ARAIM Continuity and Availability Assertions, Assumptions, and Evaluation Methods

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ABSTRACT

In this paper, we describe assumptions and assertions needed to account for continuity and availability risks in Advance Receiver Autonomous Integrity Monitoring (ARAIM). We identify key differences with current RAIM-based operations, including the potential of ARAIM to not require systematic pre-flight availability screening. The assumptions and assertions provide a rationale on why fault exclusion is needed in horizontal-ARAIM (H-ARAIM), but is not needed in vertical ARAIM (V-ARAIM). We implement existing methods and design new approaches to predict continuity and availability in the presence of satellite faults and outages, and we analyze performance sensitivity to risk requirements and measurement models.

1. INTRODUCTION

Global Navigation Satellite System (GNSS) measurements are vulnerable to rare-event faults including satellite failures, which can lead to major integrity threats for users. To mitigate their impact, fault-detection algorithms such as Receiver Autonomous Integrity Monitoring (RAIM) can be implemented. RAIM exploits redundant ranging signals to achieve self-contained fault detection at the user receiver. With the modernization of the United States' Global Positioning System (GPS) and the emergence of Europe's Galileo, the number of redundant dual-frequency ranging signals increases dramatically, which opens the possibility to fulfill stringent navigation integrity and continuity requirements using RAIM. Researchers in the European Union and in the U.S. have been investigating horizontal Advanced RAIM (H-ARAIM) and vertical ARAIM (V-ARAIM) for worldwide en-route positioning and vertical guidance of aircraft, respectively.

Of primary concern in safety critical navigation applications is integrity, which is a measure of trust in sensor information. The integrity risk is the probability of a sensor system providing hazardously misleading information without timely warning [1]. A widely used representation of integrity comes in the form of protection levels, which are probabilistic bounds on positioning errors set to guarantee a predefined integrity requirement. Ensuring integrity is straightforward if it is done regardless of conflicting requirements on continuity and availability. For example, a trivial way to achieve high integrity is to send alerts at all times. Thus, continuity and availability are essential: continuity is the probability of unscheduled mission interruptions; availability is the fraction of time where integrity and continuity requirements are met [1].

Two conflicting aspects of RAIM-based fault detection arise from the addition of new redundant ranging signals in multi-constellation GNSS. On the one hand, the integrity monitoring performance using ARAIM is improving. On the other hand, the heightened likelihood of satellite faults because of the larger number of SVs causes more occurrences of mission interruptions due to faults being detected, thereby increasing the continuity risk. In response, fault-exclusion algorithms can be implemented. Thus, a primary motivation for implementing exclusion is to reduce continuity risk. The interpretation of continuity risk requirements must therefore be clarified as it can vary depending on aircraft operation.

In response, in this paper, we describe the assumptions and assertions made in the E.U./U.S. ARAIM Working Group-C to account for continuity and availability. These assumptions and assertions are needed (a) to set detection and exclusion thresholds that limit the risk of false alerts and failed exclusions, and (b) to predict and analyze ARAIM availability and continuity performance.

The second section of this paper provides the background and motivation for this paper. It explains that ARAIM continuity analyses are not only concerned with loss of continuity (LOC) due to false alerts as in RAIM, but also with other sources of mission interruptions including true alerts (i.e., detection of faults that actually occurred). The third section of the paper lists the ARAIM continuity and availability definitions, assertions, and assumptions that are foundational to ARAIM performance evaluation. Key findings include the fact that H-ARAIM airborne algorithms should consider a continuity risk rate requirement ranging from 10^{-4} to 10^{-8} per hour. H-ARAIM is therefore likely to require an exclusion function whereas V-ARAIM may not.

The fourth and fifth sections focus on dealing with satellite outages. Satellite outages occur, for example during satellite station-keeping maneuvers. We derive upper bounds on the probability LOC and loss of availability (LOA) that account for potential satellite outages. In Section 5, for performance analysis, as an alternative to considering all possible outages for each individual satellite geometry, we derive a new predictive integrity risk bound and an equivalent predictive protection level equation.

In Section 6, worldwide availability is quantified for nominal GPS and Galileo constellations, for nominal ARAIM measurement error and fault model parameters, and for variations thereof. Both fault-detection (FD) and FD-and-exclusion (FDE) are evaluated for performance comparison under our new assumption versus prior RAIM assumptions and previous ARAIM documentation [2-4]. Performance analyses explore the potential of ARAIM to meet continuity requirements without systematic availability prediction prior to aircraft departure.

2. BACKGROUND AND MOTIVATION

This paper provides a rationale for the treatment of continuity and availability, both in ARAIM analyses and in the ARAIM reference airborne Algorithm Description Documents (ADD) [3-5]. This paper explains ADD modifications in [5] as compared to [3, 4] that more accurately account for the risk of loss of continuity (LOC) and loss of availability (LOA). Preliminary availability evaluations will be presented that quantify the impact of these modifications.

Availability Analysis and Satellite Outages. The procedure, and the design and simulation parameters needed to carry out ARAIM availability analyses are described in Appendix A of [4] (in particular in Section A.V), and in [5] (in particular in Section 6). Sections 4 and 5 of this paper focus on the treatment of satellite outages, which has been discussed in the ARAIM Technical Subgroup of Working Group C from 2016 to 2019.

RAIM versus ARAIM. ARAIM is intended for both en-route navigation and vertical guidance. It is therefore held to a higher level of scrutiny than RAIM. ARAIM continuity analyses are not only concerned with loss of continuity (LOC) due to false alerts as in RAIM, but also with other sources of mission interruptions. For example, when fault detection is performed without an exclusion function, LOC is caused by both false alerts and true alerts (i.e., detection of faults that actually occurred, which will happen more frequently in multi-constellation ARAIM than in GPS-based RAIM). Also, unlike RAIM, ARAIM has the potential to not require systematic availability prediction prior to aircraft departure. This development could have an impact on ARAIM continuity and availability that the following assertions and assumptions help quantify.

FD versus FDE. Future ARAIM availability and coverage analyses will include performance predictions under both RAIM and ARAIM continuity and availability assumptions. We may use the labels FD for fault-detection-only (RAIM assumptions) versus FDE for fault detection and exclusion (ARAIM assumptions). FD-availability can be used as reference for comparison with existing RAIM implementations and with prior ARAIM performance evaluation, including in [3, 4]. FDE-availability is needed to analyze aspects of continuity that are not captured in FD-availability. Figure 1 illustrates differences in availability obtained for FD (in Figure 1(a)) versus FDE (in Figure 1(b)) when using H-ARAIM to achieve RNP 0.3 requirements [1, 6], for nominal constellations of 24 GPS and 24 Galileo satellites [4]. Availability is color-coded from red to blue corresponding to 95% to 100% availability range. In this example, the FDE availability map is established under continuity assertions and assumptions outlined in Section 2 of this paper. Figure 1(b) also accounts for the risk of wrong exclusions [5, 7], for the number of effectively uncorrelated samples during the exposure period [8], and for satellite outages using the method developed in Sections 4 and 5 of this paper.

Link to Other ARAIM Documents. This paper complements the ARAIM Fault Assertions in Appendix B of [4]. It provides a rationale for aspects of the Reference Airborne Algorithm Description Documents (ADD) [5] that include using continuity and accuracy versus integrity error models, allocating continuity risk for threshold setting, addressing satellite outages in availability evaluation, requiring (or not) an exclusion function in H-ARAIM versus V-ARAIM. In addition, this paper helps support the transition from availability analyses under FD assumptions in [3, 4] to more realistic FDE assumptions for availability analyses including the treatment of satellite outages in current and future documents.

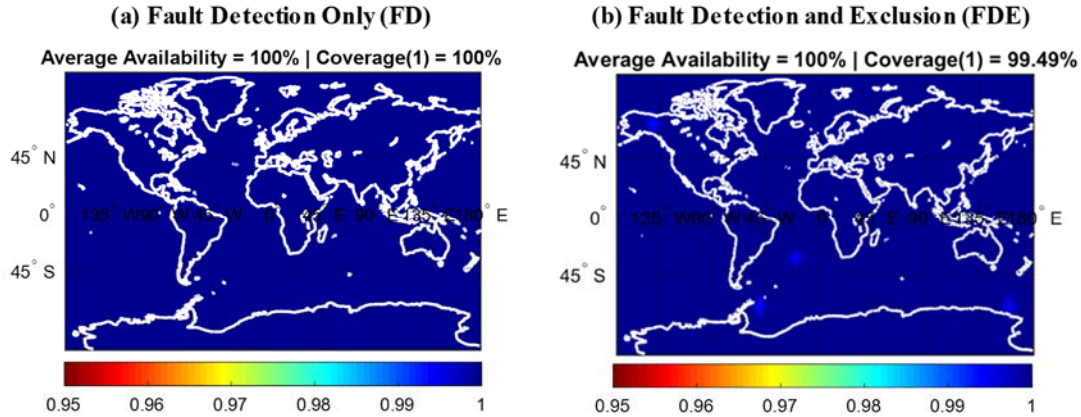

Figure 1. H-ARAIM RNP 0.3 Availability Maps (a) for Fault Detection Only (FD) under RAIM Assumptions, and (b) for Fault Detection and Exclusion (FDE) under ARAIM Assumptions that Include the Continuity Assertions Given in this Paper (assuming a continuity risk requirement of 10^{-6} per hour).

3. DEFINITIONS, ASSERTIONS, AND ASSUMPTIONS

The following Tables 1, 2, and 3 outline key definitions, assertions, and assumptions that are considered to be foundational for ARAIM in order to:

i. set detection and exclusion thresholds at the aircraft receiver (when thresholds are used), based on continuity requirements (e.g., as specified in [6] for SBAS); thresholds can either be fixed or configurable depending on receiver manufacturer choices

ii. simulate, predict, and analyze system-level continuity and availability performance under clearly identified conditions

These definitions, assertions, and assumptions are based on a current perspective of ARAIM, with special emphasis on continuity and availability. Assertions are statements supported by strong evidence, including experimental data and/or precedents in other comparable systems. Assumptions are statements made with lower confidence but required to carry out ARAIM analysis and system design. Their complete description including rationales and notes can be found in Appendix. They are published for the first time in this paper. It is expected that they will be amended or revised as the ARAIM concept evolves over time. Assertions and assumptions are referred to in the remainder of the paper that focuses on satellite outages.

Under Definition 3, the continuity risk rate requirement can be allocated for V-ARAIM and H-ARAIM as illustrated in Figure 2. The risk rate requirement allocated to 'other' sources of LOC, i.e., sources that are not at the receiver, is consistent for V-ARAIM and H-ARAIM (although respectively given in per 15 s and per hour). This allocation is merely indicated as a placeholder to acknowledge the presence of other causes for operational interruptions, e.g., due to environmental factors (ionosphere scintillation, radio frequency interference, aircraft banking, etc.).

Table 1 Definitions of Terms Relevant to ARAIM Continuity

1	Detection occurs at the airborne receiver when a detection test statistic exceeds a threshold. • A false alert is a detection event under fault-free conditions. • A true alert is the detection of an actual fault.
2	Exclusion can be implemented after detection. Exclusion is the process of identifying and removing one or more potentially faulty measurements. Exclusion occurs if the subset of remaining measurements is deemed fault-free, i.e., if this subset's test statistic is lower than a threshold. Conversely, if the exclusion test statistic exceeds the threshold, and if no candidate subset is found to be fault-free, then: • a false alert (with no exclusion) occurs under fault-free conditions, and • a true alert (with failed exclusion) occurs when a fault is actually present.
3	The maximum allowable rate of mission interruptions due to alerts at the airborne receiver is noted R_{alert}. R_{alert} is a fraction of the overall continuity risk rate requirement.
4	The protection level (PL) is a probabilistic bound on the positioning error computed at the airborne receiver at each time epoch. The PL accounts for nominal measurement errors, for potentially undetected faults, and for potential wrong exclusions if exclusion is implemented.

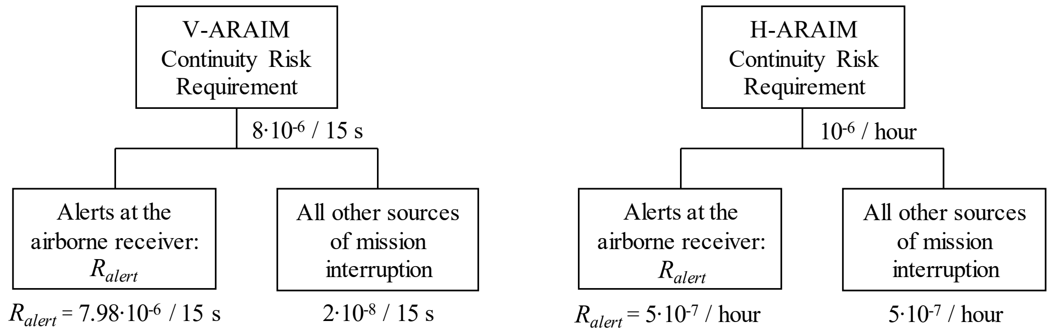

Figure 2. Example Continuity Risk Rate Requirement Allocations for H-ARAIM and V-ARAIM (assuming an H-ARAIM continuity risk requirement of 10^{-6} per hour).

Table 2 ARAIM Continuity and Availability Assertions

1	In both H-ARAIM and V-ARAIM, loss of continuity (LOC) is considered a 'minor' severity event.
1'	(Corollary to Assertion 1) For accuracy and continuity evaluation, ARAIM measurement error models are based on user range error (URE), whereas for integrity analysis, they rely on constellation service provider (CSP) commitments and on an Integrity Support Message (ISM), validated by the ISM Generator.
2	H-ARAIM and V-ARAIM continuity is measured 'in an average sense', over time.
3	The average rate of GPS narrow satellite faults is no greater than 10^{-8} / 15 s / SV. The average rate of GPS wide constellation faults is no greater than $3{\cdot}10^{-8}$ / 15 s.
4	For continuity evaluation, the average rate of scheduled and unscheduled GPS satellite outages, R_{out}, is no greater than $2{\cdot}10^{-4}$/ h / SV.

Table 3 ARAIM Continuity and Availability Assumptions

1	For GNSS constellations other than GPS, the average rate of narrow faults is expected to range from $6{\cdot}10^{-8}$ / 15 s / SV to $3.6{\cdot}10^{-7}$ / 15 s / SV. The rate of non-GPS constellation wide faults is expected to be smaller than $4.2{\cdot}10^{-7}$ / 15 s.
2	For continuity evaluation, and for GNSS constellations other than GPS, the average rate of effective scheduled and unscheduled GPS satellite outages, R_{out}, is expected to range from $1{\cdot}10^{-4}$ / h / SV to $2{\cdot}10^{-3}$ / h / SV. NOTE of Assumption 2 in Appendix adds that: "*Preliminary ARAIM continuity performance evaluations assume a nominal rate of $2{\cdot}10^{-4}$ / h / SV for all constellations* […]"
3	Under nominal ARAIM performance assumptions, the aircraft receiver may not be required to perform H-ARAIM availability prediction.
4	In both V-ARAIM and H-ARAIM, R_{alert} is chosen to be a fraction of the specified continuity risk rate requirement (e.g., following the example allocation in Figure 1). The remaining fraction is assumed to bound the average risk rate of LOC due to all sources other than receiver alerts.
5	For dual-constellation implementations under Assumption 1, the V-ARAIM R_{alert} requirement of $7.98{\cdot}10^{-6}$ per 15 seconds can be achieved without implementing an exclusion function at the aircraft.
6	H-ARAIM airborne algorithms should consider a continuity risk requirement ranging from 10^{-4} to 10^{-8} per hour. NOTE 1 of Assumption 6 in Appendix adds that: "*Preliminary H-ARAIM performance analyses and receiver algorithm design may assume a tentative value of 10^{-6} per hour for the continuity risk requirement*. […]"
6'	(Corollary to Assumption 6) An H-ARAIM airborne exclusion function is needed to meet a $5{\cdot}10^{-7}$ per hour R_{alert} requirement.
7	At the airborne receiver, it is sufficient when setting detection and exclusion thresholds to consider the following sources of LOC: • in V-ARAIM when no exclusion function is implemented: false and true alerts due to detection • in V-ARAIM when exclusion is used: false alerts with no exclusion, and true alerts with failed exclusion • in H-ARAIM when exclusion is used: false alerts with no exclusion, and true alerts with failed exclusion of GPS narrow faults, and of narrow and wide faults for the other constellations Detection and exclusion thresholds must be determined for the set of narrow and wide faults that are monitored in the airborne PL computation algorithm [9].
8	For ARAIM continuity performance simulation, prediction, and analysis, the following sources of LOC are considered: • false and true alerts in V-ARAIM when no exclusion function is implemented, • false alerts with no exclusion and true alerts with failed exclusion in H-ARAIM and V-ARAIM when exclusion is used, • occurrences of PPL > AL, where PPL is the predictive PL accounting for the impact of scheduled and unscheduled single-satellite outages

The complete description of Assumptions and Assertions in Tables 2 and 3, including rationales and notes, can be found in Appendix. Assumption 5 relies on Assumption 1 being validated. Corollary to Assumption 6 relies on Assumptions 1 and 6 being validated. Assumption 3 marks a potential evolution in ARAIM continuity performance interpretation by stating that H-ARAIM does not require systematic availability prediction. Under Assumption 3, the continuity risk, or risk of unpredicted mission interruptions, includes occurrences of PL>AL transitions, which are considered loss of availability (LOA) events in existing RAIM and in previous ARAIM documents [3, 4]. Updated H-ARAIM performance analyses have been carried out to quantify both FD (under RAIM assumptions) and FDE (under new continuity assumptions) [10, 11, 12]. The most recent analyses are reported below for example baseline configurations. They tend to support Assumption 3 for RNP 0.3 H-ARAIM at more than 95% of worldwide locations. But, no availability coverage requirement has yet been specified for H-ARAIM. Therefore, until the risk of LOA can be considered low enough, availability and continuity are analyzed separately considering that PL>AL transitions cause LOA.

Assumptions and assertions are needed to enable availability and coverage performance prediction. They must be validated, for example using steps detailed in Appendix. They are cited repeatedly in next section's H-ARAIM outage analysis.

4. ANALYZING SATELLITE OUTAGES IN ARAIM

Pre-GEAS and Pre-ARAIM Approach

Prior to the GNSS Evolutionary Architecture Study (GEAS), which was a precursor to ARAIM, SBAS and RAIM availability analyses accounted for satellite outages considering Table A.7-2 in [13], and using the method described in [14]. This method evaluates availability at each location and time by considering all outage cases, factoring in the probability of a specific combination of satellites operating out of all satellites visible at that time and location, within the total number of satellites in that constellation.

ARAIM Approach, Until Milestone 3 Report (till 2018)

This approach was deemed cumbersome in the early days of GEAS and ARAIM, which instead assumed "depleted constellations" of 23 GPS and 23 Galileo satellites for analysis [3, 4]. The depleted constellations captured the fact that if an outage occurred, then a single satellite in the constellation would be impacted. (In contrast, in [14], the worst-case outage at any time and location is always accounted for, which is conservative, and often overly-conservative.) Also, depleted constellations account for single satellite outages, because the impact of dual satellite outages on availability is negligible as illustrated in the rightmost column of Table 4. Table 4 is established assuming varying values of R_{out} , including the nominal value given in Assertion 4 and Assumption 2 of $2 \cdot 10^{-4}$ / h / SV, i.e., R_{out} of $2 \cdot 10^{-4}$ /h/ SV, which over one hour exposure periods corresponds to outage probabilities P_{out} of of $2 \cdot 10^{-4}$ /h/ SV for both GPS and Galileo. The justification for these numbers is given in Appendix under Assertion 4 and Assumption 2.

Table 4 Dual-Constellation State Probability Model for a One Hour Exposure Period, for Varying Values of the Prior Rate of Satellite Outages for GPS and Galileo

Number of baseline constellation slots	Binomial model: Rout_GPS = $2 \cdot 10^{-4}$ /h/SV Rout_GAL = 0.13 /h/SV	Binomial model: Rout_GPS = 0.043 /h/SV Rout_GAL = 0.043 /h/SV	Binomial model: Rout_GPS = $2 \cdot 10^{-4}$ /h/SV Rout_GAL = 0.043 /h/SV	Binomial model: Rout_GPS = $2 \cdot 10^{-4}$ /h/SV Rout_GAL = $2 \cdot 10^{-4}$ /h/SV
All 48 Slots	0.035187	0.12128	0.34658	0.99044
47 or More Slots	0.16154	0.38284	0.72198	0. 99996
46 or More Slots	0.37899	0.65902	0.9169	1
45 or more Slots	0.61764	0.84929	0.98146	1
44 or more Slots	0.80518	0.94548	0.99678	1
43 or more Slots	0.91749	0.98351	0.99955	1
42 or more Slots	0.97074	0. 99575	0.99995	1
41 or more Slots	0.99125	0.99906	1	1
40 or more Slots	0.99778	0.99982	1	1

Current ARAIM Approach

Using depleted constellations rather than the method in [14] has facilitated the development of other aspects of the airborne ARAIM multiple hypothesis solution separation (MHSS) algorithm, including the treatment of fault hypotheses, exclusion options, the development of optimal estimators, the inclusion of the number of effective samples, etc. [5, 8]. The reference algorithm has reached a level of maturity such that we can now revisit the accounting of satellite outages.

We have established that satellite outages had a minor impact on V-ARAIM, in part due to the shorter exposure periods during landing as compared to en-route operations [10, 12, 15-17]. Satellite outages do not impact integrity because when a satellite is flagged as not operational, the receiver unambiguously knows not to use its signals. This is in contrast with a fault that may impact a satellite without the receiver's knowledge. The integrity risk bound, or equivalently, the protection level may increase as a result of an outage, which impacts availability (while preserving integrity). Thus, for H-ARAIM, satellite outages impact both availability and continuity.

Because ARAIM does not yet have a firm availability coverage requirement (although the value of 90% coverage of 99.5% availability is used in [4]), ARAIM availability and continuity performance are analyzed separately, considering that PL>AL transitions cause LOA. Thus, Assumption 3 will only hold if (or, at locations where) the risk of LOA is smaller than the LOC requirement.

First, when both detection and exclusion are used, and when only accounting for LOC due to alerts at the receiver (following Assumption 7), we can express a bound on the probability of loss of continuity $P(LOC)$ as [7, 16-18]:

$$P(LOC) \leq P_{other} + \sum_{i=1}^{h} P(D \cap \overline{E} \mid H_i) \breve{P}_{Hi} + P(D \cap \overline{E} \mid H_0) \breve{P}_{H0} + \sum_{i=h+1}^{2^n-1} \breve{P}_{Hi} \tag{1}$$

where

P_{other} is a place-holder term that accounts for LOC due to sources of operational interruption that are not at the receiver (including environmental factors described in Appendix and accounted for in Figure 2)
D is the detection event
$\overline{E}$ is the no-exclusion event (causing either false or true alerts)
i is the fault hypothesis index
h is the number of monitored fault hypotheses
n is the number of satellites in view
H_i is the *i*th fault hypothesis (H_0 is the outage-free hypothesis)
$\breve{P}_{Hi}$ is the prior probability of occurrence of the *i*th fault hypothesis for continuity, derived from the satellite and constellation fault rates in Assertion 3 and Assumption 1, for $i = 0,..,2^n - 1$

The second right hand side term in equation (1) accounts for failed exclusions, and the third term for no exclusion as defined in [6]. These terms are used to derive detection and exclusion thresholds [5, 7]. For example, if $\breve{P}_{Hi}$ is large, then fault hypothesis H_i must be monitored. The larger $\breve{P}_{Hi}$ is, the larger the detection and exclusion thresholds and PL become. Occurrences of PL>AL transitions impact LOC. It is worth noting that the two rightmost terms are much smaller than the continuity risk requirement, and can be budgeted out of the continuity risk requirement when determining detection and exclusion thresholds in [5].

Second, the impact of outages on LOA can be accounted for using the method in [14], which accounts for all possible outage cases. In the next section of this paper, we present an alternative method based on a predictive bound on the probability of hazardously misleading information (HMI), or equivalently, a predictive protection level.

5. PREDICTIVE INTEGRITY RISK BOUND ACCOUNTING FOR SATELLITE OUTAGES

In this section, to facilitate availability evaluation, we derive a predictive integrity risk bound, or equivalently, a predictive protection level (PPL). This predictive bound is not derived at the receiver, but is derived to predict the performance that will be achieved at the receiver under different possible outage conditions. It is worth noting that the bound is computed for a given outage condition (e.g., for the "worst-case" outage event that maximizes the bound), and it is not averaged over outage events.

Unavailability, or probability of loss of availability (LOA), is the risk averaged over time that the computed bound on the probability of HMI $\hat{P}_{HMI}$ exceeds the integrity risk requirement I_{REQ} (or integrity risk requirement allocation when performing exclusion as described in [4]). Equivalently, the probability of LOA can be expressed as the risk that the protection level exceeds the alert limit.

It must be noted upfront that this derivation was initially made under Assumption 3, considering that outages impacted LOC instead of LOA. The derivation therefore assumes that R_{out} is $2 \cdot 10^{-4}$/ h / SV for both GPS and Galileo, which may be true for continuity, but may not be true for availability. The derivation will be revisited in future work.

For a given time and location, the risk of LOA is defined as:

$$P(LOA) \equiv P(\ \hat{P}_{HMI} > I_{REQ}\ ) \tag{2}$$

The objective of this derivation is not to evaluate $P(LOA)$, but it is to derive a predictive $\hat{P}_{HMI}$ that accounts for outages. Considering a set of mutually exclusive, exhaustive outage cases, and of detection versus no detection events, we can use the law of total probability to rewrite equation (2) as:

$$\begin{aligned} P(LOA) &= \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ}\ \cap O_{j\ out}) + P(\ \hat{P}_{HMI} > I_{REQ}\ | O_{\geq 2\ out})P(O_{\geq 2\ out}) \\ &= \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_{j\ out} \cap \bar{D}\ ) + \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_{j\ out} \cap D\ ) \end{aligned} \tag{3}$$

where

- $O_{j\ out}$ is the event of a an outage simultaneously impacting j satellites, for $j=0$, $j=1$, $j \geq 2$
- D is the detection event
- $\bar{D}$ is the no detection event

The following predictive $\hat{P}_{HMI}$ bound derivation makes assumptions that are true for GPS, and are reasonable for Galileo. We assume that the probability of two or more simultaneous outages $P(O_{\geq 2\ out}) \approx 0$, or more specifically, that $P(O_{\geq 2\ out}) << A_{REQ}$, where A_{REQ} is the availability requirement expected to be larger than 10^{-4}. If it is not true that $P(O_{\geq 2\ out}) << A_{REQ}$, then the derivation must be modified. But if it is, we can assume that $P(O_{\geq 2\ out}) = 0$ with no significant impact on $P(LOA)$ evaluation.

Then, $P(LOA)$ can be upper-bounded using conditional forms of the joint probabilities in equation (3). The conditions only impact the $\hat{P}_{HMI}$ variable since I_{REQ} is constant. Using the bound $P(\bar{D} \cap O_{j\ out}) \leq P(O_{j\ out})$, we obtain the following inequality:

$$\begin{aligned} P(LOA) \leq \sum_{j=0}^{1} \Big[P(\ \hat{P}_{HMI|\bar{D} \cap j\ out} > I_{REQ}\ )P(O_{j\ out}) \Big] &+ P(\ \hat{P}_{HMI|D \cap 0\ out} > I_{REQ}\ )P(D \cap O_{0\ out}) \\ &+ P(\ \hat{P}_{HMI|D \cap 1\ out} > I_{REQ}\ )P(D \cap O_{1\ out}) \end{aligned} \tag{4}$$

The three terms on the right hand side in (4) are addressed as follows. For the first term, we can write the following inequalities:

$$\begin{aligned}\sum_{j=0}^{1}\Big[P(\ \hat{P}_{HMI|\bar{D}\cap j\ out} > I_{REQ}\ )P(O_{j\ out})\Big] &\le \max_j\{P(\ \hat{P}_{HMI|\bar{D}\cap j\ out} > I_{REQ}\ )\}\sum_{j=0}^{1}P(O_{j\ out}) \\ &\le P(\ \max_j\{\hat{P}_{HMI|\bar{D}\cap j\ out}\} > I_{REQ}\ )\end{aligned} \tag{5}$$

where we used the bound $\sum_{j=0}^{1}P(O_{j\ out}) \le 1$. For the second term in (4), we use the following inequalities:

$$P(D\cap O_{0\ out}) \le P(O_{0\ out}) \le 1 \tag{6}$$

The third term in (4) is rewritten assuming that outage and detection events are independent and considering mutually exclusive, exhaustive hypotheses of fault-free H_0 and faulted events $H_{\ge 1}$. Using the bound $P(D\cap H_{\ge 1}) \le P(H_{\ge 1})$, equation (4) can be upper-bounded with the following inequalities:

$$\begin{aligned}P(\ \hat{P}_{HMI|D\cap 1\ out} > I_{REQ}\ )P(D\cap O_{1\ out}) &\le P(\ \hat{P}_{HMI|H_0\cap D\cap 1\ out} > I_{REQ,0}\ )P(D\cap H_0)P(O_{1\ out}) \\ &\quad + P(\ \hat{P}_{HMI|H_{\ge 1}\cap D\cap 1\ out} > I_{REQ,\ge 1}\ )P(H_{\ge 1})P(O_{1\ out}) \\ &\le P(D\cap H_0)P(O_{1\ out}) + P(H_{\ge 1})P(O_{1\ out})\end{aligned} \tag{7}$$

where $I_{REQ,\ge 1}$ and $I_{REQ,\ge 1}$ are integrity risk requirements allocated to the fault free and faulted hypotheses [4]. The upper bound in equation (7) is a sum of two very small values that can be budgeted out of the availability requirement. First, detection thresholds are set to limit the probability of false alerts, so that $P(D\cap H_0) \le C_{REQ,D}$, where $C_{REQ,D}$ is a continuity risk allocation for detection under H_0. This number is small compared to the availability requirement, and is multiplied by another small number, the prior probability of any single-SV outage $P(O_{1\ out})$. Second, the prior probability of satellite faults (as evaluated for continuity and availability using rates given in Assertion 3 and Assumption 1) is a small number that also gets multiplied by another small number $P(O_{1\ out})$. Therefore, we may write:

$$P(\ \hat{P}_{HMI|D\cap 1\ out} > I_{REQ}\ )P(D\cap O_{1\ out}) \approx 0 \tag{8}$$

Substituting equations (5), (6) and (8) into (4), we obtain the following inequality:

$$P(LOA) \le P(\ \max_j\{P_{HMI|\bar{D}\cap j\ out}\} > I_{REQ}\ ) + P(\ P_{HMI|D\cap 0\ out} > I_{REQ}\ ) \tag{9}$$

To distinguish terms where exclusion is implemented and where it is not (under no detection event $\bar{D}$), we use the notations:

$$\hat{P}(HMI_{FD}\mid O_{j\ out}) \equiv \hat{P}_{HMI|\bar{D}\cap j\ out} \quad \text{and} \quad \hat{P}(HMI_{FDE}\mid O_{0\ out}) \equiv \hat{P}_{HMI|D\cap 0\ out} \tag{10}$$

Furthermore, we obtain an expression of the predictive integrity risk by gathering all $\hat{P}_{HMI}$ terms, and rewrite (9) using (10) as:

$$P(LOA) \le 2\cdot P\Big(\ \max\Big\{\ \max_j \hat{P}(HMI_{FD}\mid O_{j\ out}),\ \hat{P}(HMI_{FDE}\mid O_{0\ out})\ \Big\} > I_{REQ}\ \Big) \tag{11}$$

Thus, using the same outage case notations as in equation (1) for specific single outage cases j, $j = 1,..,n$, the predictive probability of HMI, $P\hat{P}(HMI)$, can be written as:

$$P\hat{P}(HMI) \le \max\left\{ \max_j \hat{P}(HMI_{FD} \mid O_j),\ \hat{P}(HMI_{FDE} \mid O_0) \right\} \tag{12}$$

where O_j is the *j*th outage case (O_0 is the outage-free case). The equivalent PPL equation is:

$$PPL = \max\left\{ \max_j PL_{FD,j},\ PL_{FDE,0} \right\} \tag{13}$$

where $PL_{FD,j}$ is the protection level (PL) for detection only [5] under outage of satellite *j*, and $PL_{FDE,0}$ is the outage -free PL for detection and exclusion [5].

Summary for this Section

Availability can be evaluated at each location and time either using the constellation state model described in [14], or by comparing $P\hat{P}(HMI)$ in equation (12) to I_{REQ}. The latter approach may facilitate availability evaluation if [14] was not implemented to start with, but is more conservative because the multiple inequalities that were used in equations (3) to (8). Steps to bound the risk of LOA were detailed in equations (2) to (11). An equivalent approach to $P\hat{P}(HMI)$ is to compute the PPL in (13) and to compare it to the alert limit to determine availability at a given location and time.

6. PRELIMINARY AVAILABILITY ANALYSIS ACCOUNTING FOR SATELLITE OUTAGES

Worldwide FD and FDE availability is evaluated in Figure 1 for H-ARAIM RNP 0.3 following the process described in Appendix A of [4] and in [5]. A list of key parameters and parameter values is given in Table 2. This availability performance assessment included:

- the continuity assertions and assumptions described in Section 2 of this paper
- an implementation of the method in [8] to account for the number of effectively uncorrelated samples for both continuity and integrity risk evaluation
- a treatment of satellite outages using the predictive integrity risk bound ($P\hat{P}(HMI)$) derived in Section 4 of this paper

At the October 2019 Working Group C meeting of the ARAIM Technical Subgroup, two groups independently evaluated availability for multiple configurations under the assumptions listed in Table 4 and in [10, 11]. The availability evaluation presented in Table 5 was performed for two example joint constellations of 24 GPS and 24 Galileo satellites, and of 27 GPS and 27 Galileo satellites. (The 27 GPS 27 Galileo is an illustrative example of an optimistic joint constellation, it is not intended to capture different outage conditions.) Group 1 obtained slightly higher availability as compared to Group 2, but assumed a lower Galileo URA than Group 2. Both groups are working on refining these analyses. Preliminary availability evaluation are not yet sufficient to determine whether, for RNP 0.3 using H-ARAIM, we can make Assumption 3 according to which pre-flight availability screening is not needed.

Table 4 Dual-Frequency GPS/Galileo H-ARAIM Availability Analysis Parameters

Parameter Description	**Parameter Value**
Overall performance criterion	coverage of 100% availability
RNP 0.3 Horizontal Alert Limit (HAL)	HAL = 556 m
Temporal resolution	24 hours with 600 second steps
Spatial resolution	10 by 10 degree user grid
Integrity risk requirement	$I_{REQ} = 10^{-7}$
Continuity risk requirement	$C_{REQ} = 10^{-6}$
GPS User Range Accuracy (URA)	URA = 1m, 2.4m, 5m
Galileo User Range Accuracy (URA)	URA = 1m, 2.4m, 5m
Prior probability of satellite fault for integrity	$P_{sat,GPS} = P_{sat,GAL} = 10^{-5}$
Prior probability of constellation fault	$P_{const,GPS} = 10^{-8}$, $P_{const,GAL} = 10^{-4}$
Nominal bias values (for integrity)	$b_{nom} = 0.75$ m

Table 5 Sensitivity of Coverage of 100% Availability
(assuming an H-ARAIM continuity risk requirement of 10^{-6}/h, and R_{out} of $2{\cdot}10^{-4}$/ h / SV for both GPS and Galileo)

Constellations	24 GPS, 24 Galileo	27 GPS, 27 Galileo
Group 1 [10]: GPS URA = 2.4 m, Galileo URA = 2.4 m	FD: 100 %	FD: 100 %
	FDE: 99.3%	FDE: 100%
Group 2 [11]: GPS URA = 2.4 m, Galileo URA = 4 m	FD: 100 %	FD: 100 %
	FDE: 95.06%	FDE: 99.2%

7. CONCLUSION

This paper first described continuity and availability assertions and assumptions that are foundational to ARAIM algorithm development and analysis. It then focused on the treatment of satellite outages. Methods for evaluating the impact of outages on continuity and availability were presented. In particular, we derived a new method using predictive integrity risk bound that can facilitate availability evaluation.

Preliminary availability evaluations were presented and are being refined to determine whether GPS/Galileo constellations are strong enough to avoid having to predict availability before aircraft departure. In the next steps of this work, we will assess the looseness of the predictive integrity risk bound as compared to using the constellation state probability model.

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APPENDIX: RATIONALE FOR ARAIM CONTINUITY & AVAILABILITY ASSERTIONS & ASSUMPTIONS

Definition 1. Detection occurs at the airborne receiver when a detection test statistic exceeds a threshold.

- A false alert is a detection event under fault-free conditions.
- A true alert is the detection of an actual fault.

NOTE 1 — The most commonly used detection test statistics are the solution separations, the chi-squared norm of the measurement residual vector, and the norm of the parity vector.

NOTE 2 — The detection threshold is set in accordance with a continuity risk requirement allocation to limit the risk of false alerts.

Definition 2. Exclusion can be implemented after detection. Exclusion is the process of identifying and removing one or more potentially faulty measurements. Exclusion occurs if the subset of remaining measurements is deemed fault-free, i.e., if the subset's test statistic is lower than a threshold. Conversely, if the exclusion test statistic exceeds the threshold, and if no candidate subset is found to be fault-free, then:

- a false alert (with no exclusion) occurs under fault-free conditions, and
- a true alert (with failed exclusion) occurs when a fault is actually present.

NOTE 1 — Similar to detection, the exclusion threshold is set in accordance with a continuity risk requirement allocation to limit the risk of failed exclusion.

NOTE 2 — Exclusion provides a means to improve continuity by allowing to pursue a mission when a fault is (truly or falsely) detected.

NOTE 3 — Relevant text and figure can be found in RTCA DO-229E, including Figure 1.3: Diagram of Fault Detection and Exclusion (FDE) Conditions [6]. Reference [6] distinguishes failed exclusion from 'wrong exclusion', which occurs when a satellite subset is removed, but the subset does not include all faulted satellites. 'Wrong exclusion' does not impact continuity.

Definition 3. The maximum allowable rate of mission interruptions due to alerts at the airborne receiver is noted R_{alert}. R_{alert} is a fraction of the overall continuity risk rate requirement.

NOTE 1 — R_{alert} is a pre-defined input parameter to the airborne receiver algorithm used to set detection and exclusion thresholds. The allowable probability of mission interruptions, P_{alert}, is expressed as: $P_{alert} = R_{alert}\, T_E$, where T_E is the exposure period. Consistent with [1] and with our interpretation of the requirements in [8], we opt to fix T_E for continuity analysis to 15 s for V-ARAIM, and to 1 hour for H-ARAIM (the rationale is explained in [8]).

NOTE 2 — Assertions are made below to determine R_{alert} values for H-ARAIM and V-ARAIM based on service performance and requirements of current operational systems (SBAS, GBAS, RAIM).

Definition 4. The protection level (PL) is a probabilistic bound on the positioning error computed at the airborne receiver at each time epoch. The PL accounts for nominal measurement errors, for potentially undetected faults, and for potential wrong exclusions if exclusion is implemented.

NOTE 1 — At the receiver, occurrences of PL unexpectedly transitioning from below to above the alert limit are perceived as loss of continuity events regardless of their root causes (depleted geometry due to satellite outage, loss of signal due to ionosphere scintillation, unusually high nominal error, etc.).

NOTE 2 — For ARAIM analysis purposes, a distinction is made between the 'instantaneous' PL (referred to as PL) computed at the receiver and the 'predictive' PL (or PPL) used in continuity and coverage performance analysis and prediction.

Assertion 1. In both H-ARAIM and V-ARAIM, loss of continuity (LOC) is considered a 'minor' severity event.

Rationale: From en-route through Category I precision approach, the loss of navigation function is typically considered to be a major failure condition for the aircraft [19]. However, LOC for a given flight operation does not necessarily constitute a loss of navigation, because the receiver may continue to output a valid position estimate with protection levels that support a different flight operation (e.g. a missed approach procedure) or an alternate source of navigation may be available (e.g. terrestrial navigation aids or INS). ARAIM has a strong similarity with RAIM and SBAS, in terms of purpose and design. These currently operating systems were designed by considering LOC as a 'minor' severity event; therefore, ARAIM should do the same.

NOTE — The following assertions do not take credit for the fact that many aircraft navigation systems in operation mitigate LOC using INS.

Corollary to Assertion 1. For accuracy and continuity evaluation, ARAIM measurement error models are based on user range error (URE), whereas for integrity analysis, they rely on constellation service provider (CSP) commitments and on an Integrity Support Message (ISM), validated by the ISM Generator.

Rationale: In ARAIM, loss of continuity and accuracy are considered 'minor' severity events (following Assertion 1). Positioning errors caused by minor severity events are currently evaluated in WAAS using historical performance rather than CSP commitments. ARAIM follows the WAAS precedent when sufficient historical data is available (i.e., for GPS), and otherwise considers CSP commitments.

Assertion 2. H-ARAIM and V-ARAIM continuity is measured 'in an average sense', over time.

Rationale: Relevant text from ICAO Annex 10, Note 4 of Table 3.7.2.4-1 [1]:
"Continuity requirements for APV and Category I operations apply to the average risk (over time) of loss of service, normalized to a 15-second exposure time."

NOTE — The 'average rate' of LOC is obtained by dividing the number of LOC occurrences over a predefined period (e.g., over one year) by the period itself, and then normalizing to the appropriate exposure time. The 'average rate' of LOC shall not be averaged over multiple locations.

Assertion 3. The average rate of GPS narrow satellite faults is no greater than 10^{-8} / 15 s / SV. The average rate of GPS wide constellation faults is no greater than $3 \cdot 10^{-8}$ / 15 s.

Rationale

1. In ARAIM, LOC is considered a 'minor' severity event (following Assertion 1). Risks caused by minor severity events are currently evaluated in WAAS based on historical performance rather than CSP commitments. ARAIM follows the WAAS precedent when sufficient historical data is available (i.e., for GPS), and otherwise considers CSP commitments (see Assumption 1 below).
2. For GPS narrow faults, the average rate of 10^{-8} / 15 s / SV is based on:
    - five observed GPS narrow faults over eight years (2010-2017) in a 31 GPS satellite constellation [20]
    - once a fault is detected, it remains detected until out of view or set unhealthy
    - a given fault only impacts a single 15-second exposure time-period
3. For GPS wide faults, the average rate of $3 \cdot 10^{-8}$ / 15 s is based on:
    - no observed GPS wide fault over the past eight years [20]; this corresponds to an observed rate of $7 \cdot 10^{-6}$ / hour, or equivalently $3 \cdot 10^{-8}$ / 15 s.

Assertion 4. For continuity evaluation, the average rate of scheduled and unscheduled GPS satellite outages, R_{out}, is no greater than $2 \cdot 10^{-4}$ / h / SV.

Rationale: In ARAIM, the aircraft is not expected to consult NANUs (Notice Advisory to Navstar Users) prior to departure. We define the outage rate R_{out} , for scheduled and unscheduled outages, as the number of 'outage onset' events over time, i.e., the rate of transitions of satellite status from operational to not-operational. R_{out} primarily impacts continuity. In contrast, the percentage of downtime over operation time primarily impacts availability. Performance Analysis Reports [21] make an account of 964 outages over 19 years, which, assuming 30 satellites operational at any time, corresponds to R_{out} of $1.93 \cdot 10^{-4}$ / h /SV. It is safe to consider an upper bound of $2 \cdot 10^{-4}$ / h / SV, which is consistent with other corroborating evidence.

NOTE 1 — Other references provide R_{out}-values that are lower or equal to $2 \cdot 10^{-4}$ / h / SV. First, the total number of outages observed from 2012 to 2016 is 160 outages over 1,245,645 satellite hours [22], which corresponds to a rate of $1.28 \cdot 10^{-4}$ / h / SV. Second, the observed rate of unscheduled outages is $6.5 \cdot 10^{-5}$ / h / SV [23]. Third, Table 3.6-1 of [13] can be interpreted as a commitment that the unscheduled outage rate will not exceed $2 \cdot 10^{-4}$ / h / SV.

NOTE 2 — The outage probability used in the constellation availability model of Table A.7-2 of [13] is 0.043. This number is specified for availability analysis and is consistent with downtime over operation time. It is not an average rate of observed outages, and is not directly relevant to ARAIM continuity analysis.

NOTE 3 — If one assumes that aircraft consult NANUs before take-off (e.g., potentially, in future ARAIM analyses), then, as pointed out in NOTE 1, the rate of unscheduled outages is Rout-≤ 6.5·10-5 / h / SV [18].

Assumption 1. For GNSS constellations other than GPS, the average rate of narrow faults is expected to range from $6 \cdot 10^{-8}$ / 15 s / SV to $3.6 \cdot 10^{-7}$ / 15 s / SV. The rate of non-GPS constellation wide faults is expected to be smaller than $4 \cdot 10^{-7}$ / 15 s.

Rationale

1. For narrow faults, the rate interval of [$6 \cdot 10^{-8}$ to $3.6 \cdot 10^{-7}$] / 15 s / SV corresponds to [10^{-5} to $6 \cdot 10^{-5}$] / hour / SV. This interval includes nominal values used for GNSS narrow fault rates in ARAIM analyses (10^{-5} / hour / SV is assumed in [2-5]).
2. For wide faults, an average rate of wide fault of 10^{-4} / hour is recommended for integrity in [20]: this number is an upper-bound on the rate of LOC caused by detection and no/failed exclusion of wide faults in constellations other than GPS.

3. This statement is presented as an assumption because there is currently not enough evidence to elevate it to an assertion. This assumption will have to be validated or revisited based on the observed narrow and wide fault rates of the new constellations.

Assumption 2. For continuity evaluation, and for GNSS constellations other than GPS, the average rate of effective scheduled and unscheduled GPS satellite outages, R_{out}, is expected to range from $1 \cdot 10^{-4}$ / h / SV to $2 \cdot 10^{-3}$ / h / SV.

Rationale: In the absence of historical data on outages for GNSS constellations other than GPS, we consider a range of R_{out}-values including that of GPS. The range allows for both lower and higher rates as compared to GPS.

NOTE — Preliminary ARAIM continuity performance evaluations assume a nominal rate of $2 \cdot 10^{-4}$ / h / SV for all constellations, and analyze sensitivity to R_{out} for non-GPS constellations.

Assumption 3. Under nominal ARAIM performance assumptions, the aircraft receiver may not be required to perform H-ARAIM availability prediction.

Rationale: The definition of continuity in RTCA DO-229E [6] stipulates that:
"[…] continuity is the probability that the specified system performance will be maintained for the duration of a phase of operation, presuming that the system was available at the beginning of that phase of operation and was predicted to operate throughout the operation."
Aircraft operations may be scheduled based on pre-flight availability prediction by the air navigation service provider (ANSP) given the current states of the core constellations. For example, using the WAAS precedent, high ARAIM availability will be established and proved through analysis. No systematic predictions of ARAIM outages will be provided for operational purposes.
Nominal ARAIM assumptions are described in [5] and in Appendix A of [4]. At the 10/09/2019 ARAIM Working Group-C meeting in El Segundo, CA, two groups independently evaluated RNP 0.3 performance under the continuity and availability assumptions presented in this paper [10, 11]. They showed that for nominal constellations of 24 GPS and 24 Galileo satellites, for URA values of 2.4 m, coverage of 100% availability exceeded 95%. I.e., more than 95% of a worldwide grid's locations achieved 100% availability of integrity and continuity. Under these assumptions, pre-flight ARAIM availability predictions are not needed at 95% of locations, and PL>AL transitions would then cause LOC.

NOTE 1 — For events which might result in lower availability than predicted in analysis, NOTAMs may be issued, warning users that performance may be degraded in areas of interest. Thus, at the receiver, all occurrences of (PL>AL) transitions would cause LOC.

NOTE 2 — Availability prediction may show that the multi-constellation satellite geometry is strong enough to support high availability (appropriate value to be defined) at all locations, so that availability does not need to be predicted for each individual flight.

NOTE 3 — If the decision is taken to consider all occurrences of (PL>AL) transitions as causes of LOC, then the definition of R_{out} in Assertion 4 and Assumption 2 may need to be refined and availability analyses .

Assumption 4. In both V-ARAIM and H-ARAIM, R_{alert} is chosen to be a fraction of the specified continuity risk rate requirement (e.g., following the example allocation in Figure 1). The remaining fraction is assumed to bound the average risk rate of LOC due to all sources other than receiver alerts.

NOTE 1 — Sources of LOC in the navigation system include:
- false and true alerts (whether an exclusion function is implemented or not)
- all other causes for LOC, including occurrences of PL unexpectedly transitioning to above the alert limit (AL)

Root causes for unexpected transition events of (PL > AL) include:
- unpredicted nominal measurement errors (high *URA* and b_{nom} values)
- scheduled and unscheduled satellite outages (in ARAIM operations, the receiver is not required to check availability at dispatch)
- loss of signal and Hatch filter re-initialization due to detected cycle slips occurring during ionosphere scintillation, aircraft banking, radio-frequency interference (RFI), etc.

The inclusion of environmental factors (ionosphere, RFI) in the continuity allocation tree is to acknowledge their presence, it is not intended to set a strict requirement on receiver design.

NOTE 2 — V-ARAIM further allocates R_{alert} (or P_{alert}) between horizontal and vertical positioning performance during approach as described in Annex A of [4], whereas H-ARAIM focuses on horizontal positioning performance for en-route navigation.

NOTE 3 — In Figure 1, the average rate of LOC due to all sources other than receiver alerts is assumed to be lower than $5 \cdot 10^{-7}$ / hour = $2 \cdot 10^{-8}$ / 15s.

NOTE 4 — This assumption will have to be validated or revisited using continuity risk allocation trees, continuity and availability coverage simulations, and operational service performance analysis.

Assumption 5. For dual-constellation implementations under Assumption 1, the V-ARAIM R_{alert} requirement of $7.98 \cdot 10^{-6}$ per 15 seconds can be achieved without implementing an exclusion function at the aircraft.

Rationale

1. The continuity risk requirement for V-ARAIM is $8 \cdot 10^{-6}$ per 15 seconds [1, 4, 24]. A R_{alert} of $7.98 \cdot 10^{-6}$ per 15 seconds is selected in the example allocation of Figure 1 and Assumption 2.
2. As described in Assertion 4 and Assumption 1, the average LOC rates due to true alerts without subsequent exclusion are assumed to be:
   - 10^{-8} / 15s / SV for GPS narrow faults,
   - $6 \cdot 10^{-8}$ / 15s / SV for non-GPS constellations narrow faults,
   - $3 \cdot 10^{-8}$ / 15 s for GPS wide faults, and
   - smaller than $4 \cdot 10^{-7}$ / 15s for non-GPS constellation wide faults.

   For an example dual-constellation ARAIM implementation with 14 visible satellites from GPS and 14 from the other constellation, the average rate of LOC experienced by the airborne receiver is less than $1.5 \cdot 10^{-6}$ / 15s.

NOTE — The second point of the above 'Rationale' describes a bound on the probability of true alerts in V-ARAIM, which can be subtracted from R_{alert}. The remaining allocation ($6.48 \cdot 10^{-6}$ / 15s) can then be used to set detection thresholds that limit the risk of false alerts.

Assumption 6. H-ARAIM airborne algorithms should consider a continuity risk rate requirement ranging from 10^{-4} to 10^{-8} per hour.

Rationale: Continuity requirements for en-route, terminal, initial approach, intermediate approach, non-precision approach, departure operations are $1\text{-}10^{-4}$/hour to $1\text{-}10^{-8}$/hour.

Relevant text from [6], Note 4 of Table 3.7.2.4-1 and Appendix D:

> "Ranges of values are given for the continuity requirement for en-route, terminal, initial approach, NPA and departure operations, as this requirement is dependent upon several factors including the intended operation, traffic density, complexity of airspace and availability of alternative navigation aids. The lower value given is the minimum requirement for areas with low traffic density and airspace complexity."
>
> "The navigation system continuity requirement for a single aircraft is $1\text{-}10^{-4}$ per hour."
>
> "The highest value given (i.e., $1\text{-}10^{-8}$ per hour) is suitable for areas with high traffic density and airspace complexity, where a failure will affect a large number of aircraft. This value is appropriate for navigation systems where there is a high degree of reliance on the system for navigation and possibly for dependent surveillance."
>
> "Intermediate values of continuity (e.g., $1\text{-}10^{-6}$ per hour) are considered to be appropriate for areas of high traffic density and complexity where there is a high degree of reliance on the navigation system but in which mitigation for navigation system failures is possible. Such mitigation may be through the use of alternative navigation means or the use of ATC surveillance and intervention to maintain separation standards."

NOTE 1 — Preliminary H-ARAIM performance analyses and receiver algorithm design may assume a tentative value of 10^{-6} per hour for the continuity risk requirement. The following references support the selection this intermediate value for H-ARAIM:

- Relevant text in RTCA DO-229E, §R.2.2.2 Note 2: "The required continuity risk depends on operational considerations and is expected to be in the range 10^{-7}/hour to 10^{-5}/hour.".
- Relevant WAAS System specifications in FAA-E-2892D, §3.1.3.3.1-2: The continuity risk requirement is 10^{-6} per hour for LNAV.

Under our current assumptions, the tentative 10^{-6} per hour requirement used in [3, 4] is conservative as compared to the LOC 'minor' severity requirement, which ranges from 10^{-3} to 10^{-5} per hour [25]. Reference [10] showed that availability sensitivity to the continuity risk requirement was low.

NOTE 2 — For analysis purposes, following NOTE 1 and Assumption 2, the resulting R_{alert} requirement for H-ARAIM is $5 \cdot 10^{-7}$ per hour.

NOTE 3 — In an average sense, the continuity risk requirement for en-route navigation (for example, 10^{-6} per hour) can be more stringent than for LPV200 approach (the requirement of $8 \cdot 10^{-6}$ per 15 seconds can be interpreted as a requirement as large as $2 \cdot 10^{-3}$ per hour).

NOTE 4 — The airborne receiver is unaware of the phase of flight it is in. To address different V-ARAIM and H-ARAIM requirements, the receiver may constantly provide two HPL's (one for H-ARAIM, one for V-ARAIM), and let the flight management system use the relevant one. (V-ARAIM requires a VPL in addition to HPL.)

Corollary to Assumption 6. An H-ARAIM airborne exclusion function is needed to meet a $5 \cdot 10^{-7}$ per hour R_{alert} requirement.

Rationale: Detection of narrow faults on GPS satellites can occur with a rate of $2.3 \cdot 10^{-6}$/SV per hour according to observed data [20]. In the best case for continuity, if five satellites are visible and if exclusion is not implemented, the mission interruption rate due to detection alerts exceeds 10^{-5} per hour, which exceeds the R_{alert} requirement of $5 \cdot 10^{-7}$ per hour (even without accounting for other sources of loss of continuity such as false alerts).

NOTE — The rationale holds true for the entire range of continuity risk requirement in Assumption 6. Assuming that 11 or more satellites are visible and assuming prior probabilities $P_{sat} = 10^{-5}$, $P_{const} = 0$, which are fairly favorable values for continuity, then the risk of a true alert alone, i.e., the risk of fault occurrence, exceeds the entire continuity risk budget, even for a continuity risk rate requirement of 10^{-4}/h, and without even accounting for false alerts and other source of LOC.

Assumption 7. At the airborne receiver, it is sufficient when setting detection and exclusion thresholds to consider the following sources of LOC:

- in V-ARAIM when no exclusion function is implemented: false and true alerts due to detection
- in V-ARAIM when exclusion is used: false alerts with no exclusion, and true alerts with failed exclusion
- in H-ARAIM when exclusion is used: false alerts with no exclusion, and true alerts with failed exclusion of GPS narrow faults, and of narrow and wide faults for the other constellations

Detection and exclusion thresholds must be determined for the set of narrow and wide faults that are monitored in the airborne PL computation algorithm [9].

Rationale: The following points are considered:

1. The R_{alert} requirements are established following Assumptions 3 and 4. Other sources of LOC are accounted for using the remainder of the continuity risk requirement.
2. In V-ARAIM, the probability of true alerts is bounded by the average probability of faults, which can be subtracted from R_{alert}. The remaining probability defines a requirement on the risk of false alerts. Detection thresholds are then set to meet this requirement.
3. H-ARAIM integrity analysis and PL computation assume a probability of GPS constellation wide fault $P_{const} \leq 10^{-8}$ (see Annex B in [4]). Similar, H-ARAIM continuity analysis and threshold determination use a value of $P_{const} \leq 10^{-8}$ for GPS.
4. At the receiver, the R_{alert} requirement can be applied as an instantaneous requirement if measurement time-correlation is properly accounted for [8, 26]. Average continuity can be assessed at the ground, either by simulation for system design purposes, or by service history analysis for operational systems.
5. In practice, similar threshold-setting approaches are implemented in existing GPS-based RAIM methods without outstanding issues regarding continuity risk.

Assumption 8. For ARAIM continuity performance simulation, prediction, and analysis, the following sources of LOC are considered:

- false and true alerts in V-ARAIM when no exclusion function is implemented,
- false alerts with no exclusion and true alerts with failed exclusion in H-ARAIM and V-ARAIM when exclusion is used,
- occurrences of PPL > AL, where PPL is the predictive PL accounting for the impact of scheduled and unscheduled single-satellite outages

Rationale: The above sources of LOC are the ones for which historical data is available for GPS.

NOTE 1 — According to Assumption 7, the continuity risk caused by events in the first two bullets is accounted for in detection and exclusion threshold setting. The impact on PPL of satellite outages and of missed alerts (i.e., 'missed detection when no exclusion is implemented', or 'missed detection and wrong exclusion when exclusion is used') can be accounted for in the PPL equation [4, 5, 15]. As an alternative, the impact of satellite outages can be accounted for using the method described in [14].

NOTE 2 — As mentioned in Sections 1 and 2 of this paper, ARAIM availability and coverage analyses will include performance predictions under both RAIM and ARAIM (FD and FDE) continuity assumptions. RAIM only considers the impact of false alerts in detection threshold setting.

NOTE 3 — At present time, Assumption 8 is a placeholder for a future, more detailed description of the continuity prediction process. ARAIM prototyping and testing will help refine this process.

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