

# Using GPS to Synthesize A Large Antenna Aperture When The Elements Are Mobile

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## BIOGRAPHY

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## ABSTRACT

This paper presents a design for a transmitting antenna array comprised of mobile antenna elements, where the Global Positioning System (GPS) is used to estimate the current location and velocity of those elements. GPS is also used to synchronize the clocks carried by the mobiles. With this information, a central algorithm can control the phase of the radio signal radiated from each element so that the multiple signals add constructively at the desired receiving site. The algorithm can also control the elemental phases to cause destructive interference at any undesired receiving site. In this way, GPS is used to synthesize an antenna aperture larger than any single robot or human could carry. This enhanced system is able to communicate over longer distances and have the capability to avoid communication to undesired listeners.

To evaluate the results, the GPS-based array is compared to a more conventional adaptive antenna array. The latter array uses signal strength measurement feedback from the desired

receiving direction and the undesired receiving direction. This conventional approach will be our baseline to evaluate the efficacy of GPS. When some of the mobile antenna elements are not reporting GPS, the two algorithms will be combined.

## INTRODUCTION

Our paper assumes that the distributed antenna array includes a finite number of mobile antenna elements, as shown in Fig. 1. Each of these will include a radiating element operating at single common frequency, a GPS receiver, means to communicate GPS information back to a central processor, and means to control the phase of the radiated signal. The network is also aware of the bearings to a desired receiving site and an undesired receiving site.

The central processor processes the received GPS measurements and estimates the relative position of the mobile antenna elements. The central processor also controls the phase of the signal radiated from each element. The control commands will enable the signals from the individual mobile to combine so that the desired pattern is achieved.

Several control algorithms are illustrated in this paper, and they are divided into three categories.

1. **Signal Strength Feedback Only:** A closed loop control algorithm based solely on signal strength measurement feedback without using GPS positioning, a technique that is the closest to the conventional use of an adaptive antenna array. The conventional array will be our baseline

to evaluate the efficacy of our GPS-based array, as shown in Fig. 2.

2. **GPS Positioning Only:** An open loop control algorithm using GPS positioning without applying signal strength measurement feedback, a technique that we designed to control the phases of the signal from the mobile antenna elements, as shown in Fig. 3.
3. **GPS Positioning and Signal Strength Feedback:** A closed loop control algorithm combining GPS positioning and signal strength measurement feedback. This is a new technique to control the mobile antenna elements, as shown in Fig. 4.

This combination is used to mitigate GPS position errors and timing errors, or the absence of GPS measurements from some of the mobile antenna elements.

Our evaluation of the different control algorithms is based on the following three performance goals:

1. Maximizing the directivity of the combined signal toward the desired receiver.
2. Maximizing the  $SIR$  ( $S/I$ ), where  $S$  is the field strength at the desired receiving location, and  $I$  is the field strength at the undesired receiving location.
3. Finding the positions of the mobile antenna elements.

The evaluations will be based on computer simulation of an example with 8 mobile antenna elements.

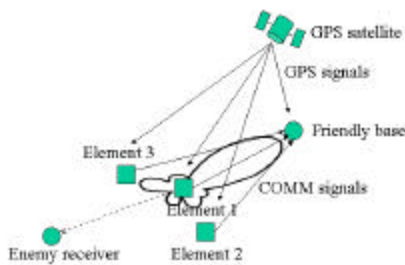


Figure 1. Configuration

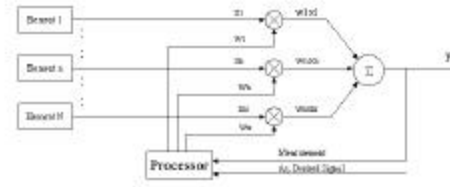


Figure 2. Feedback from the desired direction only and feedback from the desired and undesired directions.

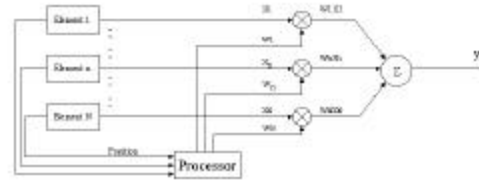


Figure 3. Using GPS positioning only (open loop), no feedback from the desired or undesired directions.

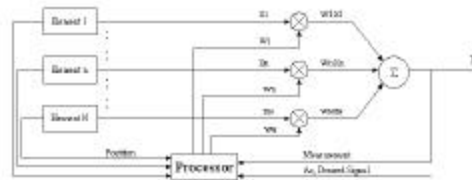


Figure 4. GPS and feedback from the desired direction, GPS and feedback from the desired and undesired directions, partial GPS and feedback from the desired direction.

## BASICS OF ANTENNA ARRAYS

In this paper, we present a potentially simpler scheme using GPS for obtaining antenna gain with an array of mobile antenna elements. We make the following assumptions.

1. After a radio wavefront has left the transmitting antenna, its power density is independent of azimuth, and so we neglect any complexity in the radio propagation environment. Despite the diffraction, refraction and multiple reflections, the propagation environment can be represented as a linear system.
2. All mobile antenna elements are identical and each is a point source of radiation.
3. Far field observation gives us a simple geometric interpretation, “parallel ray approximation”. It is a good approximation for radiation calculations, and each mobile antenna element is considered as a point source based on this assumption.

The signal vector  $X$  of the mobile antenna elements is

$$X = \begin{bmatrix} A_1 e^{(jf_1)} \\ A_2 e^{(jf_2)} \\ \cdot \\ \cdot \\ \cdot \\ A_n e^{(jf_n)} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

where,

$A_i$  = radiation amplitude of each mobile element, it is a function of the distance and elevation of the mobile antenna element. We let  $A_i = 1$ , which implies that isotropic antennae are considered.  
 $\phi_i$  = radiation phase of each mobile element  
 $i = 1, 2, 3, \dots, n$ , number of mobile elements

The weight vector  $W$  of the phase commands can be represented as

$$W = \begin{bmatrix} e^{(jq_1)} \\ e^{(jq_2)} \\ \cdot \\ \cdot \\ \cdot \\ e^{(jq_n)} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{bmatrix}$$

where,

$q_i$  = designed phase for each mobile element  
 $i = 1, 2, 3, \dots, n$ , number of mobile elements

The combined radiation signal field  $Y$  is

$$Y = W^T X$$

The difference between the measurements and the desired signal field forms the error signal  $\epsilon$ :

$$\epsilon = A_r - Y$$

where,

$A_r$  = desired signal pattern

As shown in Fig. 5, the weight update equation of the least mean square (LMS) adaptive algorithm is

$$W(j+1) = W(j) + 2\mu \epsilon (X^*)$$

where,

$\mu$  = adaptive coefficient  
 $W(j)$  = weight vector before adaptation  
 $W(j+1)$  = weight vector after adaptation

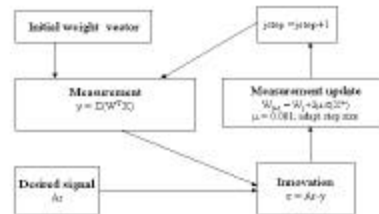


Figure 5. Block diagram for the least mean square (LMS) adaptive algorithm.

## SIGNAL STRENGTH MEASUREMENT FEEDBACK ONLY – NO GPS

This section does not use GPS. The control concept of this section is the closest to the conventional adaptive antenna and assumes signal strength feedback from the desired direction. We consider two systems: one with signal strength feedback from the desired direction only and the other with signal strength feedback from the desired and undesired directions. As shown in Fig. 6, an example antenna array used in this section includes eight mobile antenna elements, a desired receiving site, and an undesired receiving site.

For the first system, the only signal strength measurement feedback was from the desired receiving direction. The central processor based on the least mean square (LMS) adaptive algorithm compares this measured field strength to the desired field strength. The resulting commands cause the signals from the individual mobile to combine so that an approximation of the desired field strength is achieved. This fulfills the first performance goal, as shown in Fig. 7. However, this system will not be guaranteed to reduce the field strength at the undesired receiving direction because it does not have a measurement from that direction, and consequently fails in the second performance goal because of this reason. The third performance goal of positioning cannot be met, because the central processor uses the phase control command to calculate the positions of the mobile antenna elements, but it fails to do so because of the wavelength ambiguity. As shown in Fig. 9, the resulting position solutions are one wavelength apart, and thus we cannot distinguish which one of them is the true position solution.

Our second system installs another signal strength sensor on one of the mobile elements to measure the field strength in the direction of the undesired receiving site. With this measurement feedback and the one from the desired receiving direction, the central processor using the least mean square (LMS) adaptive algorithm will be able to maximize the *SIR*. This additional sensor helps us achieve the second performance goal, as shown in Fig. 8, but it does not enable accurate positioning of the mobile antenna elements.

The systems in this section are our performance baseline and are summarized in Table 1. In the following section, we show that GPS improves

the performance of the network of the mobile antenna elements relative to this baseline.

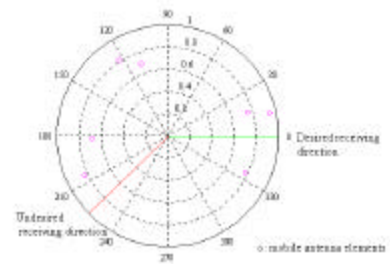


Figure 6. An example of mobile antenna elements used for evaluation and comparison.

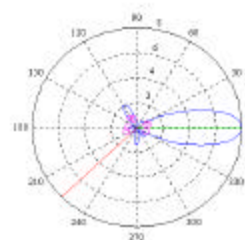


Figure 7. Result for the signal strength measurement feedback from the desired receiving direction only.

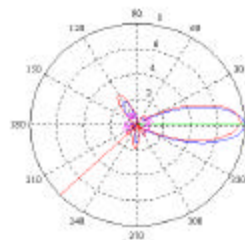


Figure 8. Result for the feedback from the desired and undesired receiving directions.

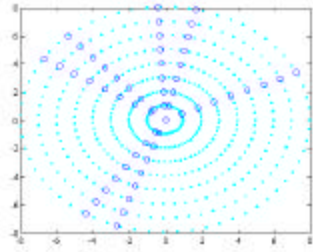


Figure 9. Position solution for using signal strength feedback only without using GPS positioning.

Table 1. Summary for using signal strength feedback only

	Maximizing the directivity toward desired receiving site	Maximizing the SRE	Forming of the mobile antenna element
Feedback from desired receiving site only	Achieved	Failed	Failed
Feedback from desired and undesired receiving directions	Achieved	Achieved	Failed

### GPS POSITIONING WITHOUT SIGNAL STRENGTH MEASUREMENT FEEDBACK

In this analysis section, signal strength measurement feedback is assumed to be unavailable, and as a result, GPS positioning is the only data available to achieve the performance goals.

The first goal is to maximize the signal strength at the desired receiving site only. GPS is used to estimate the current locations of the mobile antenna elements. With this information, the central processor controls the phase of the radio signal radiated from each mobile so that the multiple signals add constructively at the desired receiving location. Control is based on the differences of the distances from the mobile antenna elements to the desired receiving site.

Let one of the mobile antenna elements be the master element, and the others be slave elements. The distance difference  $d_i$  is defined as:

$$d_i = d_{slave} - d_{master}$$

where,

$d_i$  = distance difference

$d_{slave}$  = distance from the slave elements to the receivers

$d_{master}$  = distance from the master elements to the receivers

$i = 1, 2, 3, \dots, n$ , number of mobile antenna elements

The relation between the phase control commands and the distance differences is

$$q_i = \frac{2pd_i}{l}$$

where,

$q_i$  = phase command for mobile antenna element

$l$  = operation wavelength

$i = 1, 2, 3, \dots, n$ , number of mobile antenna elements

Now, the weight vector of the phase commands can be written as:

$$W = \begin{bmatrix} e^{jq_1} \\ e^{jq_2} \\ \cdot \\ \cdot \\ \cdot \\ e^{jq_n} \end{bmatrix}$$

The combined signal  $Y$  radiated in the field is

$$Y = W^T X$$

Then, the directivity will be maximized in the desired receiving direction by using this control algorithm. The first performance goal is achieved, and an example is shown in Fig. 10. However, we cannot achieve the second goal with this control algorithm.

A similar goal may be to maximize the signal strength toward an undesired receiving direction and to use this signal to spoof the receiver – to send a false signal. The central processor can generate another weight vector based on the same control algorithm in order to generate the

spoofing signal toward the undesired receiving direction, as shown in Fig. 11. Since GPS provides the locations of the mobile antenna elements, the third performance goal is also achieved.

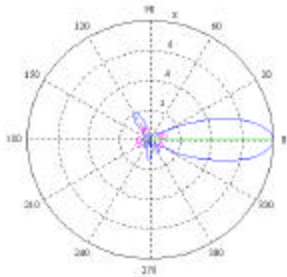


Figure 10. Result for using GPS positioning only, no signal strength feedback from the desired or undesired direction.

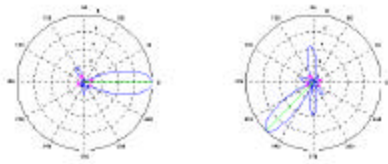


Figure 11. Result for using GPS positioning only, sending the spoofing signal in an arbitrary direction.

The second goal is to maximize SIR and to make the maximum directivity toward the desired receiving direction. These two objectives might conflict with each other. The cost function we define is

$$\text{Max. } ( F_1(\text{SIR}) + F_2(\text{Directivity}) )$$

The compromise of these two objectives gives us the optimal solution of the cost function. As shown in Fig. 12, the maximum directivity is not toward the desired receiving location exactly, because the objective is to maximize SIR. This is the result of the computer numerical optimal solution of the cost function. The second and third performance goals are achieved by using

GPS positioning with the cost function we defined.

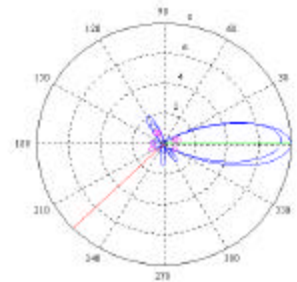


Figure 12. Result for using GPS positioning and the cost function we defined.

Table 2. Summary for using GPS positioning only

	Maximizing the directivity toward the desired receiving site	Maximizing the SIR	Positioning of the mobile antenna elements
GPS only	Achieved	Failed	Achieved
GPS with cost function feedback	Achieved	Achieved	Achieved

### COMBINED GPS POSITIONING AND SIGNAL STRENGTH MEASUREMENT FEEDBACK

While all three goals can be achieved with GPS positioning alone. This section shows how performance can be improved by combining GPS positioning and signal strength measurement feedback. It also anticipates the next section, which discusses the use of signal strength measurement feedback to recover from partial GPS outages.

The example network used in this section includes eight mobile antenna elements, a desired receiving site, and an undesired receiving site: an example is shown in Fig. 6. The different examples use different sources of field strength measurements. This section also discusses the position and timing errors of GPS.

Recall that the first goal is to maximize the directivity toward the desired receiving direction. Based on the combined algorithms, this goal is achieved, as shown in Fig. 13. This result is the same as the result for using GPS positioning alone or using signal strength measurement feedback alone.

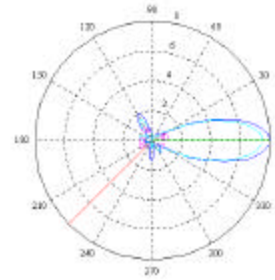


Figure 13. Result for using GPS positioning and signal strength feedback, the signal strength feedback from the desired receiving direction only.

With respect to the second goal, this section is different from the section using signal strength measurement feedback alone. The section with signal strength measurement feedback alone uses random equal weight values as an initial condition to begin the adaptive process. With the combined GPS positioning and signal strength measurement feedback, we used weight values that achieve the first performance goal by using GPS positioning only as our initial condition. This condition helped speed up convergence and avoided the divergence of the adaptation. The algorithm combining GPS positioning and signal strength measurement feedback gives the better result: SIR = 15.3dB, as shown in Fig. 14. The resulting SIR for using GPS positioning alone is 14.7dB, and the resulting SIR for using signal strength measurement feedback alone is 9.2dB. This is the advantage of combining GPS positioning and signal strength measurement feedback.

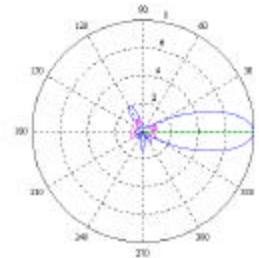


Figure 14. Result for using GPS positioning and signal strength feedback, the signal strength feedback from the desired and undesired directions.

The third goal is to locate the positions of the mobile antenna elements. GPS provides the information that lets us determine those mobile antenna elements' locations.

When the GPS position error and timing error are present, the phase commands of the central processor are not the same as those the mobile antenna elements demanded to form a desired signal pattern. As a result, larger GPS errors cause worse performance of the mobile antenna elements, as shown in Figs. 15, 16, and 17. Based on the sensitivity analysis between GPS errors and system performance, we can decide what kind of GPS we need to fulfill the specific performance, as shown in Figs. 18, 19, 20, and 21. For example, if our operation frequency is 30 MHz and we have 3dB loss in our directivity in Fig. 15. It means that the directivity is reduced from 8 to 4, and the corresponding position error in Fig. 15 is 16% of wavelength which is 1.6 meters under this operation frequency (we assumed that the light speed is  $3 \times 10^8$  meter/second). Then according to Fig. 18, we are required to use Local Area Code GPS to satisfy the performance requirement.

Table 3. Summary for using GPS positioning and signal strength feedback

	Maximizing the directivity toward desired receiving dir.	Minimizing the SIR	Positioning of the mobile antenna elements
Combined GPS and adaptive feedback from the desired receiving dir.	Achieved	Failed	Achieved
Combined GPS and adaptive feedback from the desired and undesired receiving directions	Achieved	Achieved	Achieved

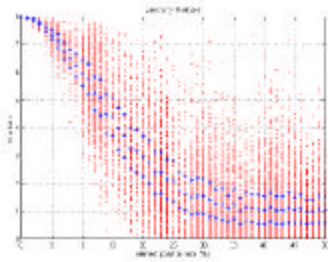


Figure 15. Sensitivity analysis, directivity, using GPS positioning only, the desired receiving site is the only destination.

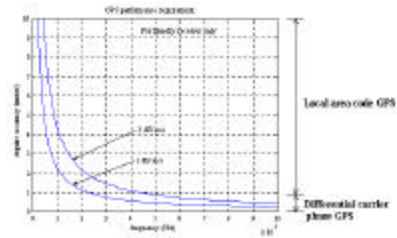


Figure 18. System requirement for the desired receiving site is the only destination.

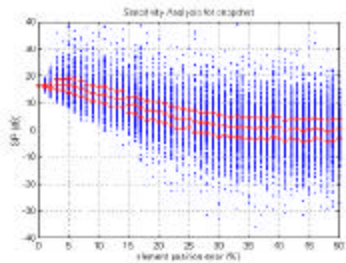


Figure 16. Sensitivity analysis, SIR, using GPS positioning only.

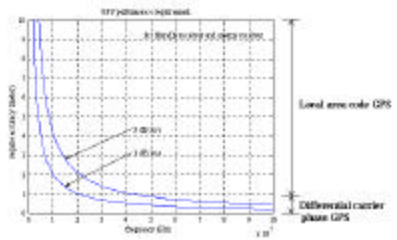


Figure 19. System requirement for using GPS positioning only.

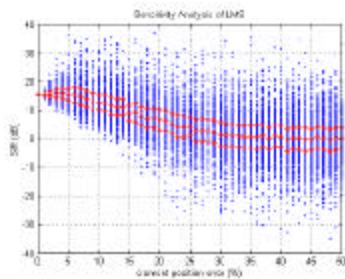


Figure 17. Sensitivity analysis, SIR, combined GPS positioning and signal strength feedback.

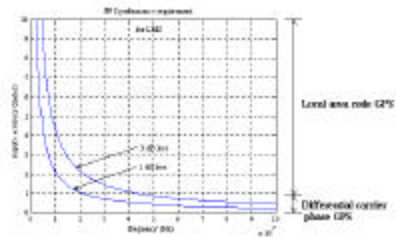


Figure 20. System requirement for using GPS positioning and signal strength feedback.



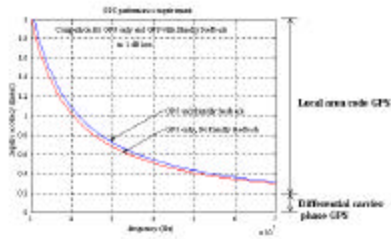


Figure 21. Comparison of system requirements: using GPS positioning only and using GPS positioning and signal strength feedback.

### PARTIAL GPS

We have redundant sensor information: some is based on GPS positioning and the rest is based on signal strength measurement feedback. Each of them can be the backup system for the other. For example, when the signal strength measurement link is lost, the system using GPS positioning can sustain the overall function. Or when GPS outages occur, the system using signal strength measurement feedback can maintain the mission.

The systems we discussed in this paper can be found when multiple mobiles are deployed in a remote and possibly hostile area. Even if one of the mobiles were lost, the control concept would enable the survivors to sustain most of the mission objectives. In this section, we discuss the situation where some of the mobile antenna elements are not reporting GPS.

When some of the mobile antenna elements are not reporting GPS, the signal strength measurement feedback will sustain the function which can control the phase of the remaining mobile antenna elements so that the multiple signals add constructively at the desired receiving location and destructively at the undesired receiving location. The first and second performance goals are thereby achieved, as shown in Fig. 22a and 23a. However, the mobile antenna elements that are not reporting GPS will not be able to determine their position, because of the wavelength ambiguity, as shown in Fig. 22b and 23b. As a result, the system can not fully meet the third performance goal.

As shown in Fig. 24, we assume that we cannot control the mobile antenna elements which are not reporting GPS. This is the worst performance of this network system. The resulting SIR of mobile antenna elements, in the case where one of them is not reporting GPS, is 14.6 dB. The resulting SIR of the previous section that combined GPS positioning and signal strength measurement feedback is 15.3 dB. And the resulting SIR of the mobile antenna elements for two of them not reporting GPS is 11.0 dB. The resulting SIR of mobile antenna elements using signal strength measurement feedback only is 9.2 dB. As a result, the system combining GPS positioning and signal strength measurement feedback is better than the system using signal strength measurement feedback only, even if the system combining GPS positioning and signal strength measurement feedback has fewer mobile antenna elements than the one using signal strength measurement feedback only. This is another advantage of using GPS positioning in the networks of mobile antenna elements.

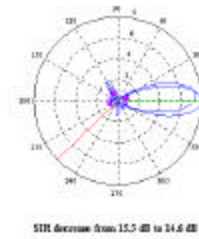


Figure 22a. Beam solution for when one of the mobile antenna elements is not reporting GPS.

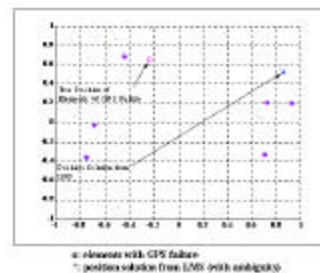


Figure 22b. Position solution for when one of the mobile antenna elements is not reporting GPS.

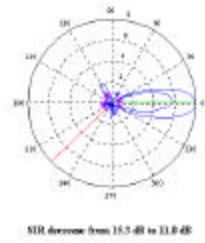


Figure 23a. Beam solution for when two of the mobile antenna elements are not reporting GPS.

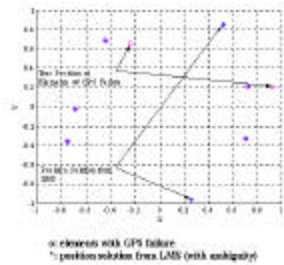


Figure 23b. Position solution for when two of the mobile antenna elements are not reporting GPS.



Figure 24. Performance result for when some of the mobile antenna elements are not reporting GPS.

## CONCLUSION

There are two advantages for using GPS. One advantage is that GPS provides better initial condition for the adaptive process, which helps speed up convergence and avoids the divergence of the adaptation. The other advantage is that the system combining GPS positioning and signal strength measurement feedback provides better performance than the system with signal strength measurement feedback alone.

When the GPS position error and timing error are present, several GPS systems such as Local Area Code GPS or Differential Carrier Phase GPS can allow us to remove the effect of these measurement errors to satisfy the required performance.

As shown in Table 4, the control algorithms presented in this paper can be applied to processing the output of the individual mobile antenna elements in the network. The techniques with GPS positioning successfully achieved all the performance goals.

Table 4. Summary

	Minimizing the diversity normal state and receiving rate	Maintaining the SIR	Protecting the mobile infrastructure
Feedback Beam Steered receiving rate only, no GPS	Achieved	Failed	Failed
Feedback Beam Steered and constant receiving rate, no GPS	Achieved	Failed	Failed
GPS only	Achieved	Failed	Achieved
GPS and the cost function via default	Achieved	Achieved	Achieved
Combined GPS and adaptation, feedback from the desired receiving rate only	Achieved	Failed	Achieved
Combined GPS and adaptation, feedback from the desired and maintained receiving rate	Achieved	Achieved	Achieved

Future work will seek control strategies that can be applied to networks of mobile elements. Each mobile element will have two functions: sensing and communication. These two functions might conflict with each other. As a result, the control strategies should be energy-efficient, able to adapt to changes in the environment or mission, and robust to the failure of one or more of the mobile elements. In order to accomplish these goals, the controllers may use position and time information from GPS. Eventually new control strategies that are fault-tolerant, energy-efficient and adaptive will be built to control networks of mobile sensors.

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