

# DESIGN AND PERFORMANCE ANALYSIS OF A LOW-COST AIDED DEAD RECKONING NAVIGATION SYSTEM

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## Abstract

This paper presents an aided dead reckoning navigation system based on the fusion of inexpensive inertial, air data and magnetic sensors aided by a skeletal network of radio-navigation aids. In the future National Airspace System of the United States—in which GPS is slated to be the primary means of navigation alongside a very small skeletal network of existing ground based radio-navigation aids—an inexpensive backup method of navigation will be required for General Aviation users. This backup method of navigation will allow users to successfully deal with scenarios where intentional or unintentional radio-magnetic interference renders GPS unusable in a given geographic area. The navigator presented in this paper is intended to be that backup navigation system for General Aviation aircraft.

Dead reckoning navigators mechanized using the classical inertial navigation techniques have traditionally been the solution to providing an alternate means of navigation that is mostly self-contained. A parametric study of navigation accuracy as a function of inertial sensor quality presented in this paper will show that this is not a practical solution for the General Aviation user. This is because the cost of the inertial sensors required to mechanize a classical inertial navigator with acceptable navigation accuracy is prohibitively expensive for the General Aviation user. A more practical solution for the General Aviation user is a heading and air speed dead reckoning navigator. A trade off study in this paper shows that the primary contributor to position drift in such a navigator is the stochastic nature of the wind field speed (i.e., the motion of the air mass in which the airplane is flying). Estimation of the wind field can be accomplished by very infrequent position fixes obtained from the skeletal network of radio-navigation aids that will be part of the future National Airspace System. An analysis of various estimator architectures for mechanizing this navigator will be presented. A low-cost sensor suite consisting of all the components required to mechanize the dead reckoning navigator was flown in a General Aviation test aircraft. The data from these flight tests shows that navigation performance comparable to that provided by current General Aviation navigation systems such as VOR and LORAN can be achieved.

## Introduction

General Aviation flying is defined as all aircraft operations except for those performed by the large commercial carriers and the military. Approximately 60% of all General Aviation operations are personal flights by private individuals [1]. A significant fraction of the remaining General Aviation operations are carried out by businesses, air taxi operators and commuter airlines. Very important niche operations such as transportation of critically ill patients to hospitals, law enforcement activities and aerial surveying make up the remaining General Aviation operations. Currently, General Aviation is undergoing a “renaissance” with a dramatic rise in the number of airframe manufactures and the total number of aircraft sold. It has been estimated that in the United States alone approximately \$11 Billion is expended annually in support of General Aviation operations [2].

One of the major changes that is occurring in General Aviation is in the area of avionics. Due to the recent availability of affordable sensor, display and computing technologies it is now possible to provide General Aviation users with integrated navigation information that is affordable. The navigation sensor that is making this possible is the Global Positioning System

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(GPS). GPS is a navigation system that provides instantaneous sub-meter accuracy position information for users worldwide. It is based on the multilateration principle where the user on the surface of the earth determines position using range information from multiple satellites. Because of its superior performance and capabilities, GPS is slated to be the primary means of navigation for all sectors of aviation replacing the existing navigation systems based on ground based radio beacons.

As implemented currently, however, GPS is susceptible to electronic interference and jamming. A deliberate or unintentional low-power radio transmission in certain frequency bands can render GPS unusable in a large geographical area[3]. The susceptibility of GPS to radio-magnetic interference does not pose a problem for military and high-end commercial users. These users have built in redundancies in the form of self contained inertial navigation systems which can be used to navigate out of large areas of interference. Equipping General Aviation aircraft with similar inertial navigators is not practical because they are prohibitively expensive for use in the acutely cost-sensitive business of General Aviation.

It is apparent that the General Aviation user will need an affordable backup navigation system in an environment where GPS is the primary means of navigation. The research in this paper addresses this problem. More specifically, it is about designing a backup navigation system for aviation use that satisfies the following constraints:

1. It is a low-cost system. In the context here, this is a system costing less than \$10,000.
2. It does not rely on GPS.
3. It has a position accuracy of 0.5 nautical after 30 minutes of operation.

The backup navigation system will have to have the capability of guiding a General Aviation airplane to the vicinity of an airport in inclement weather in the event GPS services are not available. The minimum navigation performance required to complete such a mission is equivalent to the level of performance provided by the least stringent of currently available non-precision approaches. An example of such a non-precision approach would be a VOR approach based on flying from the VOR located 30 nautical miles away from the airport of intended landing [4].

There are three options for mechanizing such a backup navigation system. The first option is to use an unaided dead reckoning navigator. The details of this option are discussed in Section 1. The second option which is discussed in Section 2 is to use position fixing based on range and bearing measurements from a skeletal network of ground based radio navigation aids. In Section 3, a mechanization based on blending dead reckoning and position fixing is presented. It will be shown that the option of blending dead reckoning with position fixing is the practical solution. In Section 4, the results of simulation studies performed to assess the performance of the blended system will be presented. In Section 5, experimental results will be presented. This will be followed by conclusions in Section 6.

## 1 Unaided Dead Reckoning

Dead reckoning is a form navigation whereby the current position of a moving vehicle is deduced by knowing speed and direction of travel since the last known position. The primary advantage of dead reckoning is that it relies on sensors contained within the vehicle and, therefore, provides a navigation system that requires no interaction with the world outside of the vehicle. A self contained navigator such as this is desirable especially as a backup navigation system.

The oldest and simplest way of implementing a dead reckoning navigator is by using speed and heading measurements. The idea of dead reckoning using speed and heading measurements is simply this: Given a measurement of heading and speed, use the heading measurement to resolve speed into north-south and east-west velocity components. By integrating the resolved velocity components once, north-south and east-west positions are obtained. A more sophisticated way of dead reckoning is that of inertial navigation. The fundamental principle behind inertial navigation is that of determining the position of a vehicle from a time history of acceleration measurements. The acceleration history can be measured by accelerometers fixed or strapped in the body of the vehicle. The primary drawback of dead reckoning is the fact that sensor errors map into unbounded position errors which grow with time. For example, in an inertial navigation system small accelerometer errors lead to position errors that grow as a function of  $t^2$ . Therefore, successful implementation of an inertial navigator requires high quality inertial sensors (i.e., accelerometers and rate gyros) that have small output errors. Since high quality inertial sensors are expensive, inertial navigation systems that employ them are prohibitively expensive for General Aviation users.

To illustrate this fact, an analysis of the performance of dead reckoning navigators which use sensors that are affordable by General Aviation users was performed. The inertial sensors used in these systems are of the quality that is labeled as "automotive grade" or "consumer grade". This term is used to describe these sensors because their primary application is in the automotive industry (active suspensions, skid control, etc.,) or the consumer hardware (camera stabilization, computer

INS Quality	Rate Gyro			Accelerometer		
	$\sigma_\omega$ (deg/hr)	$\tau$ (sec)	Noise (deg/sec)	$\sigma_f$ (g)	$\tau$ (sec)	Noise (g)
Tactical	0.35	100	0.0017	$50 \times 10^{-6}$	60	$50 \times 10^{-5}$
Automotive	180	300	0.05	$1.2 \times 10^{-3}$	100	$1 \times 10^{-3}$
Consumer	360	300	0.05	$2.4 \times 10^{-3}$	100	$1 \times 10^{-3}$

Table 1: Parameters for Error Models of Inertial Navigation Systems.

mice, etc.). These sensors range in cost from \$25 to \$1000 and are expected to drop in price in the future. Tactical grade sensors are of the quality that is used in guided munitions and are the next step up in quality from automotive and consumer grade sensors. Even though they are not in the price range of the average General Aviation user, they are included here for comparison purposes.

Table 1 shows error models for various inertial sensors that was developed as part of this analysis. For the heading and velocity dead reckoning system the heading, air speed and wind mass velocity errors are assumed to be exponentially correlated processes. For the heading system the standard deviation of the heading error is assumed to be  $2^\circ$  with a time constant of 120 seconds. For the air speed measurements, the standard deviation is assumed to be 5 knots with a time constant of 300 seconds. Finally, in accordance with a model developed in [5], the variation in the wind velocity is assumed to be governed by a Gauss-Markov process with a standard deviation of 5 m/s and a correlation time of 400 seconds.

A covariance analysis was performed using the above listed error models. Figure 1 shows the results of this analysis. This figure shows the growth in the horizontal position error as function of time for inertial navigation systems made from low cost inertial sensors of varying quality. From Figure 1 we see that the position error growth of a tactical grade inertial navigator exceeds the 1/2 nautical mile bound (i.e., 700 m) after approximately 550 seconds. However, it continues to exceed the performance of all the other dead reckoning systems until approximately 1200 seconds. At this point the errors in the inertial navigation system begins to exceed the error of the simple heading and speed dead reckoning system. For times beyond 1200 seconds the heading and speed dead reckoning system is superior to all the low cost inertial navigation systems. The primary contributor to position errors in an heading and air speed dead reckoning system is the error due to neglecting the air mass movement. If this parameter can be estimated, then the errors in a dead reckoning system can be reduced dramatically. The results of this analysis suggest that low cost dead reckoning sensors alone can not be used to make a usable backup navigation system.

## 2 Position Fixing

When navigating on the surface of the earth, if the range or bearing to three distinct points (at known geographical position) is given, it is possible to determine one's position. The process of determining position this way is called position fixing or triangulation. In the previous section it was shown that a dead reckoning navigator based on low cost sensors can not be an adequate backup navigation system for General Aviation users. This fact has led to the consideration of position fixing systems to provide backup navigation capability for General Aviation users. Range measurements needed for position fixing can be obtained from radionavigation systems such as Distance Measuring Equipment (DME) while bearing measurements can be obtained from VHF Omnidirectional Ranges (VOR) or Non Directional Beacons (NDB). NAS architectures that are based on retaining only a small subset of the existing DME as a back-up are discussed in [6]. It is the proposition of this paper that this architecture can be the basis for an efficient redundant navigation system because:

1. DME based triangulation combined with inertial navigation is a scheme of navigation that is currently used by FMS found on complex jet-lines [7]. The availability of inexpensive but high-powered micro-processors along with the recent proliferation of low cost sensors make construction of similar systems for GA users possible. This architecture, therefore, will provide navigation services for all segments of the aviation users unlike other proposed alternatives such LORAN which is used exclusively by General Aviation users.
2. VOR and NDB are systems that provide angular measurements and thus have accuracy that degrades with distance when used as part of a navigation system with area navigation capability. Increased accuracy will require a dense network of VOR or NDB sites. This is counter to the objective of reducing the upkeep cost for the National Airspace System which is predicated on maintaining as few as possible ground based navigation aids.

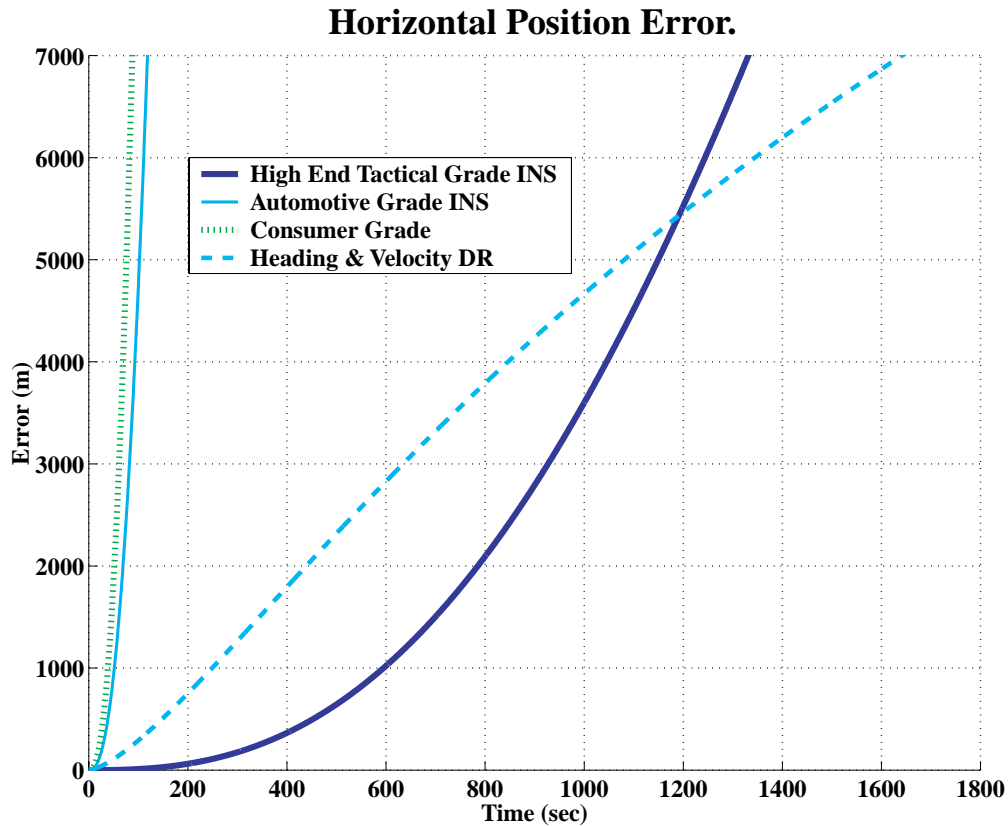


Figure 1: Error Growth Rate for Low cost Dead Reckoning Navigation System.

3. A usable skeletal network of ground based facilities may require relocating some radio-navigation aids. In comparison to a VOR facility, it is easier and less costly to install and maintain a DME facility. It is estimated that the cost of installing a DME facility is approximately 25 percent that of a VOR facility.

DME is a pulse-ranging system used in aviation which is based on the radar principle. The airborne DME radio, called an interrogator, emits a pair of pulses which are received by a ground transponder and retransmitted back to the airborne interrogator. The airborne interrogator then measures the time flight and computes the distance by multiplying the time of flight for the pair of pulses by the speed of light. It should be noted that, if high update rate DME range information can be obtained from two or more DME transponders, this information alone can be used for navigation and other sensors would not be required. However, obtaining continuous or very frequent range measurements from two or more DME transponders is difficult in practice. Most DME receivers are capable of tracking only one DME station at a time. There are specialized airborne receivers called scanning DMEs which are capable of tracking multiple stations at a time but are expensive items and used almost exclusively in the newest commercial jet liners. One solution is to carry multiple DME receivers so that two or more simultaneous range measurements from two separate DME ground stations will be available. This is also a costly solution because it requires an additional DME receiver. Another solution is use one DME to acquire the range from multiple DME ground stations intermittently. This scheme is problematic because DME is a query/response system and the potential exists for saturating the DME ground transponder if intermittent interrogations are not done carefully. This is because each time an airborne DME receiver switches from tracking one station to tracking another station, it will interrogate at a rate of approximately 135 pulses per second. This is approximately four times greater than the interrogation rate during normal tracking. If this switching is done too frequently, the number of aircraft that can be serviced by a given ground station will be reduced.

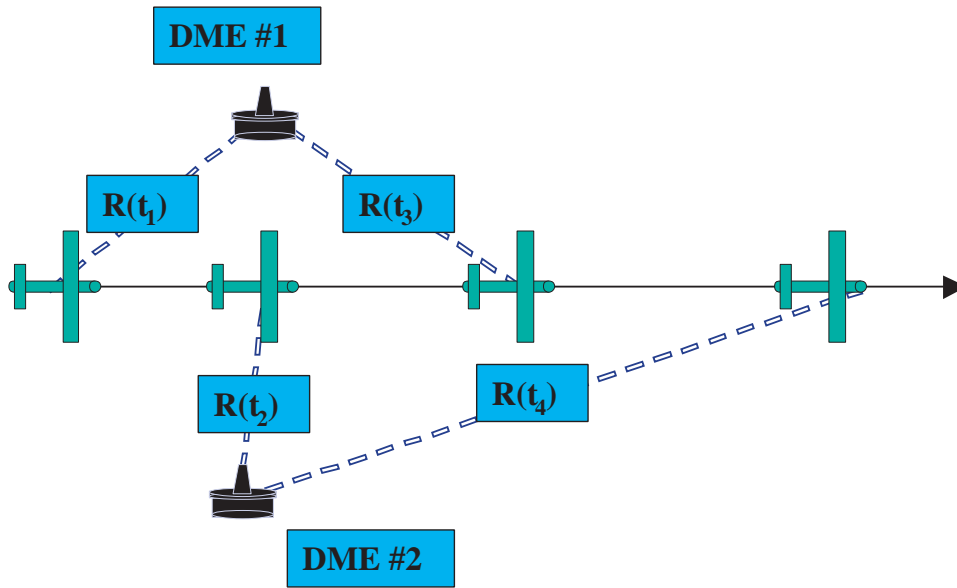


Figure 2: Sequential DME Range Updating.

This problem can be mitigated by scheduling interrogation in way that ensures that a given DME ground transponder can handle the expected traffic load. Based on a DME loading analysis in [8], DME saturation concerns can be mitigated by setting the interval between any two DME range measurements to be at least 15 seconds apart. This precludes the possibility of using position fixing alone for navigation because simultaneous range measurements cannot be obtained. Instead it leads to a navigation scheme depicted in Figure 2.

### 3 Blended Dead Reckoning and Position Fixing

In Section 1 it was shown that open loop dead reckoning schemes will develop position errors in excess of the required 0.5 nautical mile in less than 30 minutes. Thus they have to be coupled with a position fixing system of some kind to keep error growths in check. There are two choices of dead reckoning that can be employed—heading and speed based dead reckoning or classical inertial navigation. The question at this point, therefore, is what form of dead reckoning system should be used? If dead reckoning using the classical inertial navigation formulation is selected, then the sensor that have to be used are either automotive or consumer grade sensors. Tactical grade inertial sensors are out of the cost range of General Aviation. The use of these sensor with DME position fixing was explored. It was found not to work very well. The key reason for this failure is the fact that position fixes from DME are going to be intermittent. Because of the intermittent nature of the position updates, the entire fifteen-dimensional navigation state vector of an inertial navigator can not be reconstructed with a few measurements. This is because there are numerous errors that contribute to the position errors in an inertial navigator and they are not readily observable from intermittent range measurements. Thus, the logical solution is heading and speed dead reckoning. The solution proposed in this paper for constructing a low-cost backup navigation system is to blend dead reckoning sensors with intermittent DME triangulation. The intermittent DME ranges will provide a means for estimating sensor and wind errors thereby bounding the drift error in the dead reckoning systems.

### 4 Simulation Studies

In this section we present the result of simulation studies performed to assess the performance of the combined DME and dead reckoning navigator. In this simulation study, a aircraft is approaching an airport where coverage from two DME is available. The configuration of the approach track and the DME geometry is shown in Figure 3. In [9] it was shown that this DME geometry provides adequate coverage. The first question that needs to be answered is how do the errors

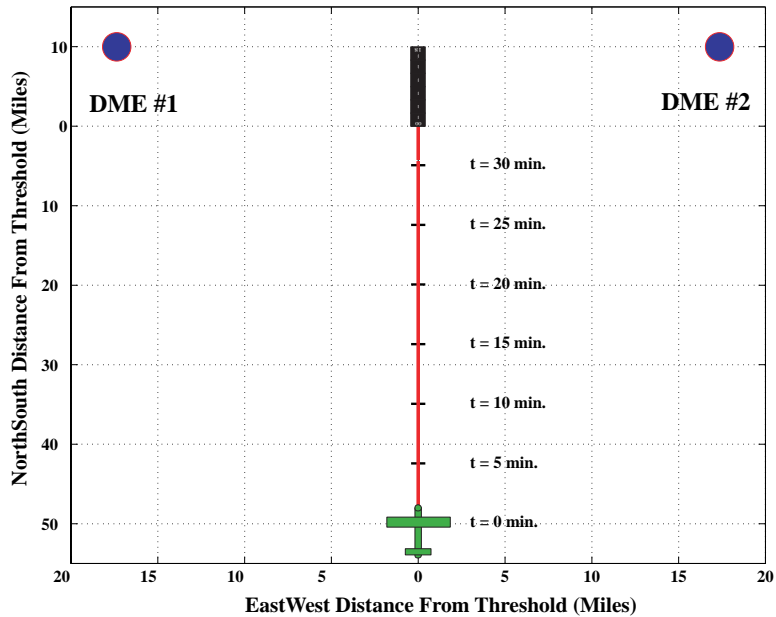


Figure 3: Simulation Ground Track and DME Geometry.

in position grow with time between the DME updates. Figure 4 shows the error ellipses that result when mechanizing a navigator with sequential position updates. The figure on the right shows the error ellipses when the system is mechanized using simultaneous DME range measurements. This is the case that would be observed when using a scanning DME or two DMEs simultaneously. This is shown for comparison purposes only because a scanning DME or multiple DMEs are not an economical alternative for General Aviation. In both the left and right subplots in Figure 4, the error ellipses are seen to grow between measurement updates. As would be expected, for the simultaneous range update case, the error ellipse shrinks uniformly at the measurement update. This is indicative of the fact that all the information needed to construct the navigation states (or compute the error in the states) is available from these simultaneous range measurements.

The situation is different when only one range measurement is available. In the case of the sequential update case, the shrinking only occurs in the direction from where the range measurement is obtained. Another way to view this is that, only partial state information is available from one range measurement. Therefore, the error ellipse can be only reduced in one directions. So it is clear that there is a control on the growth of the position errors when mechanizing the navigator using intermittent range measurements. However, Figure 4 raises one question. That is, if the time between measurement updates is allowed to get large, will there be a case where the sequential updating scheme will fail to keep the error ellipse bounded? To answer this question, the same simulation study was repeated but with different measurement update intervals. A summary of this simulation study is shown in Figure 5. What can be seen in this figure is that the covariance is kept in check for all update cases. However, for the longer update times, the error covariance will get very large and it will require a long update time before the errors are reduced down to an acceptable level.

## 5 Results

Experimental validation of the blended DME-dead reckoning navigator algorithm was conducted on post-processed flight test data collected using a Beechcraft QueenAir, a twin engine GA aircraft. The aircraft was equipped with a sensor suite which included a low cost magnetometer triad (Honeywell HMR2300), an air-data computer (Shadin ADC200) and a low cost Inertial Measurement Unit (CrossBow DMU-FOG). A very accurate record of the aircraft's trajectory was captured using GPS augmented by the Stanford University Wide Area Augmentation System (WAAS) prototype. In addition, a highly accurate record of the aircraft's heading, pitch and roll angles was captured by a navigation grade (0.01 deg/hr drift) Inertial Reference Unit (Honeywell YG1851 IRU). Although the aircraft was equipped with a low cost DME (Allied Signal KN-64), part of the study was to determine the effect of DME geometry on the navigation solution. Therefore, in the results that

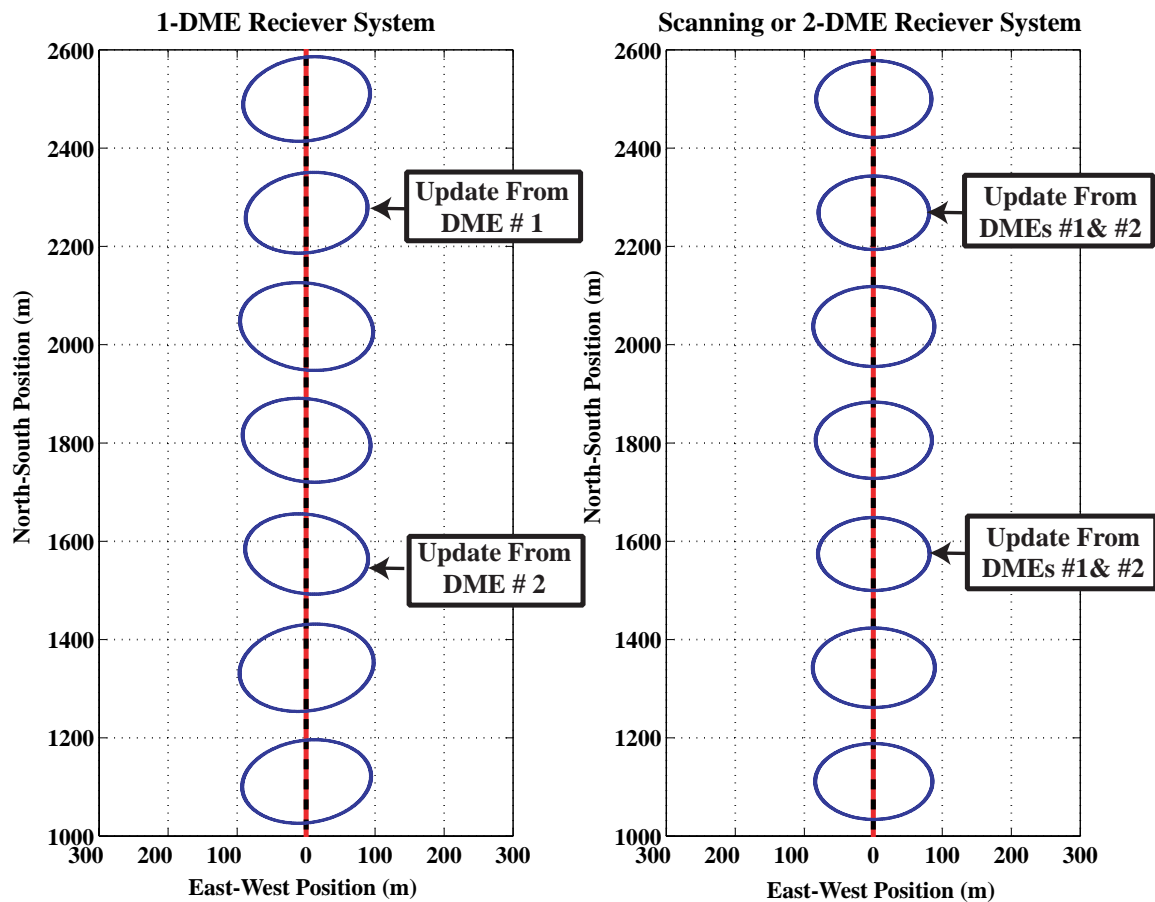


Figure 4: Position Error Ellipses for DME-DR Navigator (For Clarity Ellipse Dimensions have Been Scaled down by a factor of 2.5).

follow, the DME range measurements were simulated. Given the position coordinates of the simulated DME transponders, the accurate record of aircraft position recorded by GPS/WAAS was used to back-out the error-free DME ranges that would have been observed. The generated range measurements were then corrupted using DME range error models similar to the ones documented in [10]. The flight test trajectory consisted of a series of  $360^\circ$  turns simulating a holding pattern followed by a straight in approach to runway 25L at Livermore airport in Livermore, CA. Heading was derived using an AHRS algorithm which aided the angular rate outputs from a FOG with accelerometers and magnetometers similar to the system documented in [11] and [12]. In accordance with the simplified DME loading analysis contained in [8], DME scanning frequency was limited to once every 15 seconds to mitigate saturation.

Figure 6 shows a bird-eye view of the ground track flown by the aircraft during final approach. Note the trace generated by the open loop dead reckoning. From this trace it can be seen that, if uncompensated for, the wind would have caused drifts in the position estimate. For comparison purposes, also shown in the figure are tracks generated by various schemes for blending the range measurements with the heading and speed dead reckoning to compensate for the wind induced errors. What can be seen is that the intermittent range update method at an update rate of 15 second (0.07 Hz) has an overall error performance that is comparable to using a scanning DME or multiple DMEs.

Figure 7 shows a time history of the position errors associated with this navigator. GPS augmented by the Stanford University WAAS prototype provided the “truth” reference against which the performance of the blended DME-dead reckoning navigator was compared. For comparison purposes the errors that would be obtained using other General Aviation navigation schemes are also shown. These are LORAN and VOR. LORAN is a hyperbolic navigation system that has a nominal accuracy of 0.25 nautical miles[13]. As described earlier VOR is an angular system which provides users bearing information. In this case, it is assumed that a VOR transmitter was located approximately 10 nautical miles away from the runway threshold

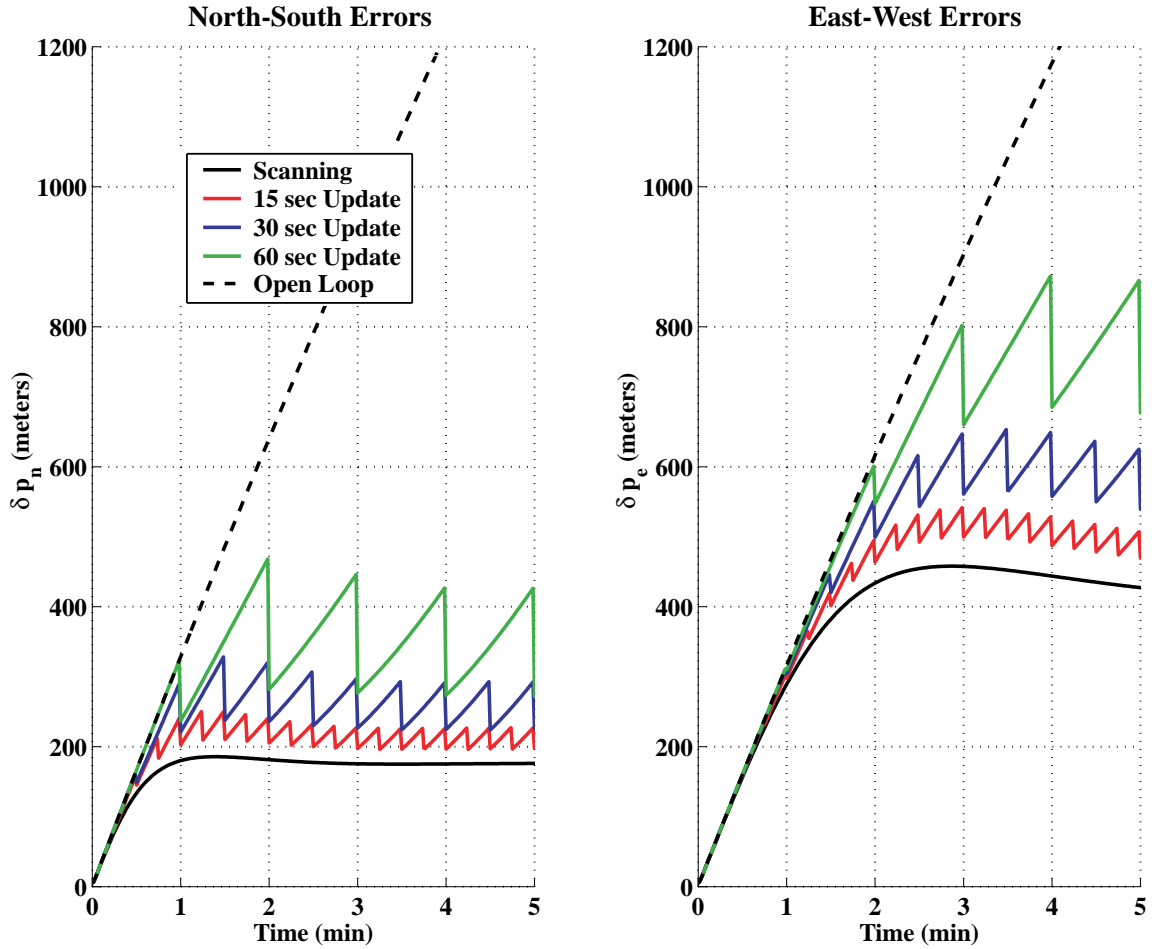


Figure 5: Growth of Error Covariances for DME-DR Navigator.

at the airport. Using a nominal VOR angular error of 2 degrees given in [14] and [15], this will translate into a position error of 0.35 nautical miles. As can be seen, the performance of the blended DME-dead reckoning navigator is within the performance bounds we established for the system in Section 2 of this paper. It is also comparable in performance to some of the existing radio-navigation aids used in aviation.

## 6 Conclusions

The work in this paper demonstrated that a GPS backup navigation system can be constructed from the fusion of low cost sensors. This system is capable of providing navigation services in the vicinity of airports that are served by DME. The navigation services provided by this system can be on the same level of accuracy of the currently used navigation systems such as LORAN and VOR. Such a system will allow the FAA to begin decommissioning a large number of existing radio-navigation aids while retaining only a small subset.

## 7 Acknowledgments

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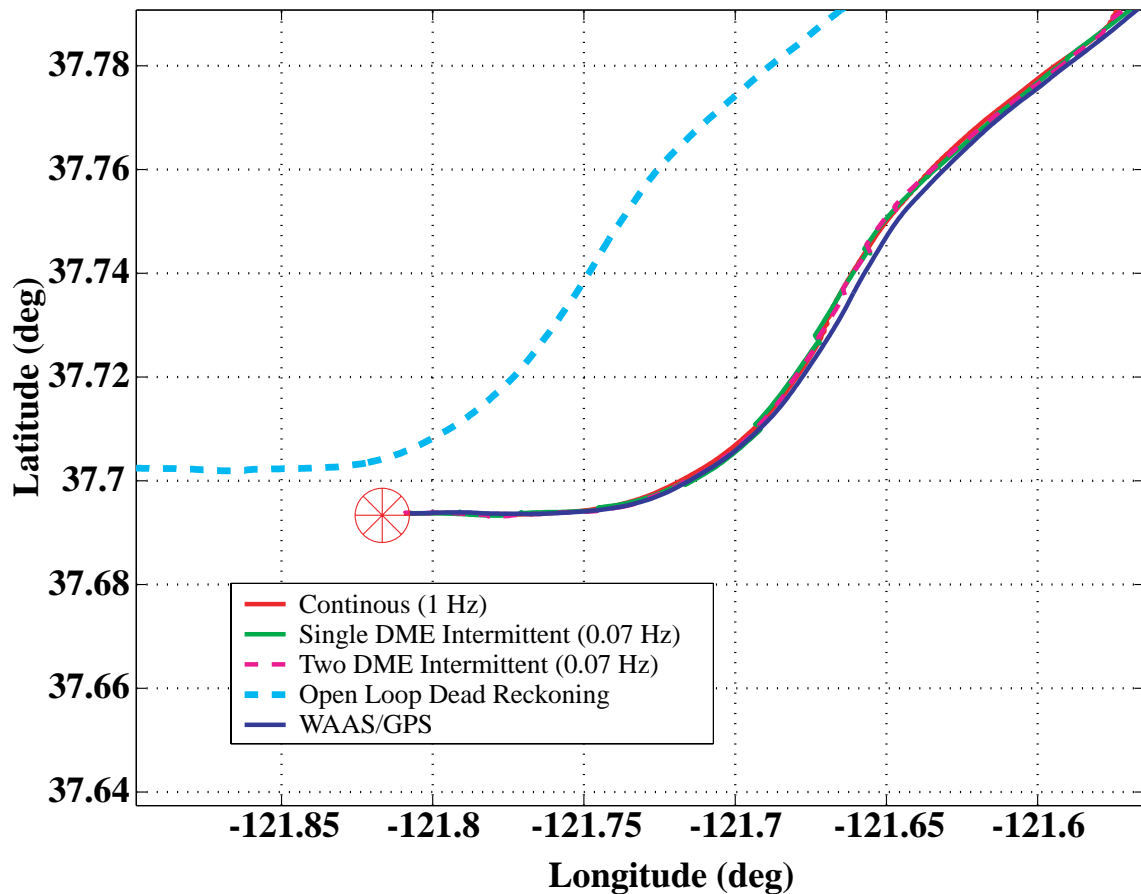


Figure 6: Ground Track and Navigation Performance of blended DME-dead reckoning Navigator.

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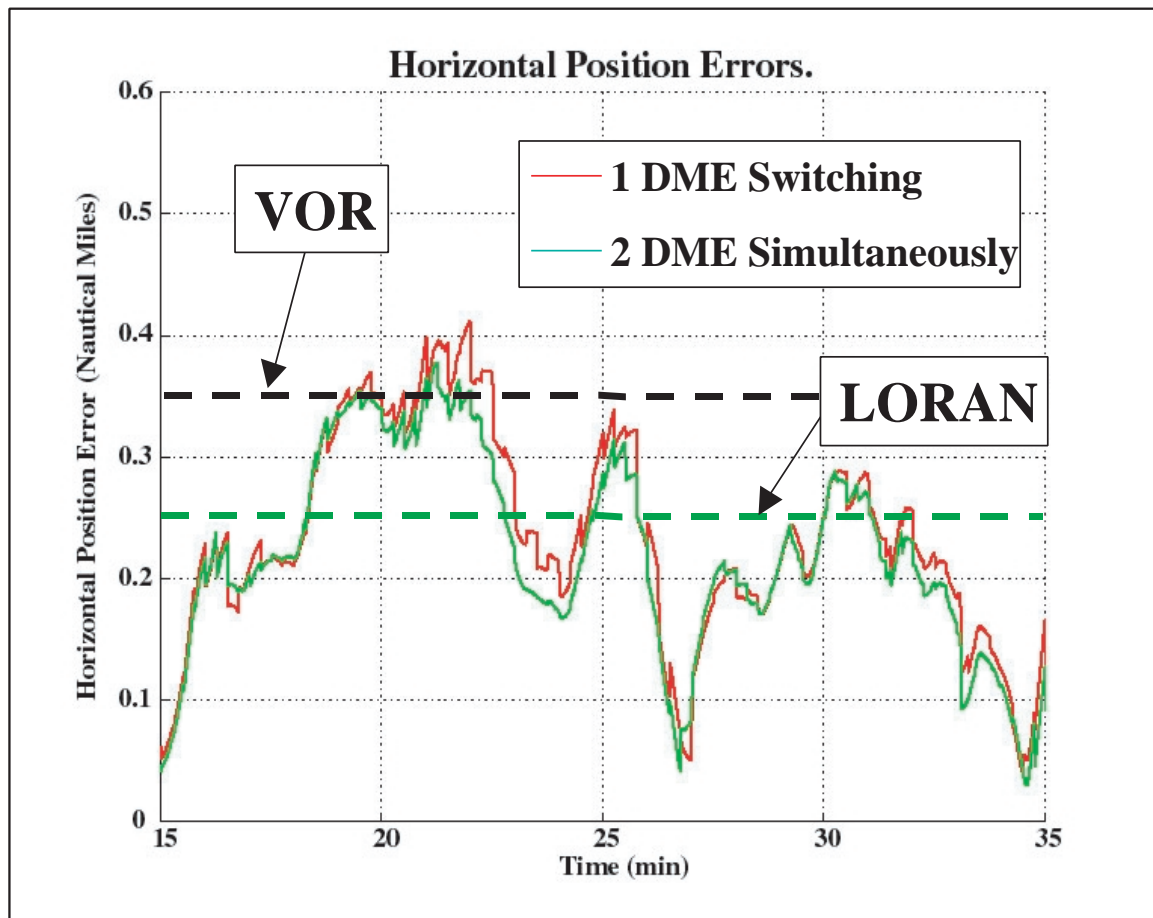


Figure 7: Navigation Errors for the blended DME-dead reckoning Navigator.

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