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Single Baseline GPS Based Attitude Heading Reference System (AHRS) for Aircraft Applications

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Abstract

Differential carrier phase GPS based attitude determination represents an attractive alternative to expensive Inertial Measurement Units (IMU's) and Attitude Heading Reference Systems (AHRS) for aviation applications. These inertial systems rely on extremely accurate accelerometers and gyros to determine attitude and therefore can be prohibitively expensive.

Ultra short baseline (less than one meter between antennas) GPS systems have been shown to provide the extremely high level of integrity required in aviation applications. Previous work with three antenna, two baseline, ultra short baseline systems has shown sub degree accuracy in pitch, roll and yaw when coupled with automotive grade solid state rate sensors. For optimal performance, mapping of antenna phase errors is required in this implementation.

An alternative method of attitude determination based on GPS and solid state accelerometers is presented. Roll and pitch are determined by using knowledge of acceleration in both the inertial and body frames. Yaw is determined using a single baseline oriented along the longitudinal axis of the aircraft. This approach, again coupled with automotive grade inertial sensors, can provide equal or better accuracy to the two baseline techniques.

The key advantage of GPS acceleration based attitude determination is that it provides for increased roll and pitch accuracy for cases in which there is a limitation on baseline length. In a three antenna configuration, the accuracy of the roll angle is inversely related to the lateral distance between the two antennas. In aircraft applications this distance may be quite small. In the acceleration based system, the accuracy is limited only by the quality of the accelerometers and accuracy of the acceleration determined from GPS measurements.

1. Introduction

Traditional GPS based attitude systems have used at least three antennas to compute differential position of each antenna from differential carrier phase measurements. This differential position is then converted to Euler angles using simple trigonometry. More complex approaches solve directly for the Euler angles but offer negligible advantages.

Detailed analysis of the error sensitivities of the GPS attitude solution were done in Ref[6]. This analysis demonstrated a significant advantage in the computation of pitch and roll were gained by using an integrated multi-antenna GPS receiver with a common oscillator. As a result of this observation, a common oscillator GPS receiver was constructed from separate Canadian Marconi OEM boards. This integrated receiver is depicted in Figure 1 and generated the data referenced throughout this paper.

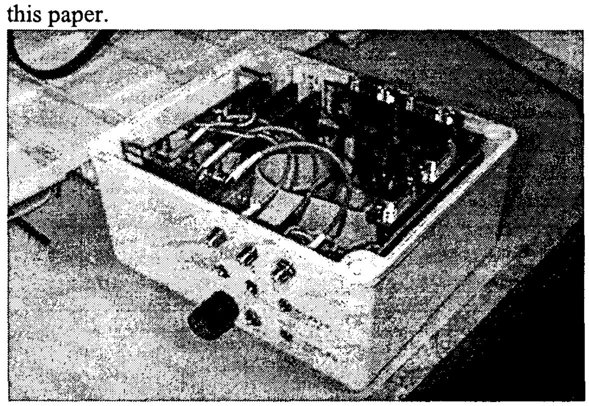

Figure 1. GPS Attitude Receiver, 15 x 20 x 10cm

This receiver architecture allows the line and clock biases between the antenna pairs to be treated as a constant and removed from the state. Using this common clock architecture, the theoretical accuracy limitations for pitch, roll and yaw can be determined. This accuracy is shown as a function of baseline length for an observed phase noise of 5 mm rms in Figure 2.

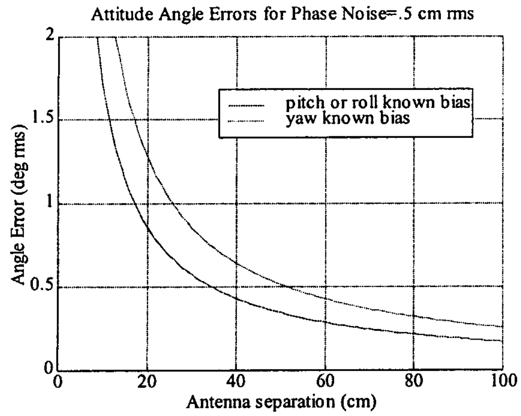

Figure 2. Theoretical GPS Attitude Accuracy

Improvements in GPS attitude solutions have been made by integration of rate gyros through a Kalman filter, Ref[7]. This approach can improve the attitude accuracy by effectively averaging GPS attitude solutions over the time constant of the Kalman filter. The time constant of the filter is limited by the quality (and therefore the cost) of the rate sensors. The limitations to this technique are long term errors such as line bias variations and non-uniformity in antenna phase delay patterns. These errors cannot be filtered out with inertial sensors, but can be calibrated out as per Ref[9] using aircraft motion and redundant measurements.

The general concept behind this new acceleration based approach is to use differential GPS for heading only, while using GPS velocity and accelerometers to compute pitch and roll. GPS is used to compute local level accelerations at 1Hz while onboard accelerometers are used to compute acceleration in the body coordinate frame.

Because this acceleration method uses differential GPS for yaw only, the requirement for a common oscillator between receivers is removed[6]. In addition, because the requirements for yaw accuracy are an order of magnitude less than for pitch and yaw the baseline can be reduced dramatically and the requirement for calibrating out line biases and antenna phase patterns is removed.

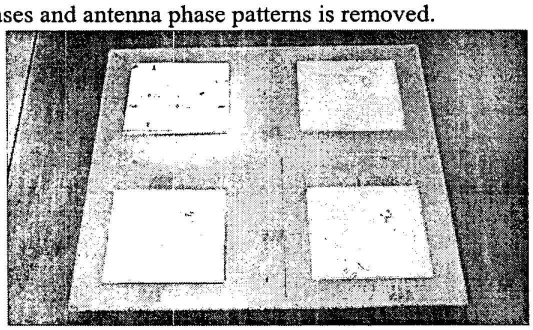

Figure 3. Four Element Patch Antenna Array, 20x20 cm

Experiments have been conducted with two and four element patch arrays with antenna spacing as small as one half wavelength. A two element array is ideally suited for this new technique where only yaw is required and the accuracy requirement is a degree or two. A four element patch antenna is shown in Figure 3. The details of the design of this patch antenna array are given in Ref[13].

2. Acceleration Vector Alignment Theory

The acceleration vector alignment technique is based on finding Euler angles that transform the acceleration vector in local level space to the body frame. It is important to note that because we are aligning vectors, the angle of rotation about the acceleration vector is unobservable. In our terrestrial one G environment this is typically the yaw angle.

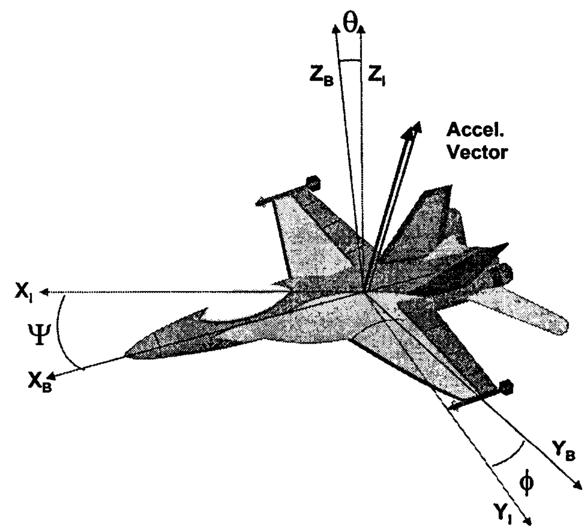

Figure 4. Acceleration Vector Alignment

Figure 4 shows the inertial and body coordinate frames and the Euler angles between the frames. Note that the specific force vectors are physically aligned but would be described differently in the different coordinate frames.

The Euler angles define a transformation matrix that transforms the acceleration vector in the inertial frame to the body frame shown in Eqn(1).

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\ -\sin(\psi)\cos(\phi)+\cos(\psi)\sin(\theta)\sin(\phi) & \cos(\psi)\cos(\phi)+\sin(\psi)\sin(\theta)\sin(\phi) & -\cos(\theta)\sin(\phi) \\ \sin(\psi)\sin(\phi)+\cos(\psi)\sin(\theta)\cos(\phi) & -\cos(\psi)\sin(\phi)+\sin(\psi)\sin(\theta)\cos(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix} \cdot \begin{bmatrix} f_N \\ f_E \\ f_D \end{bmatrix} \quad (1)$$

Where $\mathbf{f}_{xyz}$ represents specific force in the body frame and $\mathbf{f}_{NED}$ represents the specific force in the local level frame. Local level in this case refers to a north-east-down right handed coordinate frame, where θ, ϕ, and Ψ refer to roll, pitch and yaw, respectively. Eqn(1) can be expressed in more simplified form as Eqn(2a).

$$f_{XYZ}^{GPSAnt} = T \cdot f_{NED}^{GPSAnt} \qquad (2a)$$

$$f_{XYZ}^{GPSAnt} = f_{XYZ}^{Accel} - \omega_{XYZ} \times (\omega_{XYZ} \times r_{XYZ}) \qquad (2b)$$

$$f_{NED}^{GPSAnt} = a_{NED}^{GPSAnt} + g_{NED} - (2 \cdot \omega_{earth} + \rho) \times V_{NED} \qquad (2c)$$

Eqn(2a) gives the relation between the specific forces in the body and local level frames with the acceleration and rate observable. Eqn(2b) and Eqn(2c) define the specific forces in terms of the various observables. The second term in Eqn(2b) accounts for the difference in position between the accelerometers and the GPS antenna **r**. If the vector **r** and the body rates ω are both known, this correction can be applied to the body accelerometer measurements Eqn(2c). If this correction is not included and **r** is non-zero, errors will occur during maneuvering flight. The last term in Eqn (2c) gives the correction due to the Coriolis force and the transport rate. This term is on the order of .005 m/sec^2 for airspeeds on the order of 150 nautical miles per hour and is neglected.

Eqn(2a) can be solved for the ϕ and θ elements of the direction cosine matrix **T** described in Eqn(1), Ref[10]. The observations in this case are the specific force from GPS, the specific force from accelerometers and the yaw angle Ψ.

3. Flight Test Results

This solution has been applied to data taken in Stanford University's Queen Air test aircraft to validate the GPS acceleration approach. Figure 5 shows the experimental set installed in the Queen Air. A reference INS was used to measure true attitude. The receiver depicted in Figure 1 is used to record differential carrier phase for yaw, pitch and roll for comparison to the acceleration based technique.

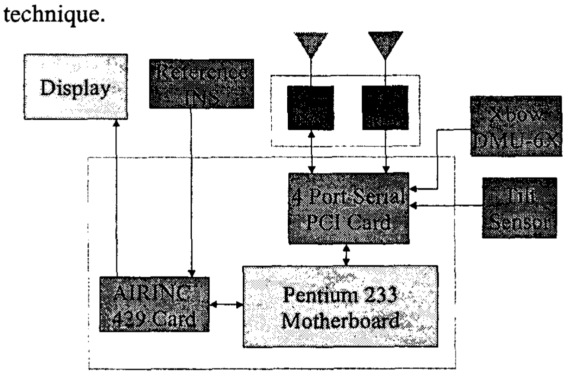

Figure 5. Block Diagram of Data Acquisition System

The initial validation of this concept is shown in Figure 6. The GPS accelerations are transformed to body coordinates for comparison with the accelerometer measurements. The transformation is done using Euler angles from the reference INS. The acceleration from both the GPS and the accelerometers agree very closely.

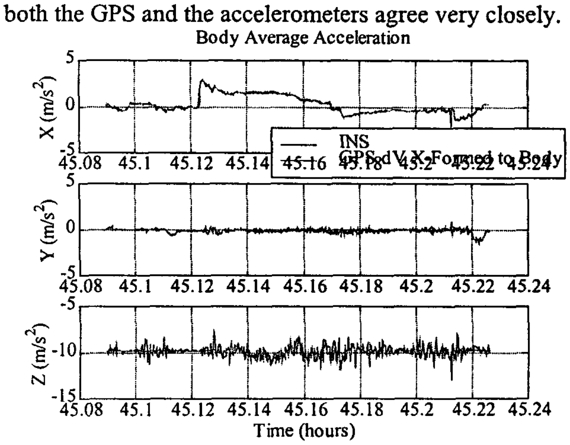

Figure 6 Acceleration in the Body Frame taken with Crossbow DMU-6X and GPS

The Euler angles can also be solved for directly given the accelerations in the two frames and the yaw angle. The results of this computation for the roll axis are given in Figure 7.

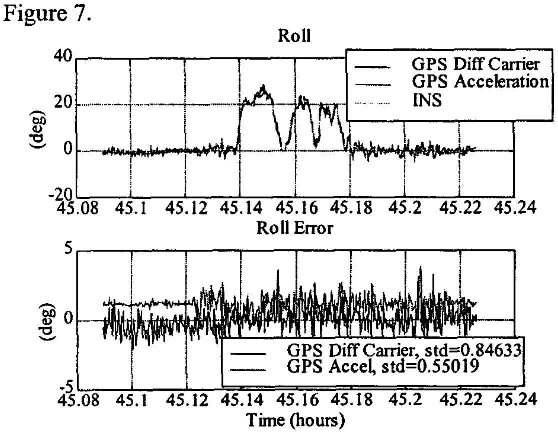

Figure 7. GPS Velocity Based and GPS Differential Carrier Based Roll

The one degree bias in the acceleration based computation is due to alignment errors between the INS and the accelerometers, with the standard deviations computed with respect to the mean. Pitch results are similar and are also given in Ref[10]. The acceleration technique performs better than the traditional differential GPS carrier phase technique. The roll baseline in this case was 36 cm.

By taking a case where the body frame is aligned with the local level frame, it is possible to do a relatively straightforward error analysis that can be generalized to non-aligned frames. In the aligned case where only roll is of interest, Eqn(1) can be solved to form Eqn(3), Ref[10].

$$\Delta\phi_{DEG} = \frac{-\Delta a_Y}{g+\Delta a_D}\cdot\frac{180}{\pi} \qquad (3)$$

This intuitively makes sense as the angle error is equal to the accelerometer or GPS acceleration error in g's times the conversion from radians to degrees.

We can also make a more general statement about the relationship between the variance of the three measurements and the variance of the computed roll angle. This is shown in Eqn(4), with details of this derivation in Ref[10].

$$\sigma_\phi = \frac{1}{g}\sqrt{\left(\sigma^2_{a_E} + \sigma^2_{a_Y}\right)}$$
$$\sigma_\theta = \frac{1}{g}\sqrt{\left(\sigma^2_{a_N} + \sigma^2_{a_X}\right)} \qquad (4)$$

Based on Eqn(4), errors in the acceleration from GPS in the inertial frame and the acceleration from accelerometers in the body frame are equally important. If one is significantly larger than the other, it will drive the error, indeed the optimal design would be to have them be approximately equal. Notably, the vertical accelerations in both frames are absent.

Based on the above analysis, system performance can be predicted based on sensor error. Errors in GPS and accelerometer accelerations were measured using the reference INS and are show in Table 1. The predicted attitude error of 0.8 degree rms compares closely to the measured rms error of 0.6 and 0.8 degrees in pitch and roll, respectively. A further increase in performance by a factor of 5 to 10 can be gained by integration of automotive grade solid state rate sensors[7]. This would lead to an overall system accuracy of 0.1 degree.

Table 1. Actual Sensor Errors and Pred Attitude Error

	GPS	Accel	Pred. Attitude	Pred. Filtered Attitude
Noise	0.14 m/sec^2 @ 1Hz	8 mg @ 1Hz	0.8 deg @ 1Hz	0.1 deg

Over the time period of the test there was no appreciable change in the bias of the CXL02LF3 accelerometers. However, due to the potential variation in bias of 30mg over a large temperature range given in the spec sheet, it would be necessary to temperature compensate the accelerometers and correct these 30mg error which corresponds to an angular error of 3 degree down to 1mg corresponding to .1 degree.

4. Integer Resolution For Single and Double Baselines

In both the acceleration based and the differential carrier phase methods of attitude determination, an unknown integer ambiguity must be resolved. We employ the instantaneous integer resolution technique based on the infinity norm of the residuals and the baseline, Ref[10]. Multi-baseline short baseline attitude systems leverage the known angle between baselines when computing integer ambiguities and provides a high level of integrity for baseline lengths up to 1 meter. The acceleration based attitude determination method requires only one baseline. A Monte Carlo simulation of the number of wrong integer solutions for different baseline lengths using the above techniques is shown in Figure 8. For single baselines, this instantaneous integer resolution algorithm can provide a 100% integrity for baseline lengths up to 30 cm.

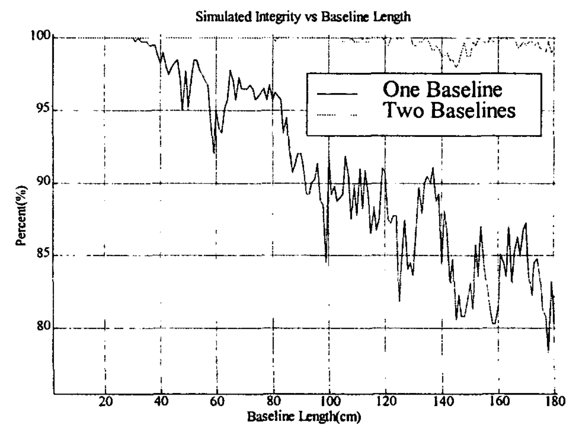

Figure 8. Integrity for Different Baseline Lengths

5. AHRS Implementation

The combination of effective integer resolution and a new technique for computation of pitch and roll make this ideally suited for aviation applications. By combining this technique with automotive grade gyros a truly robust, low cost, high accuracy AHRS can be realized, Ref[7]. Even using low cost gyros, an integrated system can provide pitch and roll accuracy of less then 0.1 degree. A block diagram of this system is shown in Figure 9.

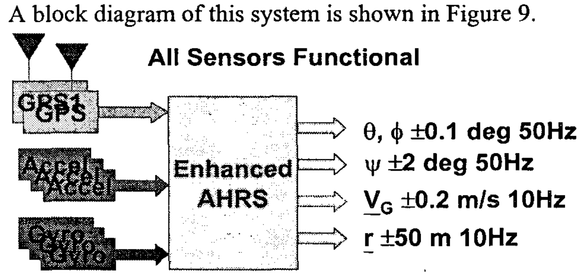

Figure 9. AHRS Implementation

This design allows for continued operation in a degraded mode if the GPS signal is completely removed. In this situation the AHRS is still capable of computing pitch and roll with 2.5 degree accuracy in turns converging to 0.5 deg accuracy in straight and level flight. The implementation of the velocity and position outputs are discussed in detail in Ref[11].

A prototype of this system has been built and flight tested. Figure 10 coupled with two of the GPS receivers shown in Figure 1 implements the system described in Figure 9.

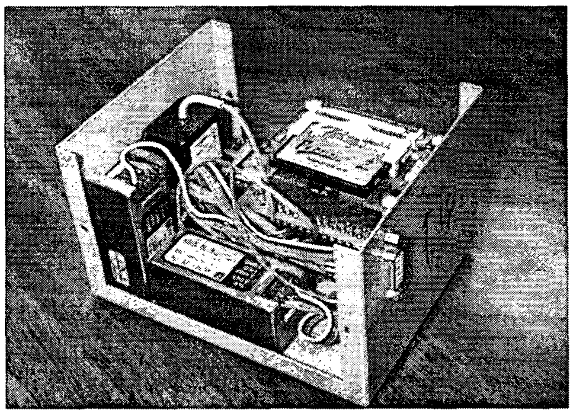

Figure 10. Prototype AHRS, 15 x 20 x 10 cm

In addition, the system is robust to any failure of a single component, degrading gracefully to a backup mode. The loss of any of the gyros typically leads to a reduced update rate and increased noise in the affected axis. Loss of the lateral accelerometer is only significant in periods of prolonged unbalanced flight. When the longitudinal accelerometer is lost, the pitch is computed by differential GPS techniques having reduced accuracy in pitch. The redundancy of the measurements also allows integrity monitoring of individual components for failure.

6. Conclusions

This GPS acceleration technique is shown to be superior to conventional differential carrier phase measurements as well as conventional inertial only attitude determination techniques. It elevates the synergy of GPS and inexpensive solid state inertial sensors to a new level. Furthermore, unlike some other GPS attitude determinations methods, Ref[8], this techniques makes no assumption about flight dynamics. In fact, this technique is not limited to aviation applications at all.

Because the basic concept behind this approach is to align acceleration or specific force vectors, the accuracy of the technique is based on the ability to resolve the angle of the specific force vector. In a one g terrestrial environment and sensors on the mg level this resolution is quite good. However, in space or low g applications the ability to resolve the specific force vector degrades dramatically.

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