td><strong>Receiver coverage</strong></td>
<td>Limited (CONUS)</td>
<td>Worldwide, but not even</td>
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<td><strong>Data availability</strong></td>
<td>Difficult to retrieve past data</td>
<td>Available</td>
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Case Studies

Planned satellite position outage, PRN 10, Day 39 of Year 2007
Ground Track of PRN 10, Day 39-40 of Year 2007
Worst Projected Ephemeris Error

PRN 10, Day 39 of 2007
Worst projected ephemeris error ($\Delta X, \Delta Y, \Delta Z, \Delta b$)

Planned Outage
Zoom in
Top-down vs. Bottom-up, 100-sec Smoothing

PRN 10 Day 39

Atlantic City NJ, 39.44° N  74.56° W
Discrepancies of Top-down vs. Bottom-up, 100-sec Smoothing

Atlantic City NJ, 39.44° N 74.56° W
Top-down vs. Bottom-up, 15-min Smoothing

Atlantic City NJ, 39.44° N  74.56° W
Discrepancies of Top-down vs. Bottom-up, 15-min Smoothing

Atlantic City NJ, 39.44° N  74.56° W
Case Studies

Unplanned clock anomaly, PRN 07, Day 229 of Year 2007
Ground Track of PRN 07, Day 229 of Year 2007
Worst Projected Ephemeris Error

PRN 07, Day 229 of 2007
Worst projected ephemeris error ( $\Delta X, \Delta Y, \Delta Z, \Delta b$ )
Top-down vs. Bottom-up, Arcata CA, 100-sec Smoothing

UTC time (hours)

Worst projected ephemeris error (meters)

Bottom-up
URA
4.42*URA
Top-down

Arcata CA, 40.97° N 124.11° W
Discrepancies of Top-down vs. Bottom-up, 100-sec Smoothing

Arcata CA, 40.97° N  124.11° W
Conclusion (1/2)

- Compared two approaches to calculate signal-in-space error
  - Top-down: strips off all other errors from the pseudo-range errors, leaves alone signal-in-space errors
  - Bottom-up: builds up signal-in-space errors from satellite position errors and clock errors
- Top-down and bottom-up both have pros and cons

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Conclusion (2/2)

• Two case studies

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<td>Planned outage?</td>
<td>Yes</td>
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<td>Outage type</td>
<td>Satellite position</td>
<td>Satellite clock</td>
</tr>
<tr>
<td>Site investigated</td>
<td>Atlantic City, NJ</td>
<td>Arcata, CA</td>
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• Top-down and bottom-up match well for both normal and abnormal cases
• The discrepancies are independent of the filter length of carrier smoothing
• The discrepancies are due to
  – Inaccurate estimate of iono/tropo/multipath/receiver clock errors
  – Other error sources, e.g. code-carrier incoherence, signal deformation, Inter-signal errors, satellite antenna phase center variation, satellite antenna group delay center variation, relativistic correction errors, etc
  – Inaccuracies in precise ephemerides
  – Incorrect choice of active broadcast ephemeris
• The discrepancies are within +/-4 meters as a starting point
• Near term goal: better than 1 m
The authors acknowledge Tom McHugh from the FAA Tech Center for providing the WAAS/NSTB data of the 2007 outages.
Back-up Slides
Top-down Methodology in Detail: Removing Ionosphere Error

Top-down

Signal in space error
- Satellite position
- Clock

Propagation error
- Ionosphere
- Troposphere

User receiver error
- Receiver clock
- Multipath

Dual-frequency iono-free combination:

\[
\rho_{IF} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2} \rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2} \rho_{L2},
\]

\[
\Phi_{IF} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2} \Phi_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2} \Phi_{L2},
\]

\(\rho\): Code measurement
\(\Phi\): Carrier measurement
\(\rho_{IF}\): Iono-free combination of code measurements
Top-down Methodology in Detail: Removing Troposphere Error

Signal in space error
- Satellite position
- Clock

Propagation error
- Ionosphere
- Troposphere

User receiver error
- Receiver clock
- Multipath

Troposphere delay for satellite $i$
Troposphere mapping function for satellite $i$
Troposphere Vertical Error

Estimate and removal of troposphere error based on WAAS Minimum Operational Standard (MOPS):

$$\sigma_{i,tropo} = \sigma_{TVE} \cdot m(E_{l_i})$$
Top-down Methodology in Detail: Removing Receiver Multipath Error

Carrier smoothing:

\[
\rho(t_i) = \frac{1}{M} \rho(t_i) + \frac{(M - 1)}{M} \left[ \rho(t_{i-1}) + (\Phi(t_i) - \Phi(t_{i-1})) \right],
\]

\[
\bar{\rho}(t_i) = \rho(t_i).
\]

\( \rho(t) \): Code measurement  
\( \Phi(t) \): Carrier measurement  
\( \bar{\rho}(t) \): Smoothed pseudo-range measurement
Carrier smoothing using a recursive filter of length M:

Signal in space error
- Satellite position
- Clock

Propagation error
- Ionosphere
- Troposphere

User receiver error
- Receiver clock
- Multipath

Top-down Methodology in Detail: Removing Receiver Clock Error

- Receiver clock error is a common error for pseudo-ranges from all satellites
- For healthy satellites, the signal in space error is zero-mean i.i.d.
- Averaging the remaining errors from those healthy satellites cancels out satellite in space errors and leaves alone the user clock bias

Different among satellites, cancels out after averaging

Common among satellites, remains after averaging
Bottom-up Methodology, Data Sources

- **International GNSS Service (IGS) network**
  - Provide broadcast ephemeris
  - 350+ receivers worldwide
  - Output pseudo-range measurements and navigation data in RINEX format
  - Data update every 2 hours

- **National Geospatial-Intelligence Agency (NGA) network**
  - Provide post-processed true ephemeris
  - 10+ receivers worldwide
  - Output satellite position and clock information
  - Data update every 15 minutes


http://earth-info.nga.mil/GandG/sathtml/StationMap.gif
Bottom-up Methodology in Detail

1. **Start**
2. Pick proper broadcast ephemerides based on the time of the truth
3. Propagate broadcast satellite positions to the time of the truth
4. Propagate broadcast satellite clock error to the time of the truth
5. Calculate the difference between the propagated broadcast ephemerides and the truth
6. Project the ephemeris error to a certain receiver on earth

**Flowchart Steps:***
- Choose the most recent TTOM
- Use Kepler’s equations
- Based on the clock drift and the drift rate
- The broadcast and true ephemerides are of the same time stamp for fair comparison
- Project along line-of-sight between the satellite and the receiver

**End**
The ephemeris anomaly of PRN 10 on Day 39 is due to satellite position errors.
Ephemeris Error – Clock

PRN 07, Day 229 Year 2007

The clock error is the cause of the anomaly.